

Question 18

(9 marks)

(a) A sequence is defined by $T_{n+1} = 0.75T_n$, $T_1 = 160$.

(i) Calculate T_4 .

(1 mark)

$$T_4 = 67.5$$

(ii) Determine how many terms of the sequence are larger than 1.

(1 mark)

18 terms

(iii) State whether the sequence contains at least one negative number, explaining your answer.

(2 marks)

No. All terms will be positive and getting closer and closer to zero.

(b) The first two terms, in order, of a geometric sequence are $\frac{1}{2}$ and $\frac{2}{3}$.

(i) Calculate the next term of this sequence, leaving your answer as a fraction in simplest form.

(2 marks)

$$r = \frac{2}{3} \div \frac{1}{2} = \frac{4}{3}$$

$$T_3 = \frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$$

(ii) State a rule for the n^{th} term of this sequence.

(2 marks)

$$T_n = \frac{1}{2} \times \left(\frac{4}{3}\right)^{n-1}$$

(iii) Determine the minimum number of terms of this sequence that are required to have a sum of at least 10.

(1 mark)

8 terms

Section Two: Calculator-assumed

65% (97 Marks)

This section has **twelve (12)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 8

(7 marks)

After the failure of a computer containing the details of 312 clients, Chris was given the job of re-entering all the client information into a new computer. On the first day he managed to re-enter the details of 15 clients. On each subsequent day, he was given more and more time for this job and managed to add two more clients each day than on the previous day.

- (a) How many clients did Chris re-enter on the third day? (1 mark)

15, 17, 19: **19 clients.**

- (b) How many clients had Chris re-entered altogether after five days? (2 marks)

$15 + 17 + 19 + 21 + 23 = 95$ clients

- (c) Deduce a rule for T_n , the number of clients re-entered on day n in the form $T_n = an + b$. (2 marks)

$$T_n = 15 + (n - 1) \times 2$$

$$= 15 + 2n - 2$$

$$= 2n + 13$$

- (d) On which day did Chris finish the job, and how many clients did Chris re-enter on this day? (2 marks)

Using CAS

On day 12, when he re-entered the details of 37 clients.

$a_{n+1} = a_n + 2$		
n	a_n	Σa_n
1	15	15
2	17	32
3	19	51
4	21	72
5	23	95
6	25	120
7	27	147
8	29	176
9	31	207
10	33	240
11	35	275
12	37	312
13	39	351

Question 14

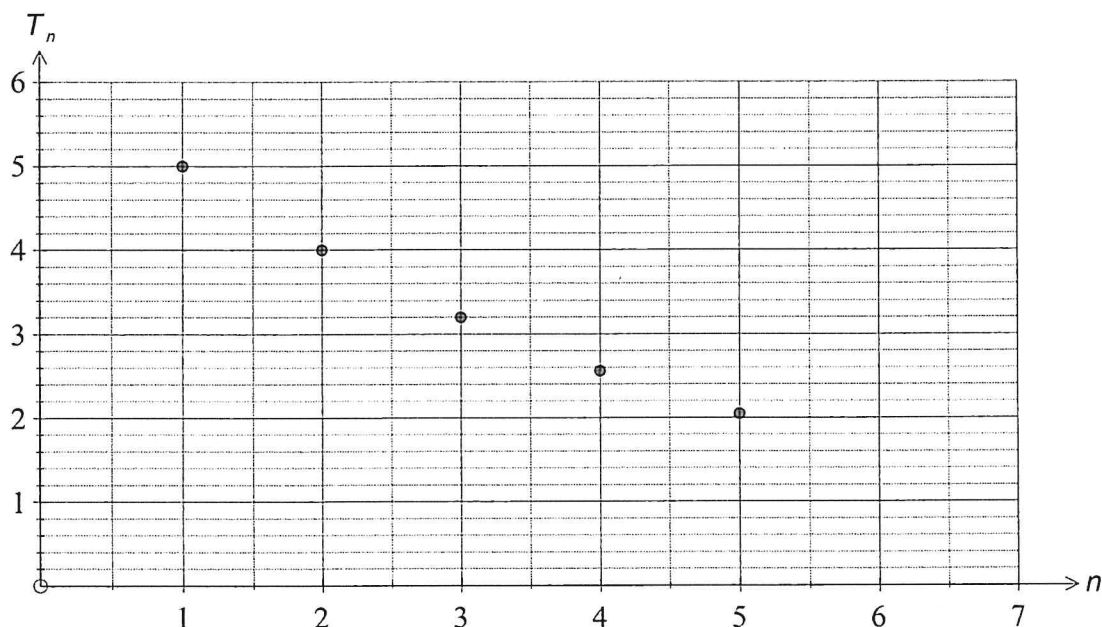
(8 marks)

Sequence A is defined given by $T_{n+1} = 0.8T_n$, $T_1 = 5$.

- (a) Use the rule to complete the first five terms of Sequence A in the table below. (2 marks)

n	1	2	3	4	5
T_n	5	4	3.2	2.56	2.048

- (b) Graph the first five terms of sequence A on the axes below. (2 marks)



- (c) How many terms of Sequence A are greater than 1? (1 mark)

8 terms.
($T_9 = 0.8\dots$)

- (d) The terms of the sequence can also represent the value of a secondhand car (in thousands of dollars) at the start of each year (year n).

- (i) Determine the value of the car at the start of the sixth year. (1 mark)

$$T_6 = 1.6384 \Rightarrow \text{value is } \$1638.40$$

- (ii) By what percentage is the value of the car decreasing each year? (1 mark)

20%

- (iii) The value of the car is written off when it falls below \$500. At the start of which year will this occur? (1 mark)

Year 12. (Value has fallen to \$429...)

Question 14

(8 marks)

A company purchases equipment at a cost of \$44 000 and expect the equipment to be used in the business for eight years. At the end of this time they expect to sell the equipment for \$7 000.

- (a) Calculate the total loss in value of the equipment. (1 mark)

$$44000 - 7000 = \$37\,000$$

Under the straight line depreciation method, the loss in value is spread equally over the eight years.

- (b) Calculate the annual loss in value of the equipment. (1 mark)

$$37000 \div 8 = \$4\,625 \text{ per year}$$

- (c) State a recursive rule for the value, V_n , of the equipment at the end of year n . (2 marks)

$$V_{n+1} = V_n - 4625, V_0 = 44000$$

- (d) Determine the value of the equipment at the end of the fourth year. (1 mark)

$$V_4 = \$25\,500$$

- (e) After an accounting review, it was found that the equipment was actually to be used for ten years, after which time it could be sold for \$5 000. Use this information to revise your answer to (d), assuming the straight line depreciation method is still used and showing all working. (3 marks)

$$\frac{44000 - 5000}{10} = 3900$$

$$V_{n+1} = V_n - 3900, V_0 = 44000$$

$$V_4 = \$28\,400$$