Week 1

Question

sally

scie

Sequences

A

A sequence is any group or set of items, not necessari numbers, listed in a particular order,

e.g. {5, 9, 13, 17} or {a, e, i, o, u} Finite

e.g. {1, 3, 5, 7, 9 ...} Infinite

Terms are the items in of a sequence, usually denoted as T_1, T_2, T_3 et

Writing Rules

Recurrence Relations (Recursive Rules)

Explicit Rules (General Rules)

The terms of a sequence are denoted by using subscript. ie. T_2 is the second term in a sequence.

T_n is called the n^{th} term, T_{n+1} is the next term to T_n.

Performing Operations (use BIMDAS)

In a sequence 22, 20, 18, 16, 14...

a. Find $T_1 - T_4$

$= 22 - 16$

$= 6$

b. Find $3T_4$

$= 3(16)$

$= 48$

Recurrence Relations

A recurrence relation is one in which each successive term depends on the previous term.

3, 8, 13, 18 . . . . $T_{n+1} = T_n + 5$, where $T_1 = 3$

rule   term number

Order

T_{n-1}, T_n, T_{n+1}, T_{n+2} . . .

To define a sequence recursively you need:

- the relationship between the terms

  and

- the term value

12, 8, 4, 0, −4, −8 . . . $T_{n+1} = T_n - 4$, where $T_1 = 12$

Using a recurrence relation

a. Find the first 4 terms given the sequence:

$T_n = 5T_{n-1}$, $T_1 = 2$

$= 2$

$= 5(2) = 10$

$= 5(10) = 50$

$= 5(50) = 250$

Use the recursive rule to find T_5

$_{n+1} = 3T_n - 4$, $T_3 = -1$

$= 3(-1) - 4 = -3 - 4 = -7$

$= 3(-7) - 4 = -21 - 4 = -25$

Writing the recurrence relation $T_{n+1} = T_n + 2$ in the form $T_{n+1} - T_n = 2$ gives us the difference between any term and the preceding term and hence is called the difference rule.

Given the recurrence relation $T_{n+1} = T_n + 5$, $T_1 = 9$

$$T_{n+1} - T_n = 5$$

Fibonacci Sequence

1, 1, 2, 3, 5, 8, 13, 21 ... $T_{n+2} = T_{n+1} + T_n$

+

Lucas Sequence

1, 4, 5, 9, 14, 23 ...

Squares

1, 4, 9, 16, 25, 36, 49, 64 ... $T_n = n^2$

First Order Linear Sequence

1, 3, 7, 15, 31, 63 ... $T_{n+1} = 2T_n + 1$

Cannot divide

$\div 2 = \times 0.5$

Writing Recursive Rules

Determine the first six terms of a sequence for which:

The first term of the sequence is 11
Each term is 5 more than the previous term

$T_{n+1} = T_n + 5$ or $T_n = T_{n-1} + 5$

same thing

T_n = current term
T_{n+1} = next term
T_{n-1} = previous term

$T_1 = 11$

$T_{1+1} = T_1 + 5$
$T_2 = T_1 + 5$
$= 11 + 5$
$= 16$

$T_{2+1} = T_2 + 5$
$T_3 = T_2 + 5$
$T_3 = 16 + 5$
$T_3 = 21$

Determine the first six terms of a sequence for which:

The first term of the sequence is 4
Each term is the previous term doubled then take 5.

$T_{n+1} = 2T_n - 5$ or $T_n = 2T_{n-1} - 5$

$T_1 = 4$

$T_2 = 2 \times 4 - 5$
$T_2 = 3$

$T_3 = 2 \times 3 - 5$
$= 1$

$T_4 = 2 \times 1 - 5$
$= -3$

$T_5 = 2 \times -3 - 5$
$= -11$

$T_6 = 2 \times -11 - 5$
$= -27$

Classpad note:
Always T_{n+1} format. Have to rearrange if set out like this.

Recursive Rules

Given that $T_{n+1} = 2T_n + 2$ and $T_3 = 54$
Find the first 4 terms.

$T_{4} = 2T_3 + 2$
$(3+1)$
$= 2 \times 54 + 2$
$= 110$

Working Backwards

$T_3 = 2T_2 + 2$
$(2+1)$
$54 = 2T_2 + 2$
$52 = 2T_2$
$26 = T_2$

Think of T_n as x

$T_2 = 2T_1 + 2$
$26 = 2T_1 + 2$ $T_1 = 12$
$24 = 2T_1$

A sequence is defined by the recurrence relation:
$T_{n+1} = aT_n + b, \; T_1 = 5$

If $T_2 = 3$ & $T_3 = -3$, determine a & b.

$n = 1$

$T_2 = aT_1 + b$
$3 = 5a + b$ ①

$n = 2$

$T_3 = aT_2 + b$
$-3 = 3a + b$ ②

$3 = 5a + b$
$-(-3 = 3a + b)$
$6 = 2a$
$3 = a$

$3 = 5 \times 3 + b$
$3 = 15 + b$
$-12 = b$

Substitution & Elimination

$Ap = T_n + 2$ = Linear
$Gp = 2T_n$ = Exponetial

Shapes of Graphs

$T_{n+1} = T_n - 2$ Linear Graph (AP) 

$T_{n+1} = 2T_n$ Exponetional ↑ (GP) 

$T_{n+1} = 0.4T_n$ Exp ↓ (GP) 

$T_{n+1} = 2T_n + 2$ F.O.L

$T_{n+1} = 0.4T_n - 2$ F.O.L Recurrence

$T_{n+1} = (-1)^n \times 2T_n$ Diverging Graph

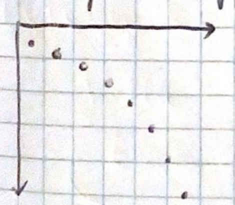
$T_{n+1} = (-1)^n \times 0.5T_n$ Converging Graph

Week 2

Graphs

Describe the shape of each graph & think of a recursive rule that would fit the progression.

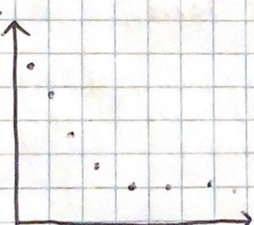
a.



- First Order Linear Recurrence

$T_{n+1} = 2T_n - 4, T_1 = -2$

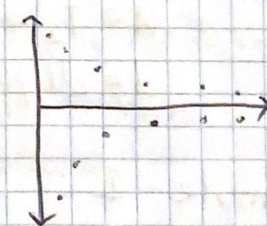
b.



- Decreasing GP
- Exp. decay

$T_{n+1} = 0.75T_n, T_1 = 50$

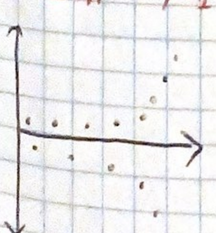
c.



- Converging

$T_{n+1} = -0.5T_n, T_1 = 20$

d.



- Diverging

$T_{n+1} = -2T_n, T_1 = 5$

e.

- Linear ascending/increasing
- AP

$T_{n+1} = T_n + 4, T_1 = 2$

Arithmetic Progressions

This occurs when the terms in the sequence share a common difference.

e.g. 2, 6, 10, 14...

$$\left.\begin{array}{l} 6-2=4 \\ 10-6=4 \\ 14-10=4 \end{array}\right\} \text{common difference}$$

We are given the recursive rule:

common difference = d

$T_{n+1} = T_n + 4,\ T_1 = 10$

$T_1 = 10$ $\quad T_2 = 14$ $\quad T_3 = 18$ $\quad T_4 = 22$

$= a$ $\quad = a+d$ $\quad = a+2d$ $\quad = a+3d$

represents T_1

$$\boxed{T_n = a + (n-1)d}$$

General Rule for AP

$a = T_1$ or First Term

$d =$ difference

Example 1

Given the sequence $-4, -2, 0, 2 \ldots$

find

a. T_1

$T_1 = a = -4$

b. common difference

$d = +2$

c. general rule

$T_n = -4 + (n-1) \times 2$

$= 2(n-1) - 4$

d. 15^{th} term

$T_{15} = 2(15-1) - 4$

$= 28 - 4$

$= 24$

Example 2

$T_{21} = -12, \ d = 2.5$

$T_{21} = a + (21-1) \times 2.5$

$-12 = a + 20 \times 2.5$

$-12 - 50 = a$

$-62 = a$

Substitution

Example 3

$T_{25} = 34, \ a = 94$

$T_{25} = 94 + (25-1)d$

$34 = 94 + 24d$

$-60 = 24d$

$\frac{-60}{24} = d$

$-2.5 = d$

Example 4

Given the AP: 17, 21, 25...

Find which term is equal to 305.

$T_n = 17 + 4(n-1)$

$305 = 17 + 4(n-1)$

$288 = 4(n-1)$

$72 = n-1$

$73 = n$

Example 5

Given the AP: 8, 15, 22..

Find which term ~~is equal to~~ first exceeds 100.000.

$100\,000 = 8 + (n-1) \times 7$

$99\,992 = 7(n-1)$

$14284.6 = n-1$

$\lceil 14285.6 \rceil = n$

means round up

$\approx 14286^{th}$ term

$\therefore T_{14286} > 100\,000$

Find the AP

$T_6 = 25$

$T_{20} = 81$

$T_{20} = T_6 + 14d$

$81 = 25 + 14d$

$56 = 14d$

$4 = d$

$25 = a + 5d$

$25 = a + 20$

$5 = a$

OR

1. $T_6 = a + (6-1)d$

$25 = a + 5d$

2. $T_{20} = a + (20-1)d$

$81 = a + 19d$

$$\begin{array}{r} 81 = a + 19d \\ -\ 25 = a + 5d \\ \hline 56 = 14d \end{array}$$

$\frac{54}{14} = 4 = d$

$25 = a + 20$

$a = 5$

Geometric Progressions

- Graph takes on exponential form
- Recursive rule

$T_{n+1} = r \times T_n$

Given the sequence 2, 4, 8, 16
Determine the explicit & recursive rules.

We are looking for the ratio; 'r'

$a = T_1 = 2$

$\frac{T_2}{T_1} = \frac{4}{2} = 2$

$r = 2$

$\frac{T_3}{T_2} = \frac{8}{4} = 2$

$\boxed{T_{n+1} = r \times T_n, \ T_1 = a}$

Recursive rule

$\boxed{T_n = a \times r^{(n-1)}}$

General rule or Explicit

$T_1 = 2 = a$

$T_2 = 4 = a \times r$

$T_3 = 8 = a \times r^2$

$T_4 = 16 = a \times r^3$

$AP = T_n \ +/- \ d$

$GP = r \times T_n$

Example 1

We are given the GP: $\frac{1}{3}, \frac{2}{9}, \frac{4}{27}, \frac{8}{81} \ldots$

a. Determine T_1

$$a = \frac{1}{3}$$

b. Determine the common ratio

$$r = \frac{2}{3} \qquad \frac{T_2}{T_1} = \frac{2}{9} \div \frac{1}{3} = \frac{2}{\cancel{9}_3} \times \frac{\cancel{3}^1}{1} = \frac{2}{3}$$

c. State the n^{th} term] WRITE GENERAL RULE

Will state 'write recursive rule' otherwise

$$T_n = \frac{1}{3} \times \left(\frac{2}{3}\right)^{(n-1)}$$

d. Determine T_{11}

$$T_{11} = \frac{1}{3} \times \left(\frac{2}{3}\right)^{10} = \frac{1024}{177147}$$

Example 2 - Is it a gp?

a. $T_n = \frac{3^{2n}}{3^{n+1}} \leftarrow (3^n)^2$

$$T_n = \frac{\cancel{3^n} \times 3^n}{\cancel{3^n} \times 3^1} = \frac{3^n}{3}$$

$$= 3^n \times 3^{-1}$$

$$= 3^{n-1}$$

$$\therefore T_n = 1 \times 3^{n-1}$$

b. $T_n = \frac{T_{n+1}}{4}$

$\therefore T_{n+1} = 4T_n \,.\, T_1 = a$

Yes

Example 3

$T_1 = 9$ It's a GP

$T_3 = 100$

Find T_2

$T_1 = 9 = a$

$\therefore T_n = 9 \times r^{n-1}$

$T_3 = 9 \times r^2$

$100 = 9 \times r^2$

$\frac{100}{9} = r^2$

$\sqrt{\frac{100}{9}} = r$

$\frac{10}{3} = r$

$T_2 = 9 \times \left(\frac{10}{3}\right)^1$

$= \pm 30$

Know r^3

$2^3 = 8$

$3^3 = 27$

$4^3 = 64$

Example 4

$T_1 = 5$ $r = 3$ It's a GP

Find

a. T_5

$T_5 = 5 \times 3^4$

$= 405$

b. General rule

$T_n = 5 \times 3^{n-1}$

c. n for which T_n 98 415

Use classpad

$T_n = 98415$

solve$(98415 = 5 \times 3^{n-1})$

$n = 10$

d. Which term first exceeds 1 000 000

solve$(1000\,000 = 5 \times 3^{n-1})$

$n = 13$

Week 3

Steady State Solutions

Steady State

$T_{n+1} = T_n$ as $n \to \infty$

multiplier lies between
-1 & 1 for this to be
achieved

Example 1

Given the recurrence relations:

A: $T_{n+1} = \boxed{0.3}T_n + 10$, $T_1 = 15$

B: $A_{n+1} = \boxed{-1.2}T_n - 7$, $A_1 = 5$

a. State, with reason, which recurrence relation will achieve a steady state.

$T_{n+1} = 0.3T_n + 10$ because 0.3 is between -1 & 1

b. Determine the value of this steady state

$T_{n+1} = T_n = x$

$1x = 0.3x + 10$

$0.7x = 10$

$x = \frac{100}{7}$

Week 4

Misc AP $ GP Questions

Example 1

A small business records profits of $8000, $11000, $14000 $

initial amount

$\boxed{\$17000}$ over the last 4 years. What is the expected profit in 26 years from now?

$T_0 = 17000$

$T_n = 17000 + 3000(n)$

$T_{26} = 17000 + 3000 \times 26$

$= \$95000$

If we are told the inital amount

$AP = a \times d(n)$

$GP = a \times r^{(n)}$

If we know T_1

$AP = a \times d(n-1)$

$GP = a \times r^{(n-1)}$

Example 2

Population of WA is around 2.4 million in 2011. Average annual population growth is 1.9%.

a. Recursive rule

$T_{n+1} = 1.019\,T_n, \quad T_0 = 2.4$ million

b. Pop in 2020

$2020 - 2011 = 9$

$T_9 = 1.019 \times T_8$

$= 2843013$

≈ 2.84 million

c. Exceed 4 million

$4000000 = 1.019\,T_n$

$T_{28} = 4065260$ $\quad T_{27} = 3989460$

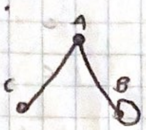
In 28 years therefore 2039 it will exceed 4 million

Week 5

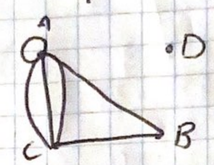
Networks & Terminology

Terminology

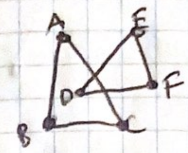
Connected graph
All vertices are connected to each other by one edge at least.



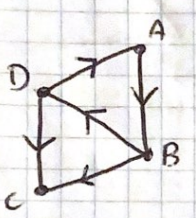
Disconnected Graph
At least one vertice is not connected by an edge.



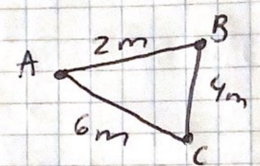
or



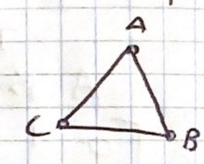
Directed Graph
The edges have directions.



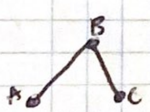
Weighted Graph
An edge has a value.



Simple Graph
Can't have direction or weights or loops or multiple edges.



Subgraph
A part of an original network.



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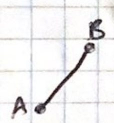
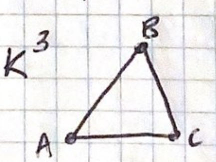


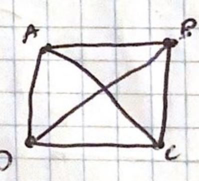
subgraph

Complete Graph
All vertices are connected to every other vertice.

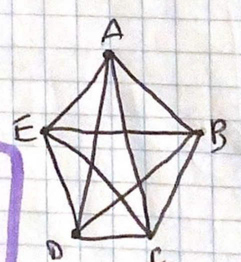
K^3



K^4

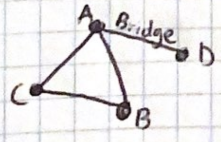


K^5



$$\boxed{\text{Number of edges} = \frac{n(n-1)}{2}}$$

Bridge
The edge connecting 2 vertices that if removed, becomes disconnected

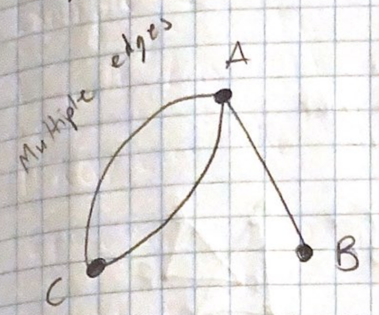


Using Vertices $ Edges

1.

a. Vertices: A, B, C

Edges: AB, AC, AC

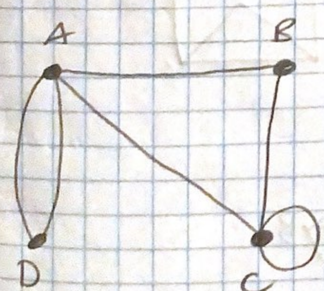


Using an Adjacency Matrice

6.

c.

$$\begin{array}{cc} & \text{To} \\ & \begin{array}{cccc} A & B & C & D \end{array} \\ \text{From} \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 0 & 1 & 1 & 2 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 2 & 0 & 0 & 0 \end{bmatrix} \end{array}$$



Given Information

2.

c. A: C C C

B: C E

C: A A A B D E

D: C E

E: B C D