

# **MATHEMATICS METHODS**

# Unit 1 and Unit 2 Formula Sheet

(For use with Year 11 examinations and response tasks)

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This document is valid for teaching and examining from 1 July 2015.

#### Measurement

Circle:  $C = 2\pi r = \pi D$ , where C is the circumference,

 $\it r$  is the radius and  $\it D$  is the diameter

 $A = \pi r^2$ , where A is the area

Triangle:  $A = \frac{1}{2}bh$ , where b is the base and h is the perpendicular height

Parallelogram: A = bh

Trapezium:  $A = \frac{1}{2} (a + b)h$ , where a and b are the lengths of the parallel sides

Prism: V = Ah, where V is the volume and A is the area of the base

Pyramid:  $V = \frac{1}{3} Ah$ 

Cylinder:  $S = 2\pi rh + 2\pi r^2$ , where S is the total surface area

 $V = \pi r^2 h$ 

Cone:  $S = \pi r s + \pi r^2$ , where s is the slant height

 $V = \frac{1}{3}\pi r^2 h$ 

Sphere:  $S = 4\pi r^2$ 

 $V = \frac{4}{3}\pi r^3$ 

#### **Functions and graphs**

Lines and Linear relationships

For points  $P(x_1, y_1)$  and  $Q(x_2, y_2)$ 

Mid-point of 
$$P$$
 and  $Q$ : 
$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Gradient of the line through *P* and *Q*: 
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Equation of the line through *P* with slope *m*:  $y - y_1 = m(x - x_1)$ 

Parallel lines: 
$$m_1 = m_2$$

Perpendicular lines: 
$$m_1 m_2 = -1$$

General equation of a line: 
$$ax + by + c = 0$$
 or  $y = mx + c$ 

Quadratic relationships

For the general quadratic equation  $ax^2 + bx + c = 0$ ,  $a \ne 0$ 

Completing the square: 
$$ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)^2$$

Discriminant: 
$$\Delta = b^2 - 4ac$$

Quadratic formula: 
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Graphs and Relations** 

Equation of a circle: 
$$(x-a)^2 + (y-b)^2 = r^2$$

where, (a,b) is the centre and r is the radius

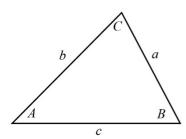
# **Trigonometric functions**

Cosine and sine rules

For any triangle ABC with corresponding length of sides a,b,c

Cosine rule: 
$$c^2 = a^2 + b^2 - 2ab \cos C$$

Sine rule: 
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



Area of 
$$\Delta$$
: 
$$A = \frac{1}{2}ab\sin C$$
 
$$A = \sqrt{s(s-a)(s-b)(s-c)} \text{ where } s = \frac{1}{2}(a+b+c)$$

Circular measure and radian measure

In a circle of radius  $\,r$  , for an arc subtending angle  $\, heta\,$  (radians) at the centre

Length of arc:

 $\ell = r\theta$  Length of chord:  $l = 2r\sin\frac{1}{2}\theta$ 

Area of sector:

 $A = \frac{1}{2}r^2\theta$  Area of segments:  $A = \frac{1}{2}r^2(\theta - \sin\theta)$ 

Trigonometric functions: (fundamentals)

$$\sin\left(-\theta\right) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\tan(-\theta) = -\tan\theta$$

$$\sin\left(\theta + \frac{\pi}{2}\right) = \cos\theta \qquad \cos\left(\theta - \frac{\pi}{2}\right) = \sin\theta$$

$$\cos\left(\theta - \frac{\pi}{2}\right) = \sin\theta$$

$$\sin^2\theta + \cos^2\theta = 1$$

Angle sum and difference identites

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

### **Counting and probability**

Combinations

Number of combinations:

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

(of r objects taken from a set of n distinct objects)

Binomial expansion:

$$(x+y)^n = x^n + \binom{n}{1} x^{n-1} y + \dots + \binom{n}{r} x^{n-r} y^r + \dots + y^n$$

Binomial coefficients:

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n \times (n-1) \times \dots \times (n-r+1)}{r \times (r-1) \times \dots \times 2 \times 1}$$

Probability

Fundamentals of probability:

$$\begin{split} P \big( \text{complement of } A \big) &= P \Big( \overline{A} \Big) = 1 - P \big( A \big) \\ P \big( A \text{ or } B \big) &= P \big( A \cup B \big) = P \big( A \big) + P \big( B \big) - P \big( A \cap B \big) \\ P \big( A \text{ and } B \big) &= P \big( A \cap B \big) = P \big( A \big) P \big( B / A \big) \\ &= P \big( B \big) P \big( A / B \big) \end{split}$$

Conditional probability:

$$P(B/A)$$
 =  $\frac{P(A \cap B)}{P(A)}$  for  $P(A) \neq 0$ 

## **Exponential functions**

Index laws:

For a, b > 0 and m, n real,

$$a^{m}b^{m} = (ab)^{m}$$
  $a^{m}a^{n} = a^{m+n}$   $(a^{m})^{n} = a^{mn}$ 

$$a^m a^n = a^{m+n}$$

$$(a^m)^n = a^{mn}$$

$$a^{-m} = \frac{1}{a^m}$$

$$a^{-m} = \frac{1}{a^m}$$
  $\frac{a^m}{a^n} = a^{m-n}$   $a^0 = 1$ 

$$a^0 = 1$$

For a>0, m an integer and n a positive integer,  $a^{\frac{m}{n}}=\sqrt[n]{a^m}=\left(\sqrt[n]{a}\right)^m$ 

### Arithmetic and geometric sequences and series

Arithmetic sequences

For initial term a and common difference d:  $T_n = a + (n-1)d$ ,  $n \ge 1$ 

$$T_n = a + (n-1)d, n \ge 1$$

$$T_{n+1} = T_n + d$$
, where  $T_1 = a$ 

$$S_n = \frac{n}{2} (2a + (n-1)d)$$

Geometric sequences

For initial term *a* and common ratio *r*:

$$T_{n+1} = rT_n$$
, where  $T_1 = a$ 

$$T_n = ar^{n-1}, n \ge 1$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_{\infty} = \frac{a}{1-r}, |r| < 1$$

#### Introduction to differential calculus

Rates of change

Difference quotient: 
$$\frac{\delta y}{\delta x} = \frac{f(x+h) - f(x)}{h}$$

Derivative (concept): 
$$\frac{dy}{dx} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Computation of derivatives: 
$$\frac{d}{dx}(x^n) = nx^{n-1}.$$

Anti-derivatives: If 
$$f'(x) = ax^n$$
 then  $f(x) = \frac{ax^{n+1}}{n+1} + c(\text{constant}), n \neq -1$ 

Note: Any additional formulas identified by the examination writers as necessary will be included in the body of the particular question.