Section One: Calculator-free 35% (51 Marks)

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

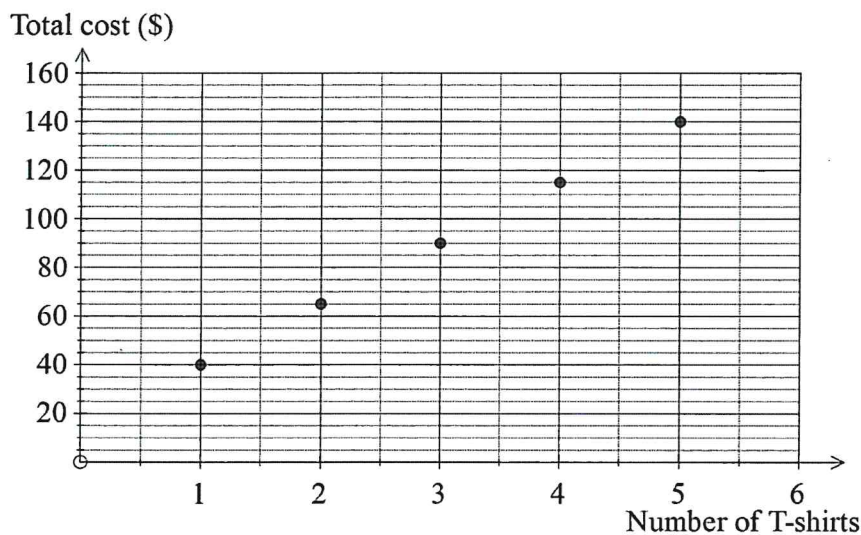Working time for this section is 50 minutes.

Question 1 (7 marks)

An online company sell T-shirts for $25 each plus a single $15 shipping fee.

(a) Complete the table below to show the total cost of purchasing up to five T-shirts. (2 marks)

Number of T-shirts	1	2	3	4	5
Total cost ($)	40	65	90	115	140

(b) Display the information in the table in graphical form on the axes below. (2 marks)

(c) State a rule for the total cost C of purchasing n T-shirts. (1 mark)

$$C = 25n + 15$$

(d) Determine how many T-shirts can be purchased for $265. (2 marks)

$$25n + 15 = 265$$
$$25n = 250 \Rightarrow n = 10$$

See next page

Question 2 **(5 marks)**

A sequence of numbers is described by the recursive equation $T_{n+1} = T_n - 8, \quad T_4 = 35$.

(a) Determine T_6. (1 mark)

$$35 - 8 - 8 = 19$$

$$T_6 = 19$$

(b) Determine T_1. (1 mark)

$$35 + 8 + 8 + 8 = 59$$

$$T_1 = 59$$

(c) State a rule for the nth term of this sequence. (2 marks)

$T_n = 59 + (n-1)(-8)$
or
$T_n = 67 - 8n$

(d) Determine T_{1001}. (1 mark)

$$T_{1001} = 59 + (1000)(-8) = 59 - 8000 = -7941$$

See next page

Question 5 **(9 marks)**

(a) Some consecutive terms of an arithmetic sequence are shown in the table below.

n	4	5	6	7
T_n	21.5	24.2	26.9	29.6

(i) Determine the eighth term of this sequence. (2 marks)

$$29.6 - 26.9 = 2.7$$

$$T_8 = 29.6 + 2.7 = 32.3$$

(ii) Determine the first term of the sequence. (1 mark)

$$21.5 - 3 \times 2.7 = 21.5 - 8.1 = 13.4$$

(iii) State a rule for the nth term of this sequence. (2 marks)

$$T_n = 13.4 + (n-1) \times 2.7$$

(iv) Determine T_{101}. (1 mark)

$$T_{101} = 13.4 + 100 \times 2.7 = 270 + 13.4 = 283.4$$

(b) The sum of the first three terms of another arithmetic sequence with a common difference of four is 45.

(i) If the first term of this sequence is a, write down an equation that shows the first three terms of this sequence have a sum of 45. (1 mark)

$$a + a + 4 + a + 8 = 45$$

(ii) Solve your equation and hence determine the sixth term of this sequence.

(2 marks)

$$3a + 12 = 45$$
$$3a = 33$$
$$a = 11$$

$$T_6 = 11 + 5 \times 4 = 31$$

See next page

Question 6 (8 marks)

(a) A sequence is defined by $T_{n+1} = T_n + 10, \quad T_1 = 45$.

(i) Determine the next two terms of the sequence. (1 mark)

$$55, 65$$

(ii) State a rule for the nth term of this sequence. (2 marks)

$$T_n = 45 + (n-1)(10)$$

(iii) Determine T_{45}. (1 mark)

$$T_{45} = 45 + 44 \times 10 = 45 + 440 = 485$$

(b) The first-order recurrence relation $T_{n+1} = aT_n + b$ was used with $T_1 = 3$ to calculate $T_2 = 4$ and $T_3 = 7$. Determine the values of a, b and T_5. (4 marks)

$3a + b = 4$

$4a + b = 7$

Subtract first from second:

$a = 3$

Substitute into first

$3(3) + b = 4$

$b = -5$

$T_4 = 3 \times 7 - 5 = 16$

$T_5 = 3 \times 16 - 5 = 43$

See next page