Question 2 **(7 marks)**

The number of laptop computers, T_n, that were brought to a school IT department for recharging during week n of the school year can be described recursively by the rule

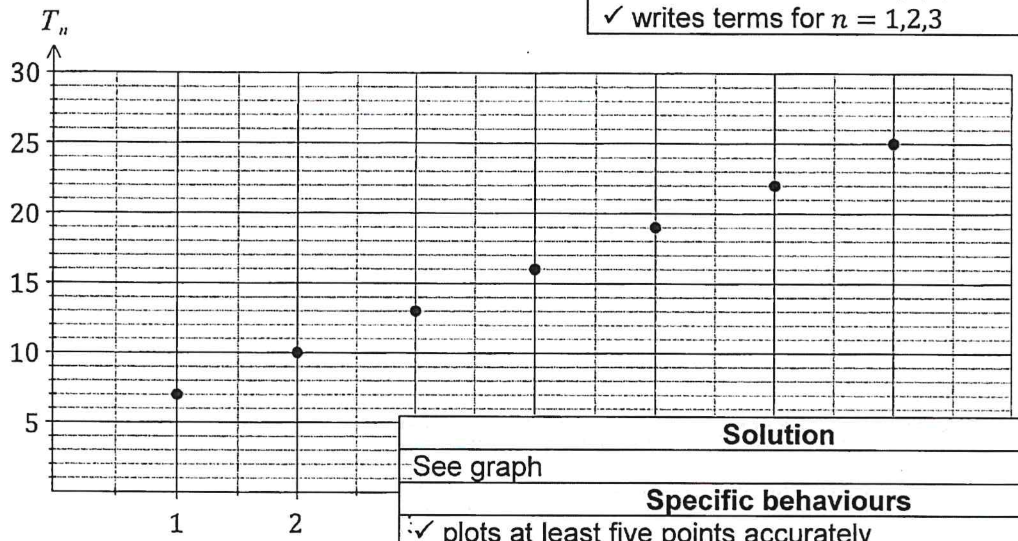
$$T_{n+1} = T_n + 3, \qquad T_4 = 16$$

(a) Use the rule to complete the table below. (2 marks)

n	1	2	3	4	5	6	7
T_n	7	10	13	16	19	22	25

Solution
See table
Specific behaviours
✓ writes terms for $n = 4,5,6,7$
✓ writes terms for $n = 1,2,3$

(b) Display the terms of the sequence from the table [...] (2 marks)



Solution
See graph
Specific behaviours
✓ plots at least five points accurately
✓ plots seven points accurately, no attempt to join them

(c) A rule to determine the number of laptops brought for recharging during week n can also be written in the form $T_n = an + b$. Determine the values of a and b. (2 marks)

Solution
$T_n = 7 + (n-1)(3) = 3n + 4$ $a = 3,\ b = 4$
Specific behaviours
✓ determines value of a
✓ determines value of b

(d) If the pattern continued, determine the number of the week during which the number of laptops brought in for recharging first exceeds 50. (1 mark)

Solution
$3n + 4 = 50, n = 15\frac{1}{3}$. During week 16.
Specific behaviours
✓ states correct week

Question 16 **(8 marks)**

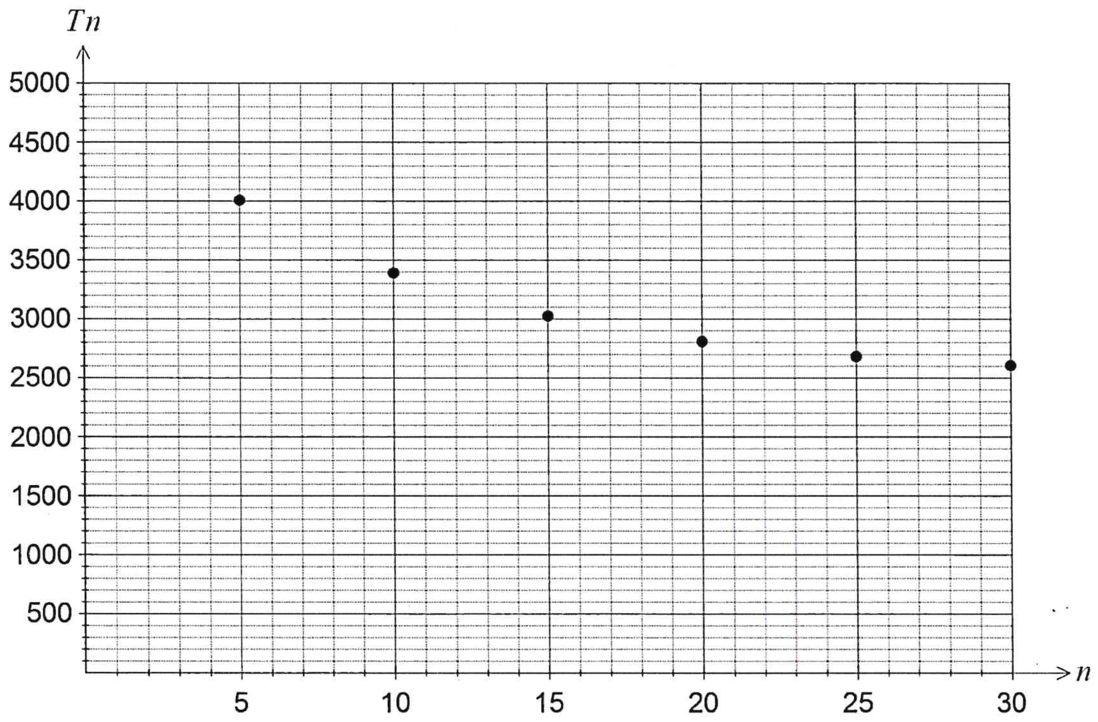
A plantation has 4 800 trees. The plantation manager is interested in modelling what would happen if each year, 10% of the existing trees were cut down for timber and another 250 new trees planted.

The number of trees, T_n, at the start of year n can be modelled by $T_{n+1} = 0.9T_n + 250,\ T_1 = 4800$.

(a) Use the recurrence relation to complete the missing values in the following table. (2 marks)

n	5	10	15	20	25	30
T_n	4009	**3391**	**3026**	**2811**	**2684**	2608

(b) Plot the values from the table on the axes below. (2 marks)



(c) Comment on how the number of trees in the plantation is changing. (2 marks)

The number of trees is decreasing, but at a slower and slower rate.

(d) Does the model predict that eventually there will be no trees left in the planation? Justify your answer. (2 marks)

No - the number will never fall below 2 500, the long term steady-state solution of the sequence.

Question 18 **(9 marks)**

(a) A sequence is defined by $T_{n+1} = 0.75T_n,\ T_1 = 160$.

(i) Calculate T_4. (1 mark)

$T_4 = 67.5$

(ii) Determine how many terms of the sequence are larger than 1. (1 mark)

18 terms

(iii) State whether the sequence contains at least one negative number, explaining your answer. (2 marks)

No. All terms will be positive and getting closer and closer to zero.

(b) The first two terms, in order, of a geometric sequence are $\frac{1}{2}$ and $\frac{2}{3}$.

(i) Calculate the next term of this sequence, leaving your answer as a fraction in simplest form. (2 marks)

$$r = \frac{2}{3} \div \frac{1}{2} = \frac{4}{3}$$

$$T_3 = \frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$$

(ii) State a rule for the n^{th} term of this sequence. (2 marks)

$$T_n = \frac{1}{2} \times \left(\frac{4}{3}\right)^{n-1}$$

(iii) Determine the minimum number of terms of this sequence that are required to have a sum of at least 10. (1 mark)

8 terms

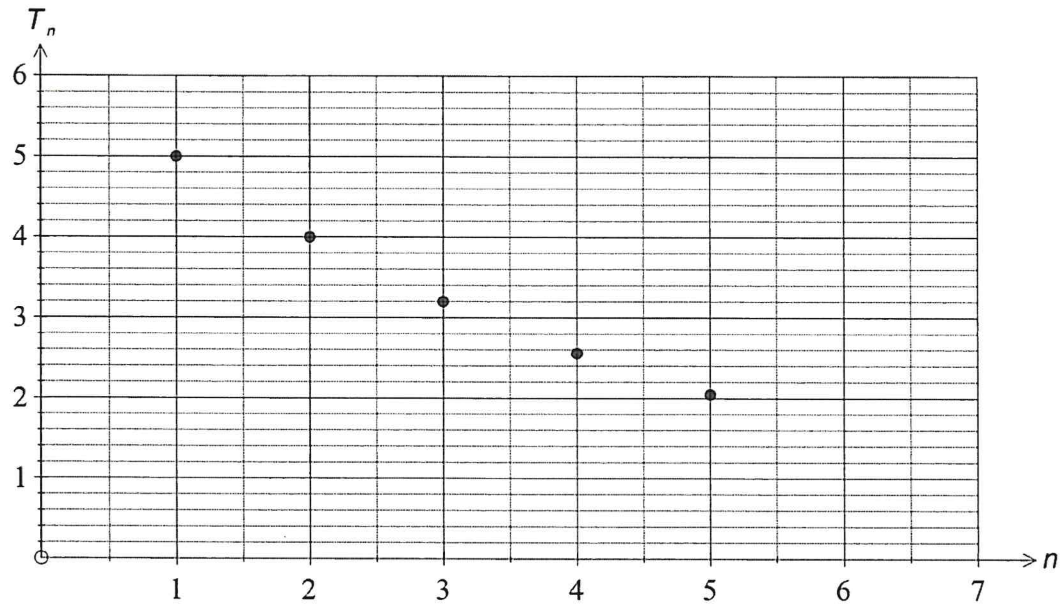
Question 14 **(8 marks)**

Sequence A is defined given by $T_{n+1} = 0.8T_n$, $T_1 = 5$.

(a) Use the rule to complete the first five terms of Sequence A in the table below. (2 marks)

n	1	2	3	4	5
T_n	5	**4**	**3.2**	**2.56**	**2.048**

(b) Graph the first five terms of sequence A on the axes below. (2 marks)



(c) How many terms of Sequence A are greater than 1? (1 mark)

8 terms.
($T_9 = 0.8...$)

(d) The terms of the sequence can also represent the value of a secondhand car (in thousands of dollars) at the start of each year (year n).

(i) Determine the value of the car at the start of the sixth year. (1 mark)

$T_6 = 1.6384 \Rightarrow$ value is \$1638.40

(ii) By what percentage is the value of the car decreasing each year? (1 mark)

20%

(iii) The value of the car is written off when it falls below \$500. At the start of which year will this occur? (1 mark)

Year 12. (Value has fallen to \$429...)