CHAPTER THIRTEEN REVIEW EXERCISE
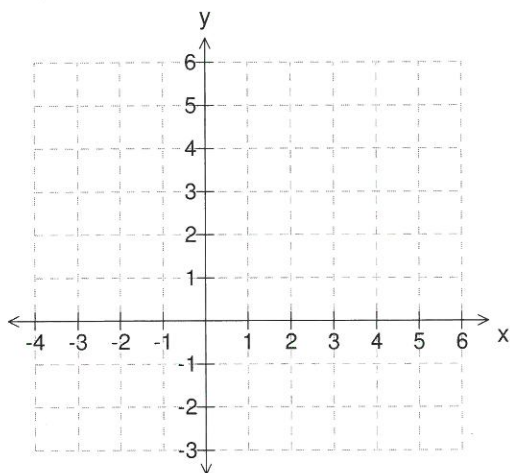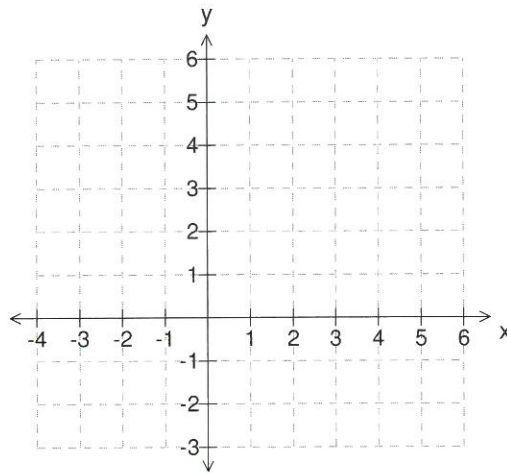
1.   Graph the following piece-wise defined equations on the given axes:

(a) $y = \begin{cases} 2x+3, & -2 \le x < 1 \\ 2-x, & 1 \le x \le 4 \end{cases}$
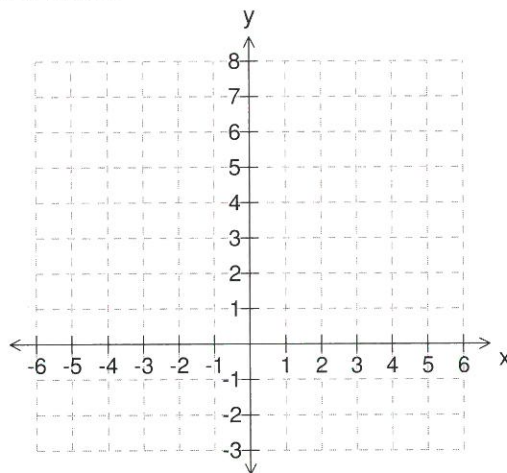
(b) $y = \begin{cases} 5, & -3 < x \le -2 \\ 2, & -2 < x \le 1 \\ -2, & x > 1 \end{cases}$

2.   (a) On the given axes draw the following piece-wise linear graph.

$y = \begin{cases} 1-x, & -5 < x < -2 \\ 3, & -2 \le x < 1 \\ 6-3x, & x > 1 \end{cases}$

(b) What is the value of y when:
(i) x = -3?   (ii) x = 1?

(iii) x = 0?   (iv) x = 10?

(c) What is the value of x when:
(i) y = 0?   (ii) y = 6?

(iii) y = 3?   (iv) y + 9 = 0?

3.   Graph on your calculator the following piece-wise linear equation.

$y = \begin{cases} -x-2, & -8 \le x < -3 \\ 1, & -3 \le x < 0 \\ 2x+1, & 0 \le x < 5 \end{cases}$

Use your calculator to determine the following:
(a) What are the coordinates of the intercepts?

(b) What is the value of y when x = 4?

(c) What is the value of y when x = 5?

(d) What is the value of x when y = 8?

(e) What is the value of x when y = 3?

(f) What is the value of x when y = 1

4.   Graph on your calculator the following piece-wise linear equation.

$y = \begin{cases} 3x+10, & -3 \le x < -1 \\ x+8, & -1 \le x < 3 \\ -2x+17, & 3 \le x \le 5 \end{cases}$

Use your calculator to find the following:
(a) What are the coordinates of the intercepts.

(b) What is the value of y when x = -2?

(c) What is the value of y when x = 3.4?

(d) What is the value of x when y = 9?

(e) What is the maximum value of y?

(f) What is the minimum value of y?

5. Consider the piece-wise linear equations:

$$y = \begin{cases} x+2, & 0 \le x \le 5 \\ 10, & 5 < x \le 10 \end{cases} \qquad\qquad y = \begin{cases} 2x, & 0 \le x < 6 \\ 18-x, & 6 \le x \le 10 \end{cases}$$

   (a) One of the given piece-wise linear equations has been graphed below. Graph the other on the axes below.
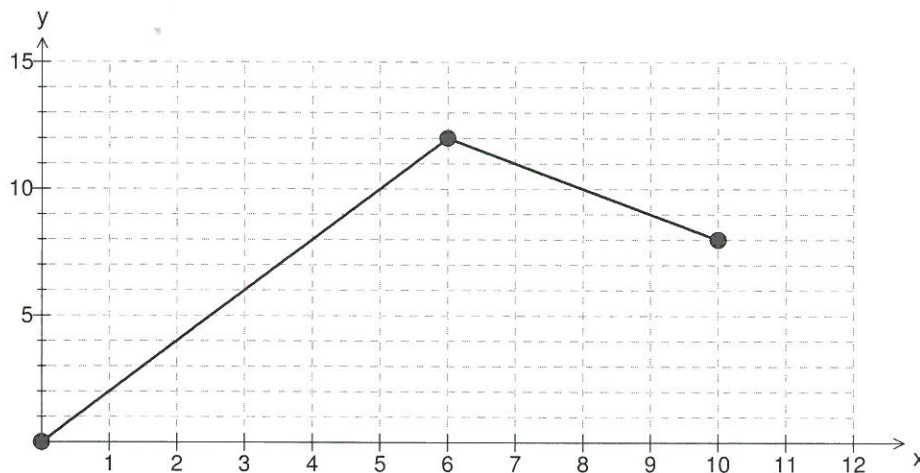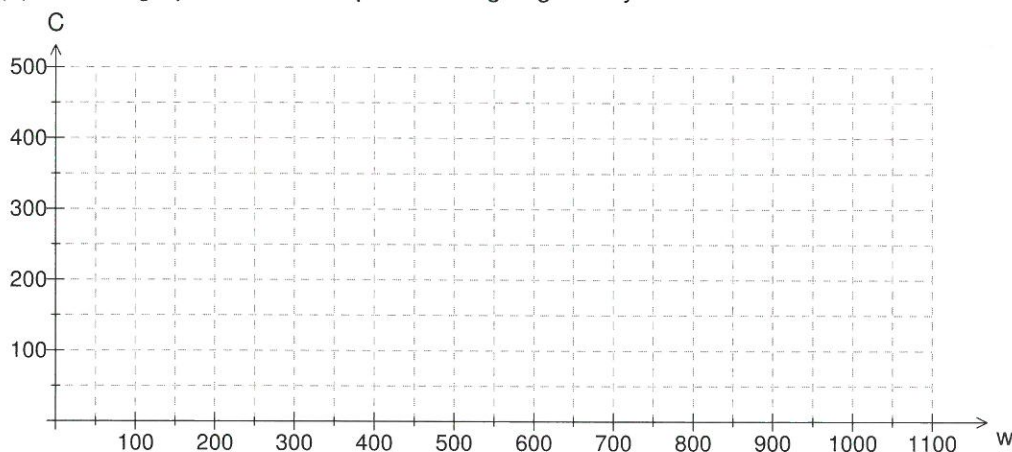

   (b) Solve the following equation: $\begin{cases} x+2, & 0 \le x \le 5 \\ 8, & 5 < x \le 10 \end{cases} = \begin{cases} 2x, & 0 \le x < 6 \\ 18-x, & 6 \le x \le 10 \end{cases}$

6. The table shows the postal charges for mailing small items up 2 kg.

Weight	Postal charge per item
Up to 50 gm	$1.00
Over 50 gm up to 100 gm	$1.50
Over 100 gm up to 250 gm	$2.50
Over 250 gm up to 500 gm	$3.00
Over 500 gm up to 1 kg	$4.50

   (a) Draw a graph to show the postal charges given by the table.

   (b) What is the cost of mailing an item weighing 155 gm?

   (c) Ian posted a package, gave the postal officer a $10 note and received $7 change. What was the weight of his package?

   (d) Five identical items each weighing 240 gm are to be posted to the same address. They could be mailed separately or by making up parcels containing more than one item. What is the cheapest way of posting these items and by how much

7.   The graph of a piecewise function is given below.
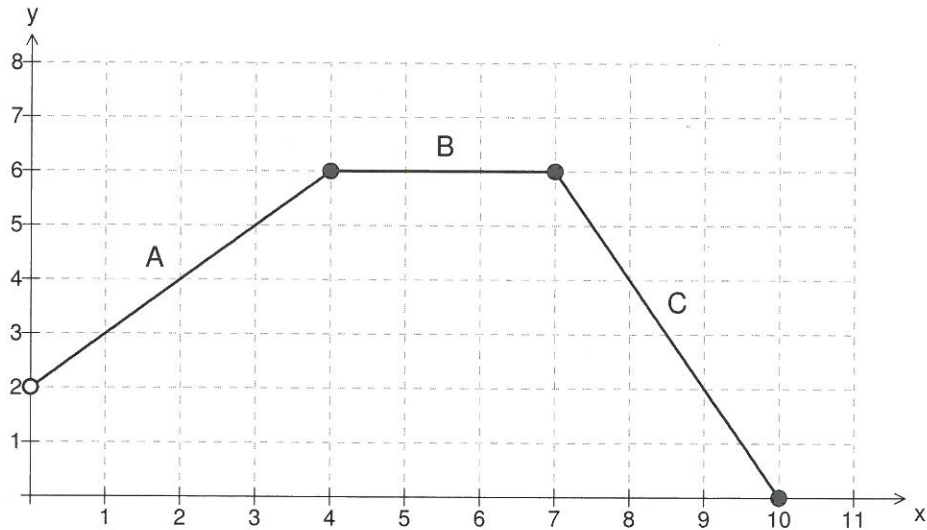
(a) Find the value of y when:
    (i) x = 6   (ii) x = 7   (iii) x = 0   (iv) x = 3.5

(b) Find the value of x when:
    (i) y = 5   (ii) y = 1   (iii) y = 2   (iv) y = 6

(c) Find the gradient of line segment C and hence its equation.

(d) Find the equation of line segment A.

(e) Complete the equation for the given piece-wise function by finding the values of a, b, c, d, e, f, and g.

$$y = \begin{cases} ax + b, & 0 < x < 4 \\ c, & 4 \le x \le d \\ ex + f, & d < x \le 10 \end{cases}$$

8.   A library charges \$0.20 per photocopy for print jobs up to 10 photocopies, \$0.15 per photocopy for print jobs up to 20 but more than 10 photocopies, and \$0.12 per photocopy for print jobs of more than 20 photocopies. Which equation best represents the cost in dollars C for n number of photocopies?

(a) $C = \begin{cases} 0.20, & 0 < n \le 10 \\ 0.15, & 11 \le n < 20 \\ 0.12, & n \ge 21 \end{cases}$
        (b) $C = \begin{cases} 0.20, & 0 < n < 10 \\ 0.15, & 11 \le n \le 20 \\ 0.12, & n \ge 21 \end{cases}$

(c) $C = \begin{cases} 0.20, & 0 < n \le 10 \\ 0.15, & 11 \le n \le 20 \\ 0.12, & n \ge 21 \end{cases}$
        (d) $C = \begin{cases} 0.20, & 0 < n < 10 \\ 0.15, & 11 < n \le 20 \\ 0.12, & n \ge 21 \end{cases}$

9.   The cost of sending parcels varies according to the weight of the parcel. The following table gives
      the postage rates for small parcels up to a maximum of 500 grams.

Weight of parcel (w)	Cost of postage (C)
Up to 50 grams	70 cents
Over 50 grams and up to 100 grams	$1.20 cents
p	q
Over 250 grams and up to 500 grams	$2.90

The step graph gives the same information as the table above.

The equation of the step graph can be written as follows: $C = \begin{cases} 0.7, & 0 < w \le 50 \\ 1.2, & 50 < w \le 100 \\ 1.9, & 100 < w \le 250 \\ r, & s < w \le 500 \end{cases}$

(a) Complete the step graph table by finding the expressions for p and q.

(b) Complete the step graph equation by finding the values of r and s.

(c) Find the cost of sending a parcel which weighs:
     (i) 40 grams   (ii) 0.24 kg

Five items, two weighting 100 grams each, two weighing 200 grams each and one weighing
300 grams need to be sent to a particular address.
(d) How much will it cost to send these items as individual packages?

(e) How much can be saved by combining these items and then posting them to the particular
     address.

10.   Sophie leaves home at 9:00 am to go shopping. She takes 15 minutes to walk to the bus stop which is 1 kilometre from her house. The bus arrives at 9:30 am, and travels at an average speed of 24 km/h arriving at the shopping centre at 9:50 am.
At 10:45 am Sophie catches a taxi and arrives home at 10:55 am.

(a) Construct a graph of the given information showing Sophie's journey to the shopping centre and return home.

(b) How long did the shopping trip take?

(c) How long did Sophie spend at the shopping centre?

(d) How far is the shopping centre from Sophie's house?

(e) Calculate the rate at which Sophie walked in:
   (i) metres per minute   (ii) kilometres per hour

(f) What was the average speed of the taxi ?

(g) What percentage of the total shopping trip time did Sophie spend:
   (i) travelling?   (ii) shopping

(h) Explain why the total of (i) and (ii) of part (g) did not add up to 100%.

11. At 5 :00 am Phil and Jack leave the boat ramp on a fishing trip with a full tank of fuel. The graph below shows the reading of the fuel gauge in litres and the time taken in hours.
The amount of fuel used by the engine depends on the speed of the boat, the faster the boat travels the more fuel is used. The men have extra fuel in jerry cans.
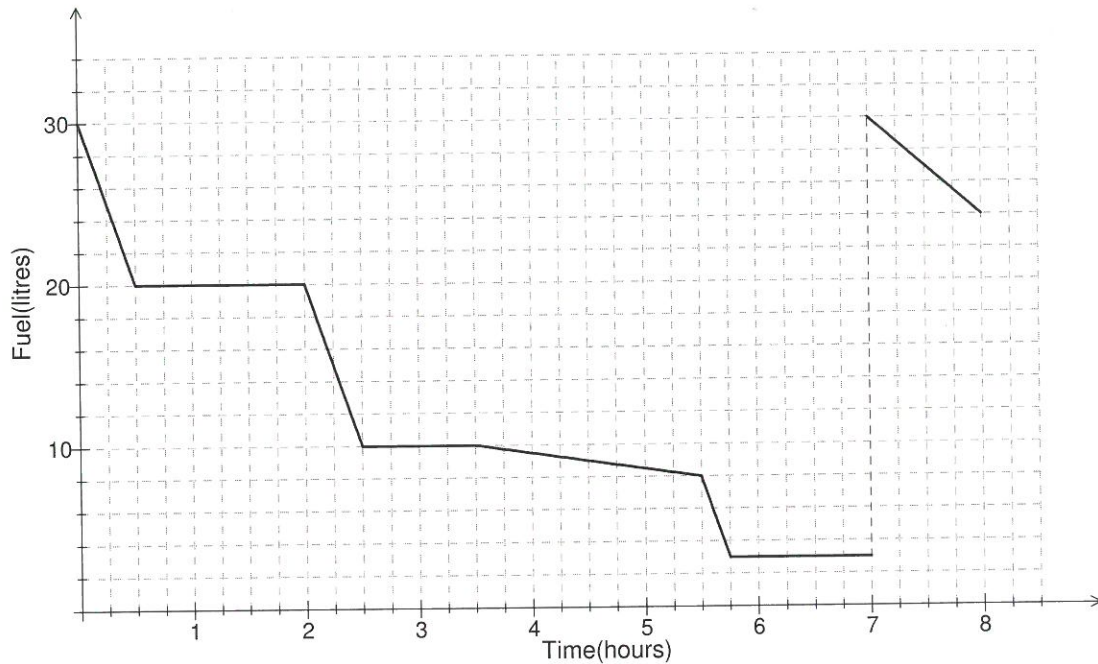

(a) How much fuel does the boat's fuel tank hold?

(b) How long did they travel before they came to their first fishing spot?

(c) How much fuel did they use in arriving at the first fishing spot?

(d) When did Jack and Phil set out for their second fishing sport?

(e) Did they use more or less fuel in travelling to the second fishing spot? Justify.

(f) At what time did Phil and Jack start trolling? (When trolling the boat moves slowly to simulate the average speed of a bait fish)

(g) How much fuel did they use while trolling for fish?

(h) When did the men arrive at the last fishing spot?

(i) What did the Jack and Phil do at 12 noon?

(j) How much fuel did they use on this fishing trip?

(k) Was the distance from the boat ramp to the first fishing spot shorter or longer than the distance back to the boat ramp after the last fishing spot? Justify your response.

(l) What was the total running time of the boat's outboard motor?

(m) At what rate, in litres per hour, was fuel being used by the motor on the trip back to the boat ramp?

(n) Calculate the hourly fuel consumption whilst trolling.

12.   Ticketmaster is selling tickets to a musical production. To encourage people to attend, the price
      ticket structure is such that they get cheaper if more are purchased. If 1 to 5 tickets are purchased
      they cost $30 each. The price schedule is outlined in the graph below, where n is the number of
   tickets purchased, and C, in dollars, is the total cost of the tickets.
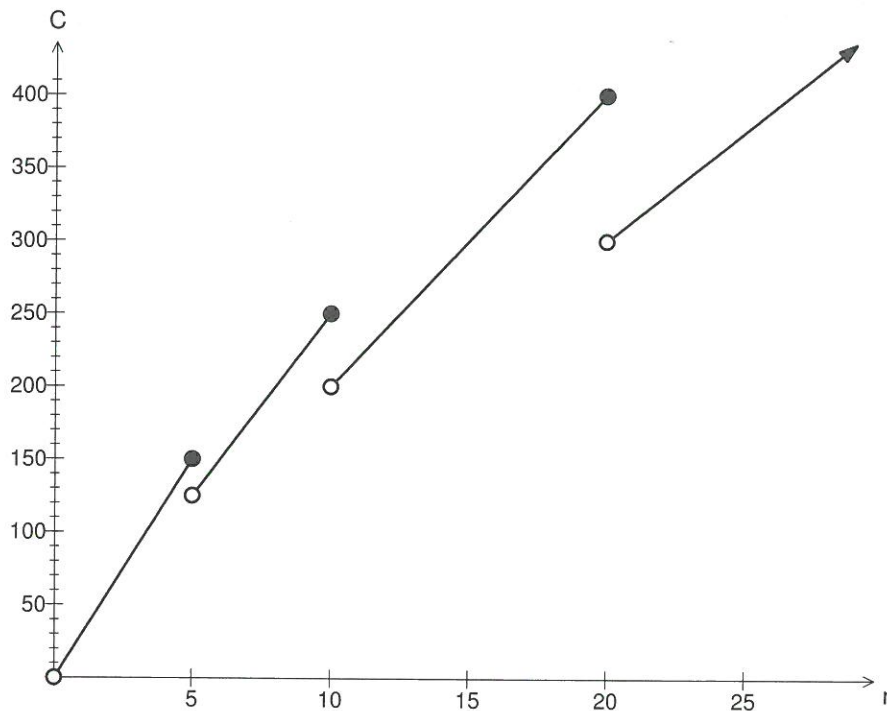
(a) What is the cost of 20 tickets?

(b) When buying 10 tickets, what is the price of
     each ticket?

(c) How many tickets can be purchased
     for $300?

(d) How many tickets can be purchased for $360?

(e) Calculate the cost per ticket if 30 tickets are purchased in one transaction.

(f) Mr Graham wants to purchase 20 tickets for his students. What would be the most economical
     way to purchase the tickets? Justify .

13.. The equation below shows the parking charges $C, for parking x hours in a hospital parking lot.

$$C = \begin{cases} 3x, & x \leq 3 \\ 10, & x > 3 \end{cases}$$

      Which of the following statements best represents the cost in dollars for parking x hours?

(a) Parking costs $3 for the initial 3 hours and a maximum of $10 per day.

(b) Parking costs $3 per hour for the initial 3 hours and $10 after this.

(c) Parking costs $3 per hour for the initial 3 hours and $10 for 3 hours after this.

(d) The parking lot charges $3 per hour for the initial 3 hours and a maximum of $10 per day.

4.(a)
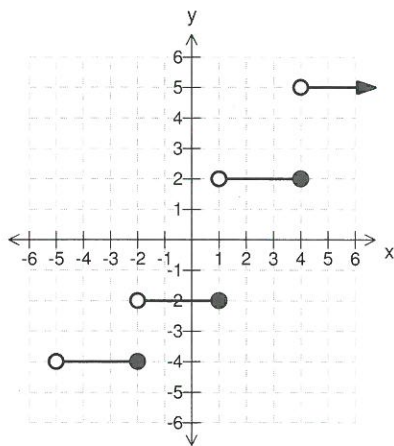
(b)
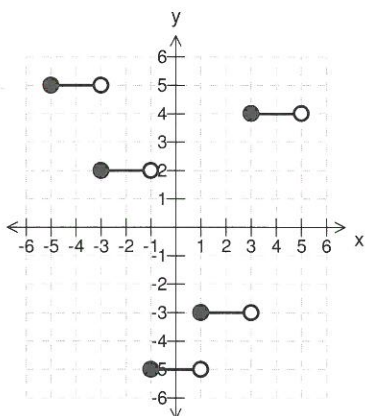
5. (a) (i) $10 (ii) $8 (iii) $15 (iv) $13
(b) Car can be parked for more than 1.5 hours but no more than 2 hours.
(c) Can park for up to 1.5 hours but no longer.
(d) $12
(e) Maximum is 12 hours for $15.
6. (a) Both companies charge the same rate on the same intervals between the endpoints.
(b) The endpoint charges are different for both companies. Company A charges the fee at the start of each interval whereas company B charges the fee at the end of each endpoint.
(c) Use company B as it will be cheaper to send parcels with weight coinciding with the endpoints of the intervals. To send a 3 kg parcel it will cost $10 with company A and only $6 for company B.
7. (a) (i) 7 mL (ii) 12 mL (iii) 15 mL
(b) 3 teaspoons
(c) 48 mL (d) 7mL x 4 = 28 Hence 5 teaspoons
8. (a)

$$C = \begin{cases} 3, & 0 < t \le 0.5 \\ 6, & 0.5 < t \le 1 \\ 9, & 1 < t \le 1.5 \\ 12, & 1.5 < t \le 2 \\ 14, & 2 < t \le 12 \end{cases}$$

(b)
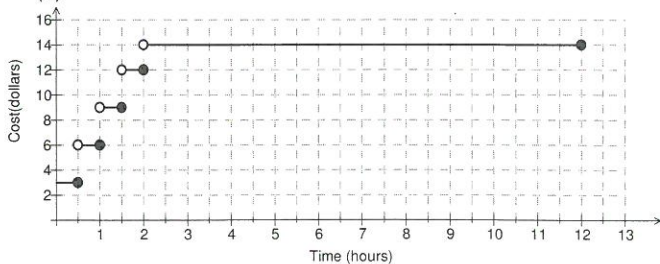
(c) (i) $12 (ii) $14 (iii) $6 (iv) $12

9. (a)
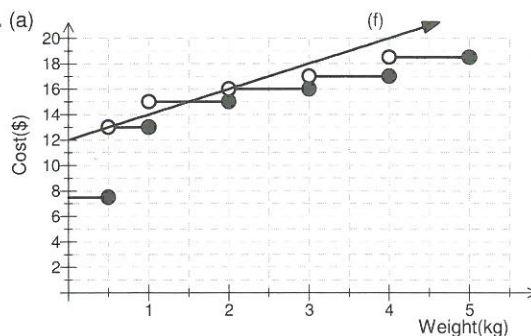
(b) $75 (c) Pack the 3.65 kg parcel with one of the 0.75 kg parcels to make one parcel of 4.4 kg. Pack the remaining 3 parcels as one parcel of 4.95 kg. Total cost $37
(d) Make up 2 parcels each comprising of a 0.75 kg and 2.1 kg and send the 3.65 kg parcel on its own. Total cost $70.05.
(e) Australia Post charge $15, Black Swan charge $16, hence Australia Post is cheaper
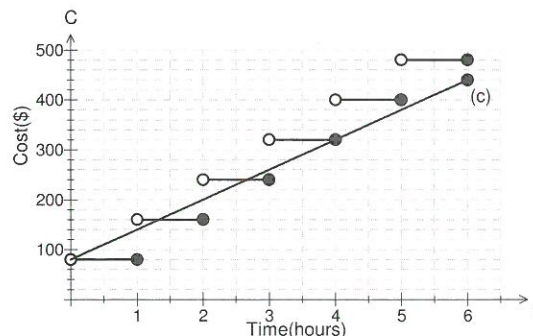(f) See graph
(g) Australia Post is the cheapest for all parcel weights up to 5 kg excluding parcels with weight between 1 kg and 1.5 kg where Black Swan Couriers provides the cheaper service.
10. (a)

Hours of labour	x	Cost($)
0 to 1 hour	$0 < x \le 1$	80
More than 1 hour to 2 hours	$1 < x \le 2$	160
More than 2 hours to 3 hours	$2 < x \le 3$	240
More than 3 hours to 4 hours	$3 < x \le 4$	320
More than 4 hours to 5 hours	$4 < x \le 5$	400
More than 5 hours to 6 hours	$5 < x \le 6$	480

(b)

(c) see graph
(d) Use Easy-Fix. $80 (e) Use Easy-Fix. $160 (f) For any work expected to take more than 3 hours.
(g) For jobs taking 1 hour 20 mins, 2 hours 40 minutes and 4 hours.
(h) The point of intersection of the two graphs in the interval $2 < t < 3$ give the required solution. We will to find the simultaneous solution to the equations C = 240 and C = 60t + 80.
By substitution 240 = 60t + 80
60t = 160
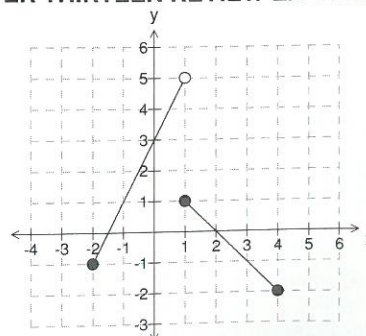t = 2.$\overline{6}$
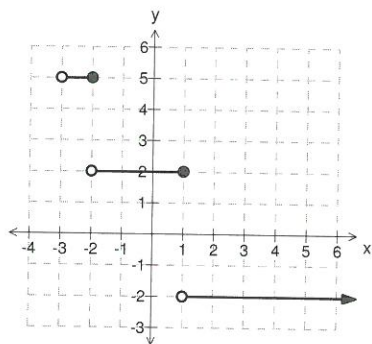Hence when time taken by a job is 2 hours 40 minutes(2.$\overline{6}$ hours) the cost by both companies is $240.

CHAPTER THIRTEEN REVIEW EXERCISE
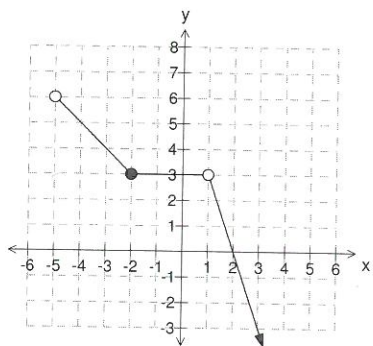1. (a)

(b)

2. (a)

(b) (i) 4 (ii) undefined (iii) 3 (iv) -24
(c) (i) 2 (ii) undefined (iii) $-2 \leq x < 1$ (iv) 5
3. (a) y intercept is (0, 1), there is no x-intercept
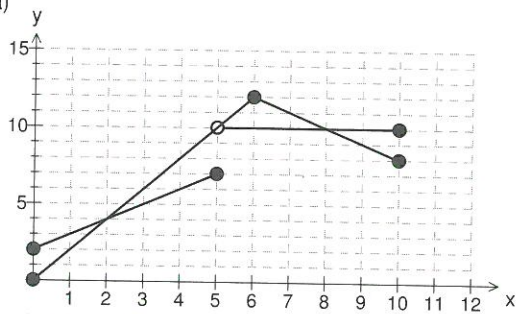(b) y = 9   (c) y is undefined
(d) x = 3.5   (e) x = -5 and x = 1
(f) $-3 \leq x < 0$

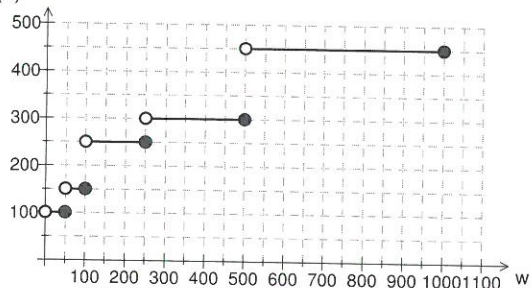**4.** (a) x intercept $(-3.\overline{3}, 0)$ and y intercept is (0, 8) (b) y = 4
(c) y = 10.2 (d) x = 1 and x = 4 (e) 11 (f) -5

5. (a)

(b) x = 2 and x = 8.
6. (a) C

(b) $2.50 (c) More than 250 gm up to 500 gm. (d) Posting
individually cost is $12.50. Making two parcels one with 4
items and the other with 1 item results in the cheapest way of
posting these items. Total cost is$7, hence cheaper by $5.50.
7. (a) (i) y = 6 (ii) y = 6 (iii) y is undefined, i.e. no value of y
exists (iv) y = 5.5
(b) (i) x = 3 and x = 7.5 (ii) x = 9.5 (iii) x = 9 (iv) $4 \leq x \leq 7$
(c) Gradient is -2, equation y = -2x + 20 for $7 < x \leq 10$
(d) y = x + 2 for $0 < x < 4$
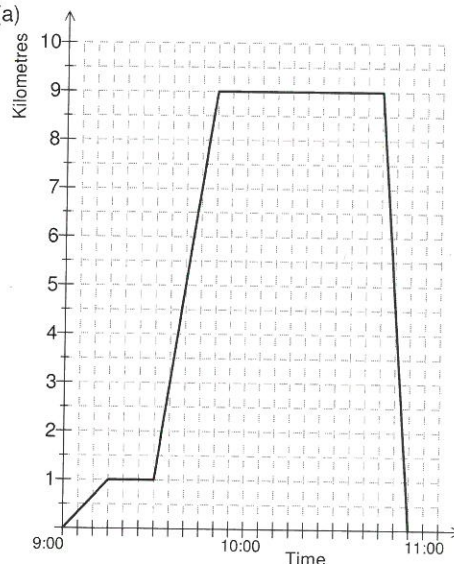(e) a = 1, b = 2, c = 6, d = 7, e = -2, f = 20
8. Equation (c)
9. (a) p = over 100 grams and up to 250 grams, q = $1.90
(b) r = 2.9, s = 250
(c) (i) 70 cents (ii) $1.90 (d) $9.10 (e) $3.30

10. (a)

(b) 1 hour 55 minutes
(c) 55 minutes
(d) 9 km
(e) (i) $66.\overline{6}$ metres / minute   (ii) 4 km/h
(f) 54 km/h
(g) (i) 39% (ii) 48%
(h) Sum not 100% as time spent waiting for the bus has not
been included.
11. (a) 30 litres
(b) 30 minutes
(c) 10 litres
(d) 7:00 am
(e) Used the same amount i.e. 10 litres
(f) 8:30 am
(g) 2 litres
(h) 10:45 am
(i) Filled the fuel tank ie replaced the 27 litres that were used
up
(j) 33 litres
(k) Longer, more fuel used in getting to the first fishing spot (10
L) than the quantity used to get back(6 L)
(l) 4 hour 15 minutes
(m) 6 litres per hour
(n) 1 litre per hour.
12. (a) $400
(b) $25
(c) 15 tickets
(d) 18 or 24 tickets
(e) $15 each
(f) Option 1: Buy 20 tickets as a group, total cost 20 x $20 =
$400. Option 2: Buy 21 tickets a group, total cost 21 x $15 =
$315 thus saving $85 and having one spare ticket. If the spare
ticket can be sold any revenue from this ticket will further
reduce the cost by that amount.
13. Statement (d)