SADLER MATHEMATICS METHODS UNIT 1

WORKED SOLUTIONS

Chapter 8 Trigonometric functions

Exercise 8A

Question 1

4

Question 2

3

Question 3

6

Question 4

5

Question 5

4

Question 6

4

Question 7

3, 6

Question 8

3, 2

Question 9

4, 5

Question 10

2, 3

Question 11

3, 2.5

Question 12

1, 4

Exercise 8B

Question 1

a 1

b 2

c 4

d 3

e 2

f 3

g 5

h 3

Question 2

a 360°

b 180°

c 360°

d 180°

e 720°

f 120°

g 90°

h 1080°

i 180°

Question 3

a 2π

b π

c 2π

d $\frac{\pi}{2}$

e $\frac{\pi}{3}$

f $\frac{2\pi}{3}$

g 4π

h π

i 0.5

Question 4

a Max at $(\frac{\pi}{2}, 1)$. Min at $(\frac{3\pi}{2}, -1)$.

b Max at $(\frac{\pi}{2}, 3)$. Min at $(\frac{3\pi}{2}, 1)$.

c Max at $(\frac{3\pi}{2}, 1)$. Min at $(\frac{\pi}{2}, -1)$.

d Max at $(\frac{\pi}{4}, 4)$ and at $(\frac{5\pi}{4}, 4)$. Min at $(\frac{3\pi}{4}, 2)$ and at $(\frac{7\pi}{4}, 2)$.

e Max at $(\frac{3\pi}{4}, 4)$. Min at $(\frac{7\pi}{4}, 2)$.

Question 5

a $3\sin x$ has a max value of 3 when $x = 90°$.

b $x - 30 = 90 \ \therefore \ x = 120$

$2\sin(x - 30)$ has a max value of 2 when $x = 120°$.

c

$x + 30 = 90 \ \therefore \ x = 60$

$2\sin(x + 30)$ has a max value of 2 when $x = 60°$.

d $-3\sin x$ has a max value of 3 when $x = 270°$.

Question 6

a $3, \frac{\pi}{4}$

b $5, \frac{3\pi}{2}$

c $2, \frac{11\pi}{6}$

d $3, \frac{\pi}{6}$

Question 7

a Amplitude = 2 $\therefore \ a = 2$

b Amplitude = 3 $\therefore \ a = 3$

c Amplitude = 3 but wave is rejected $\therefore \ a = -3$

d Amplitude $\approx$ 1.3 but wave is rejected $\therefore \ a \approx -1.3$

Question 8

a Amplitude is 3 $\therefore\ a = 3$

b Amplitude is 2 but wave is rejected $\therefore\ a = -2$

Question 9

a Period is π, $\tan 45° = 2\ \therefore\ a = 2$

b Period is $180°$, but curve is rejected.

$\tan 45 = -1$

$\therefore a = -1$

Question 10

a Amplitude = 2 $\therefore\ a = 2$

3 complete curves in $2\pi \rightarrow b = 3$

$y = 2\sin 3x$

b Wave is rejected $a < 0$

Amplitude is 3 $\therefore\ a = -3$

In 2π, 2 complete curves $\therefore\ b = 2$

$y = -3\sin 2x$

c Amplitude is 2 $\therefore\ a = 2$

In $360°$, 6 complete curves $\therefore\ b = 6$

$y = 2\sin 6x$

d Amplitude is 3 $\therefore\ a = 3$

Period is $3 \rightarrow \dfrac{2\pi}{b}$

$\therefore b = \dfrac{2\pi}{3}$

$y = 3\sin\dfrac{2\pi}{3}x$

Question 11

a Amplitude = 1 $\therefore\ a = 1$ (max at $x = 0$, $a > 0$)

Period = $\pi \rightarrow \dfrac{2\pi}{b} = \pi$

$\therefore b = 2$

$y = \cos 2x$

b Amplitude = 3

Minimum value when $x = 0\ \therefore\ a = -3$

Period : 3 curves in $2\pi, b = 3$

$\therefore y = -3\cos 3x$

c Amplitude is 3 but minimum value when $x = 0\ \therefore\ a = -3$

Period : 2 curves in $2\pi, b = 2$

$\therefore y = -3\cos 3x$

d Amplitude = 2 (max at $x = 0$, $a > 0$)

Period = $4 \rightarrow \dfrac{2\pi}{b} = 4$

$\therefore b = \dfrac{\pi}{2}$

$y = 2\cos\dfrac{\pi}{2}x$

Question 12

a Amplitude = 2 $\therefore\ a = 2$

Solid curve is 30° to the right.

$\therefore b = 30$

Period is 360°, so next value of $b = 30 + 360$

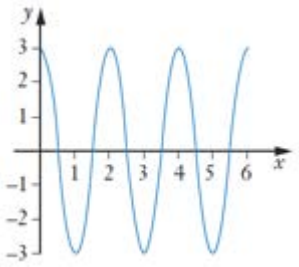
$= 390°$

b $y = -2\sin(x - 210)$

Question 13

Period: $\frac{2\pi}{\pi} = 2$

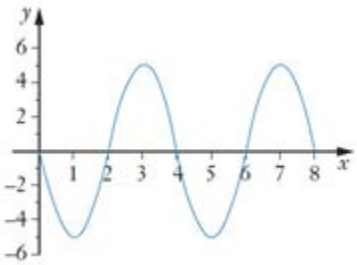
Amplitude: 3



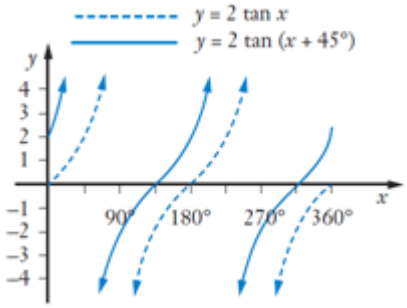
Question 14

Period: $\frac{2\pi}{\frac{\pi}{2}} = 4$

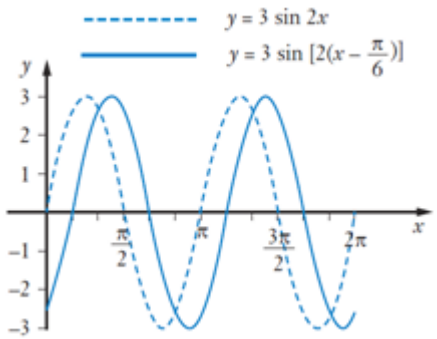
Amplitude: 5



Question 15



Question 16



Exercise 8C

Question 1

tan 190° is in the 3rd quadrant and is positive

Question 2

cos 310° is in the 4th quadrant and is positive

Question 3

tan (–190°) is in the 2nd quadrant and is negative

Question 4

sin (–170°) is in the 3rd quadrant and is negative

Question 5

sin 555° = sin 195° is in the 3rd quadrant and is negative

Question 6

cos 190° is in the 3rd quadrant and is negative

Question 7

$\tan \frac{\pi}{10}$ is in the 1st quadrant and is positive

Question 8

$\sin \frac{4\pi}{5}$ is in the 2nd quadrant and is positive

Question 9

$\cos \frac{\pi}{10}$ is in the 1st quadrant and is positive

Question 10

$\sin \left(-\frac{\pi}{5}\right)$ is in the 4th quadrant and is negative

Question 11

$\cos \frac{9\pi}{10}$ is in the 2nd quadrant and is negative

Question 12

$\tan \frac{13\pi}{5} = \tan \frac{3\pi}{5}$ is in the 2nd quadrant and is negative

Question 13

$\sin 140°$ is in the 2nd quadrant and makes a $40°$ angle with the x-axis.
$\therefore \sin 40°$

Question 14

$\sin 250°$ is in the 3rd quadrant and makes a $70°$ angle below the x-axis.
$\therefore -\sin 70°$

Question 15

$\sin 340°$ is in the 4th quadrant and makes a $20°$ angle below the x-axis.

$\therefore -\sin 20°$

Question 16

$\sin 460° = \sin 100°$ is in the 2nd quadrant and is $80°$ above the x-axis.

$\therefore \sin 80°$

Question 17

$\sin \frac{5\pi}{6}$ is in the 2nd quadrant and is $\frac{\pi}{6}$ above the x-axis.

$\therefore \sin \frac{\pi}{6}$

Question 18

$\sin \frac{7\pi}{6}$ is in the 3rd quadrant and is $\frac{\pi}{6}$ below the x-axis.

$\therefore -\sin \frac{\pi}{6}$

Question 19

$\sin \frac{11\pi}{5}$ is in the 1st quadrant and is $\frac{\pi}{5}$ above the x-axis.

$\therefore \sin \frac{\pi}{5}$

Question 20

$\sin\left(-\frac{\pi}{5}\right)$ is in the 4th quadrant and is $\frac{\pi}{5}$ below the x-axis.

$\therefore -\sin\frac{\pi}{5}$

Question 21

$\cos 100°$ is in the 2nd quadrant and is $80°$ above the negative x-axis.

$\therefore -\cos 80°$

Question 22

$\cos 200°$ is in the 3rd quadrant and is $20°$ below the negative x-axis.

$\therefore -\cos 20°$

Question 23

$\cos 300°$ is in the 4th quadrant and is $60°$ below the positive x-axis.

$\therefore \cos 60°$

Question 24

$\cos(-300°) = \cos 60°$ in the 1st quadrant.

Question 25

$\cos\frac{4\pi}{5}$ is in the 2nd quadrant and is $\frac{\pi}{5}$ above the negative x-axis.

$\therefore -\cos\left(\frac{\pi}{5}\right)$

Question 26

$\cos \frac{9\pi}{10}$ is in the 2nd quadrant and is $\frac{\pi}{10}$ above the negative x-axis.

$\therefore -\cos \frac{\pi}{10}$

Question 27

$\cos \frac{11\pi}{10}$ is in the 3rd quadrant and is $\frac{\pi}{10}$ below the negative x-axis.

$\therefore -\cos \frac{\pi}{10}$

Question 28

$\cos \frac{21\pi}{10} = \cos \frac{\pi}{10}$ and is in the 1st quadrant.

Question 29

$\tan 100°$ is in the 2nd quadrant and is $80°$ above the x-axis.

$\therefore -\tan 80°$

Question 30

$\tan 200°$ is in the 3rd quadrant and is $20°$ below the x-axis.

$\therefore \tan 20°$

Question 31

$\tan (-60°)$ is in the 4th quadrant and is $60°$ below the x-axis.

$\therefore -\tan 60°$

Question 32

$\tan(-160°)$ is in the 3rd quadrant and is 20° below the *x*-axis.

$\therefore \tan 20°$

Question 33

$\tan \frac{6\pi}{5}$ is in the 3rd quadrant and is $\frac{\pi}{5}$ below the *x*-axis.

$\therefore \tan \frac{\pi}{5}$

Question 34

$\tan\left(-\frac{6\pi}{5}\right)$ is in the 2nd quadrant and is $\frac{\pi}{5}$ above the *x*-axis.

$\therefore -\tan \frac{\pi}{5}$

Question 35

$\tan \frac{11\pi}{5} = \tan \frac{\pi}{5}$ is in the 1st quadrant and is $\frac{\pi}{5}$ above the *x*-axis.

$\therefore \tan \frac{\pi}{5}$

Question 36

$\tan\left(-\frac{21\pi}{5}\right) = \tan\left(-\frac{\pi}{5}\right)$ is in the 4th quadrant and is $\frac{\pi}{5}$ below the *x*-axis.

$\therefore -\tan \frac{\pi}{5}$

Question 37

$\sin 300°$, in 4th quadrant $\rightarrow$ negative.

Reference angle = 60°

$$\sin 60° = \frac{\sqrt{3}}{2}$$

$$\therefore \sin 300° = -\frac{\sqrt{3}}{2}$$

Question 38

$\tan 210°$, in 3rd quadrant $\rightarrow$ positive.

Reference angle = 30°

$$\tan 210° = \frac{1}{\sqrt{3}}$$

Question 39

$\cos 240°$, in 3rd quadrant $\rightarrow$ negative.

Reference angle = 60°

$$\cos 240° = -\frac{1}{2}$$

Question 40

$\cos 270° = 0$

Question 41

$\sin 180° = 0$

Question 42

$$\cos 390° = \cos 30° = \frac{\sqrt{3}}{2}$$

Question 43

$\sin(-135°)$, in 3rd quadrant $\rightarrow$ negative.

Reference angle = 45°

$$\sin(-135°) = -\frac{1}{\sqrt{2}}$$

Question 44

$\cos(-135°)$, in 3rd quadrant $\rightarrow$ negative.

Reference angle = 45°

$$\cos(-135°) = -\frac{1}{\sqrt{2}}$$

Question 45

$\sin \frac{7\pi}{6}$, in 3rd quadrant $\rightarrow$ negative.

Reference angle = $\frac{\pi}{6}$

$$\sin \frac{7\pi}{6} = -\frac{1}{2}$$

Question 46

$\cos \frac{7\pi}{6}$, in 3rd quadrant $\rightarrow$ negative.

Reference angle = $\frac{\pi}{6}$

$$\cos \frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$$

Question 47

$\tan \frac{7\pi}{6}$, in 3rd quadrant $\rightarrow$ positive.

Reference angle = $\frac{\pi}{6}$

$$\tan \frac{7\pi}{6} = \frac{1}{\sqrt{3}}$$

Question 48

$\sin \frac{7\pi}{4}$, in 4th quadrant $\rightarrow$ negative.

Reference angle = $\frac{\pi}{4}$

$$\sin \frac{7\pi}{4} = -\frac{1}{\sqrt{2}}$$

Question 49

$\cos (-\frac{7\pi}{4})$, in 1st quadrant $\rightarrow$ positive.

Reference angle = $\frac{\pi}{4}$

$$\cos (-\frac{7\pi}{4}) = \frac{1}{\sqrt{2}}$$

Question 50

$$\tan 6\pi = \tan 2\pi = 0$$

Question 51

$$\sin \frac{5\pi}{2} = \sin \frac{\pi}{2} = 1$$

Question 52

$\cos\left(-\frac{7\pi}{3}\right) = \cos\left(-\frac{\pi}{3}\right)$, in 4th quadrant $\rightarrow$ positive.

Reference angle = $\frac{\pi}{3}$

$\cos\left(-\frac{7\pi}{3}\right) = \frac{1}{2}$

Exercise 8D

Question 1

$\cos x = \frac{1}{2}$ Reference angle = 60°

$x = 60°, 300°$

Question 2

$\sin x = -\frac{1}{2}$ Reference angle = 30°

$x = 210°, 330°$

Question 3

$\tan x = 1$

$x = 45°, 225°$

Question 4

$\sin x = -\frac{1}{\sqrt{2}}$

$x = 225°, 315°$

Question 5

$\sin x = \frac{1}{\sqrt{2}}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}$

Question 6

$\cos x = -\dfrac{1}{\sqrt{2}}$ Reference angle = $\dfrac{\pi}{4}$

$x = \dfrac{3\pi}{4}, \dfrac{5\pi}{4}$

Question 7

$\tan x = -1$

$x = \dfrac{3\pi}{4}, \dfrac{7\pi}{4}$

Question 8

$\tan x = \sqrt{3}$

$x = \dfrac{\pi}{3}, \dfrac{4\pi}{3}$

Question 9

$\cos x = \dfrac{\sqrt{3}}{2}$

$x = -30°, 30°$

Question 10

$\sin x = -1$

$x = -90°$

Question 11

$\tan x = -\dfrac{1}{\sqrt{3}}$

$x = -30°, 150°$

Question 12

$\sin x = 0$

$x = -180°, 0°, 180°$

Question 13

$\sin x = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}$

Question 14

$\cos x = -\frac{1}{2}$

$x = -\frac{2\pi}{3}, \frac{2\pi}{3}$

Question 15

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

Question 16

$\cos x = 0$

$x = -\frac{\pi}{2}, \frac{\pi}{2}$

Question 17

$$\tan x = 1.5$$
$$x = \pi + 0.98$$

Question 18

$$11 + 25\cos x = 0$$
$$25\cos x = -11$$
$$\cos x = -\frac{11}{25} = -0.44$$
$$x = -116.1°, 116.1°$$

Question 19

$$\tan 2x = \frac{1}{\sqrt{3}} \qquad 0 \le x \le 180°$$
$$2x = 30°, 210° \qquad 0 \le 2x \le 360°$$
$$x = 15°, 105°$$

Question 20

$$\cos 4x = \frac{\sqrt{3}}{2} \qquad 0 \le x \le \pi$$
$$4x = \frac{\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}, \frac{23\pi}{6} \qquad 0 \le 4x \le 4\pi$$
$$x = \frac{\pi}{24}, \frac{11\pi}{24}, \frac{13\pi}{24}, \frac{23\pi}{24} \qquad \text{Reference angle} = \frac{\pi}{6}$$

Question 21

$$\sin 3x = \frac{1}{2} \qquad -90° \le x \le 90°$$
$$3x = -210°, 30°, 150° \qquad -270° \le 3x \le 270°$$
$$x = -70°, 10°, 50°$$

Question 22

$2\sqrt{3}\sin 2x = 3$ $\quad 0 \le x \le 2\pi$

$\sin 2x = \dfrac{3}{2\sqrt{3}}$ $\quad 0 \le 2x \le 4\pi$

$= \dfrac{\sqrt{3}}{2}$ $\quad$ Reference angle = $\dfrac{\pi}{3}$

$2x = \dfrac{\pi}{3}, \dfrac{2\pi}{3}, \dfrac{7\pi}{3}, \dfrac{8\pi}{3}$

$x = \dfrac{\pi}{6}, \dfrac{\pi}{3}, \dfrac{7\pi}{6}, \dfrac{4\pi}{3}$

Question 23

$2\cos 3x + \sqrt{3} = 0$ $\quad 0 \le x \le 2\pi$

$\cos 3x = -\dfrac{\sqrt{3}}{2}$ $\quad 0 \le 3x \le 6\pi$

$3x = \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{17\pi}{6}, \dfrac{19\pi}{6}, \dfrac{29\pi}{6}, \dfrac{31\pi}{6}$ $\quad$ Reference angle = $\dfrac{\pi}{6}$

$x = \dfrac{5\pi}{18}, \dfrac{7\pi}{18}, \dfrac{17\pi}{18}, \dfrac{19\pi}{18}, \dfrac{29\pi}{18}, \dfrac{31\pi}{18}$

Question 24

$(\sin x + 1)(2\sin x - 1) = 0$ $\quad$ Reference angle = $\dfrac{\pi}{6}$

$\sin x = -1$ $\quad$ or $\quad 2\sin x = 1$

$x = \dfrac{3\pi}{2}$ $\qquad \sin x = \dfrac{1}{2}$

$x = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$

$\therefore x = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{3\pi}{2}$

Question 25

$\sin^2 x = \frac{1}{2}$ $\quad 0° \le x \le 360°$

$\sin x = \pm\frac{1}{\sqrt{2}}$ $\quad$ Reference angle = 45°

$x = 45°, 135°, 225°, 315°$

Question 26

$4\cos^2 x - 3 = 0$ $\quad -\pi \le x \le \pi$

$\cos^2 x = \frac{3}{4}$ $\quad$ Reference angle = $\frac{\pi}{6}$

$\cos x = \pm\frac{\sqrt{3}}{2}$

$x = -\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}$

Question 27

$(\sin x)(2\cos x - 1) = 0$ $\quad$ Reference angle = 60°

$\sin x = 0$ or $2\cos x - 1 = 0$

$x = -180°, 0°, 180°$ $\quad \cos x = \frac{1}{2}$

$x = -60°, 60°$

$\therefore x = -180°, -60°, 0°, 60°, 180°$

Question 28

$2\cos^2 x + \cos x - 1 = 0$ $\quad -\pi \le x \le \pi$

$(2\cos x - 1)(\cos x + 1) = 0$ $\quad$ Reference angle = $\frac{\pi}{3}$

$2\cos x - 1 = 0$ or $\cos x + 1 = 0$

$\cos x = \frac{1}{2}$ $\quad \cos x = -1$

$x = -\frac{\pi}{3}, \frac{\pi}{3}$ $\quad x = -\pi, \pi$

$\therefore x = -\pi, -\frac{\pi}{3}, \frac{\pi}{3}, \pi$

Question 29

$\sin(x + \frac{\pi}{3}) = \frac{1}{\sqrt{2}}$ $\quad 0 \le x \le 2\pi$

$x + \frac{\pi}{3} = \frac{\pi}{4}, \frac{3\pi}{4}$ $\quad \frac{\pi}{3} \le x + \frac{\pi}{3} \le \frac{7\pi}{3}$

$x = -\frac{\pi}{12}, \frac{5\pi}{12}$ $\quad$ Reference angle = $\frac{\pi}{4}$

$-\frac{\pi}{12}$ is out of domain but $-\frac{\pi}{12} + 2\pi = \frac{23\pi}{12}$

$\therefore x = \frac{5\pi}{12}, \frac{23\pi}{12}$

Exercise 8E

Question 1

$\sin x = \frac{1}{4}$ $\quad -180° \le x \le 180°$

$x = 14.5°, 165.5°$

Question 2

$\sin^2 x = \frac{1}{4}$ $\quad -\pi \le x \le \pi$

$\sin x = \pm\frac{1}{2}$

$x = -\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}$

Question 3

$\sin x = \sin^2 x + \cos^2 x$

$\sin x = 1$

$\therefore x = \frac{\pi}{2}$

Question 4

$(2\sin x - 1)\cos x = 0$

$2\sin x - 1 = 0$ or $\cos x = 0$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ $\qquad x = \frac{\pi}{2}, \frac{3\pi}{2}$

$x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$

Question 5

$$\sin x + 2\sin^2 x = 0$$
$$\sin x(1 + 2\sin x) = 0$$
$$\sin x = 0 \quad \text{or} \quad 1 + 2\sin x = 0$$
$$x = 0°, 180°, 360° \qquad \sin x = -\frac{1}{2}$$
$$x = 210°, 330°$$
$$x = 0°, 180°, 210°, 330°, 360°$$

Question 6

$$(2\cos x + 1)(5\sin x - 1) \qquad 0° \le x \le 360°$$
$$2\cos x + 1 = 0 \quad \text{or} \quad 5\sin x - 1 = 0$$
$$\cos x = -\frac{1}{2} \qquad \sin x = \frac{1}{5}$$
$$x = 120°, 240° \qquad x = 11.5°, 168.5°$$
$$x = 11.5°, 120°, 168.5°, 240°$$

Question 7

$$8\sin^2 x + 4\cos^2 x = 7$$
$$4\sin^2 x + 4\sin^2 x + 4\cos^2 x - 7 = 0$$
$$4\sin^2 x + 4(\sin^2 x + \cos^2 x) - 7 = 0$$
$$4\sin^2 x + 4 - 7 = 0$$
$$4\sin^2 x - 3 = 0$$
$$4\sin^2 x = 3$$
$$\sin^2 x = \frac{3}{4}$$
$$\sin x = \pm\frac{\sqrt{3}}{2}$$
$$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

Question 8

$$\tan^2 x + \tan x = 2$$
$$\tan^2 x + \tan x - 2 = 0$$
$$(\tan x + 2)(\tan x - 1) = 0$$
$$\tan x + 2 = 0 \quad \text{or} \quad \tan x - 1 = 0$$
$$\tan x = -2 \qquad \tan x = 1$$
$$x = -63.4°, 116.6° \qquad x = -135°, 45°$$
$$x = -135°, -63.4°, 45°, 116.6°$$

Question 9

$$5 - 4\cos x = 4\sin^2 x$$
$$4\sin^2 x + 4\cos x - 5 = 0$$
$$4(1 - \cos^2 x) + 4\cos x - 5 = 0$$
$$4 - 4\cos^2 x + 4\cos x - 5 = 0$$
$$-4\cos^2 x + 4\cos x - 1 = 0$$
$$-1(4\cos^2 x - 4\cos x + 1) = 0$$
$$4\cos^2 x - 4\cos x + 1 = 0$$
$$(2\cos x - 1)^2 = 0$$
$$2\cos x - 1 = 0$$
$$\cos x = \frac{1}{2}$$
$$x = -60°, 60°$$

Question 10

$3 = 2\cos^2 x + 3\sin x$

$3 = 2(1 - \sin^2 x) + 3\sin x$

$3 = 2 - 2\sin^2 x + 3\sin x$

Reference Angle: $\frac{\pi}{6}$

$2\sin^2 x - 3\sin x + 1 = 0$

$(2\sin x - 1)(\sin x - 1) = 0$

$2\sin x - 1 = 0$ or $\sin x - 1 = 0$

$\sin x = \frac{1}{2}$ $\sin x = 1$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$ $x = \frac{\pi}{2}, \frac{5\pi}{2}$

$x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{5\pi}{2}, \frac{17\pi}{6}$

Exercise 8F

Question 1

$$\sin 2x \cos x + \cos 2x \sin x$$
$$= \sin(2x + x)$$
$$= \sin 3x$$

Question 2

$$\cos 3x \cos x + \sin 3x \sin x$$
$$= \cos(3x - x)$$
$$= \cos 2x$$

Question 3

$$\sin 5x \cos x - \cos 5x \sin x$$
$$= \sin(5x - x)$$
$$= \sin 4x$$

Question 4

$$\cos 7x \cos x - \sin 7x \sin x$$
$$= \cos(7x + x)$$
$$= \cos 8x$$

Question 5

$$\begin{aligned}\cos 15° &= \cos(45° - 30°) \\ &= \cos 45° \cos 30° + \sin 45° \sin 30° \\ &= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} \\ &= \frac{(\sqrt{3}+1)}{(2\sqrt{2})} \times \frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{\sqrt{2}(\sqrt{3}+1)}{4}\end{aligned}$$

Question 6

$$\begin{aligned}\tan 15° &= \tan(45° - 30°) \\ &= \frac{\tan 45° - \tan 30°}{1 + \tan 45° \times \tan 30°} \\ &= 1 - \frac{1}{\sqrt{3}} \div (1 + 1 \times \frac{1}{\sqrt{3}}) \\ &= \frac{\sqrt{3}-1}{\sqrt{3}} \div \frac{\sqrt{3}+1}{\sqrt{3}} \\ &= \frac{(\sqrt{3}-1)}{(\sqrt{3}+1)} \times \frac{(\sqrt{3}-1)}{(\sqrt{3}-1)} \\ &= \frac{3 - 2\sqrt{3} + 1}{3 - 1} \\ &= \frac{4 - 2\sqrt{3}}{2} \\ &= 2 - \sqrt{3}\end{aligned}$$

Question 7

$$\begin{aligned}\sin 75° &= \sin(45° + 30°) \\ &= \sin 45° \cos 30° + \cos 45° \sin 30° \\ &= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} \\ &= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} \\ &= \frac{\sqrt{3}+1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{\sqrt{2}(\sqrt{3}+1)}{4}\end{aligned}$$

Question 8

$$\begin{aligned}\cos 75° &= \cos(45° + 30°) \\ &\cos 45° \cos 30° - \sin 45° \sin 30° \\ &= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2} \\ &= \frac{\sqrt{3}-1}{2\sqrt{2}} \\ &= \frac{\sqrt{2}(\sqrt{3}-1)}{4}\end{aligned}$$

Question 9

$$\begin{aligned}\tan 75° &= \tan(45° + 30°) \\ &= \frac{\tan 45° + \tan 30°}{1 - \tan 45° \tan 30°} \\ &= 1 + \frac{1}{\sqrt{3}} \div \left(1 - 1 \times \frac{1}{\sqrt{3}}\right) \\ &= \frac{\sqrt{3}+1}{\sqrt{3}} \div \frac{\sqrt{3}-1}{\sqrt{3}} \\ &= \frac{(\sqrt{3}+1)}{(\sqrt{3}-1)} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)} \\ &= \frac{3 + 2\sqrt{3} + 1}{3 - 1} \\ &= \frac{4 + 2\sqrt{3}}{2} \\ &= 2 + \sqrt{3}\end{aligned}$$

Question 10

$$\begin{aligned}2\sin(\theta + 45°) &= 2[\sin\theta\cos 45° + \cos\theta\sin 45°] \\ &= 2\sin\theta\cos 45° + 2\cos\theta\sin 45°\end{aligned}$$

$$a\sin\theta + b\cos\theta = 2\sin\theta\cos 45° + 2\cos\theta\sin 45°$$

$$\begin{aligned}a &= 2\cos 45° & b &= 2\sin 45° \\ &= \frac{2 \times 1}{\sqrt{2}} & &= \sqrt{2} \\ &= \frac{2}{\sqrt{2}} \\ &= \sqrt{2}\end{aligned}$$

Question 11

$$8\cos(\theta - \frac{\pi}{3}) = 8\cos\theta\cos\frac{\pi}{3} + 8\sin\theta\sin\frac{\pi}{3}$$

$$c\sin\theta + d\cos\theta = 8\cos\theta\cos\frac{\pi}{3} + 8\sin\theta\sin\frac{\pi}{3}$$

$$d\cos\theta = 8\cos\theta\cos\frac{\pi}{3} \qquad c\sin\theta = 8\sin\theta\sin\frac{\pi}{3}$$

$$d = 8\cos\frac{\pi}{3} \qquad c = 8\sin\frac{\pi}{3}$$

$$= 8\times\frac{1}{2} \qquad = 8\times\frac{\sqrt{3}}{2}$$

$$= 4 \qquad = 4\sqrt{3}$$

Question 12

$$4\cos(\theta + 30°) = 4\cos\theta\cos 30° - 4\sin\theta\sin 30°$$

$$e\cos\theta = 4\cos\theta\cos 30° \qquad f\sin\theta = -4\sin\theta\sin 30°$$

$$e = 4\cos 30° \qquad f = -4\sin 30°$$

$$= 4\times\frac{\sqrt{3}}{2} \qquad = -4\times\frac{1}{2}$$

$$= 2\sqrt{3} \qquad = -2$$

Question 13

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$= \frac{5\sqrt{3}}{1} + \frac{(-\sqrt{3})}{4} \div (1 - 5\sqrt{3} \times (\frac{-\sqrt{3}}{4})$$

$$= \frac{20\sqrt{3} - \sqrt{3}}{4} \div (1 + \frac{15}{4})$$

$$= \frac{19\sqrt{3}}{4} \div \frac{19}{4}$$

$$= \frac{19\sqrt{3}}{4} \times \frac{4}{19}$$

$$= \sqrt{3}$$

$$\tan \frac{\pi}{3} = \sqrt{3}$$

$$\tan(\frac{\pi}{3} + \pi) = \sqrt{3}$$

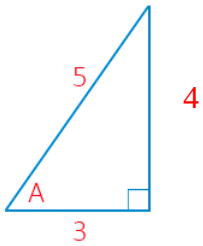
$$\therefore A + B = \frac{4\pi}{3}$$

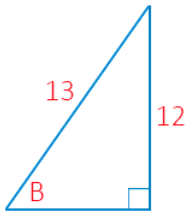
Question 14

$\sin A = \frac{4}{5} \qquad \cos B = \frac{5}{13}$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

We need to find $\cos A$ and $\sin B$





By Pythagoras, the missing length is 3

$\therefore \cos A = \frac{3}{5}$

By Pythagoras, the missing length is 5

$\therefore \sin B = \frac{12}{13}$

a

$$\begin{aligned}\sin(A+B) &= \sin A \cos B + \cos A \sin B \\ &= \frac{4}{5} \times \frac{5}{13} + \frac{3}{5} \times \frac{12}{13} \\ &= \frac{56}{65}\end{aligned}$$

b

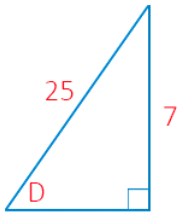
$$\begin{aligned}\cos(A-B) &= \cos A \cos B + \sin A \sin B \\ &= \frac{3}{5} \times \frac{5}{13} + \frac{4}{5} \times \frac{12}{13} \\ &= \frac{63}{65}\end{aligned}$$

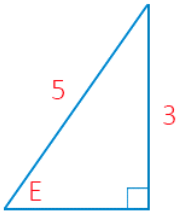
Question 15

$\sin D = \frac{7}{25}$ $\quad$ $\sin E = \frac{3}{5}$

$\sin(D-E) = \sin D\cos E - \cos D\sin E$

We need to find cos D and cos E





By Pythagoras, the missing length is 24

$\therefore \cos D = \frac{24}{25}$

By Pythagoras, the missing length is 4

$\therefore \cos E = \frac{4}{5}$

a

$$\begin{aligned}\sin(D-E) &= \sin D\cos E - \cos D\sin E \\ &= \frac{7}{25}\times\frac{4}{5} - \frac{24}{25}\times\frac{3}{5} \\ &= -\frac{44}{125}\end{aligned}$$

b

$$\begin{aligned}\cos(D+E) &= \cos D\cos E - \sin D\sin E \\ &= \frac{24}{25}\times\frac{4}{5} - \frac{7}{25}\times\frac{3}{5} \\ &= \frac{75}{125} \\ &= \frac{3}{5}\end{aligned}$$

Question 16

$$\begin{aligned}\sin\left(x+\frac{\pi}{2}\right) &= \sin x\cos\frac{\pi}{2} + \cos x\sin\frac{\pi}{2} \\ &= \sin x\times 0 + \cos x\times 1 \\ &= \cos x\end{aligned}$$

Question 17

a $\sin(x+2\pi) = \sin x \cos 2\pi + \cos x \sin 2\pi$
$= \sin x \times 1 + \cos x \times 0$
$= \sin x$

b $\sin(x-2\pi) = \sin x \cos 2\pi - \cos x \sin 2\pi$
$= \sin x \times 1 - \cos x \times 0$
$= \sin x$

Question 18

$\cos(x+2\pi) = \cos x \cos 2\pi - \sin x \sin 2\pi$
$= \cos x \times 1 - \sin x \times 0$
$= \cos x$

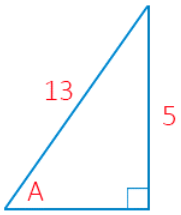
Question 19

$$\tan(x+\pi) = \frac{\tan x + \tan \pi}{1 - \tan x \tan \pi}$$
$$= \frac{\tan x + 0}{1 - \tan x \times 0}$$
$$= \frac{\tan x}{1}$$
$$= \tan x$$

Question 20

$$\tan(0-x) = \frac{\tan 0 - \tan x}{1 + \tan 0 \tan x}$$
$$= \frac{0 - \tan x}{1 + 0 \tan x}$$
$$= \frac{-\tan x}{1}$$
$$= -\tan x$$

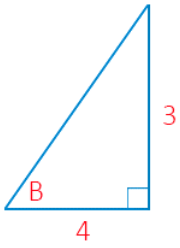
Question 21



Missing side 12

$\cos A = -\frac{12}{13}$

$\tan A = -\frac{5}{12}$



Missing side 5

$\sin B = \frac{3}{5}$

$\cos B = -\frac{4}{5}$

(Remember for obtuse angles sin > 0, cos < 0, tan < 0)

a

$$\begin{aligned}\sin(A+B) &= \sin A\cos B + \cos A\sin B \\ &= \frac{5}{13}\times\left(-\frac{4}{5}\right)+\left(-\frac{12}{13}\right)\times\frac{3}{5} \\ &= -\frac{56}{65}\end{aligned}$$

b

$$\begin{aligned}\cos(A-B) &= \cos A\cos B + \sin A\sin B \\ &= \left(-\frac{12}{13}\right)\times\left(-\frac{4}{5}\right)+\frac{5}{13}\times\frac{3}{5} \\ &= \frac{63}{65}\end{aligned}$$

c

$$\begin{aligned}\tan(A+B) &= \frac{\tan A+\tan B}{1-\tan A\tan B} \\ &= \left(-\frac{5}{12}+(-\frac{3}{4})\right)\div\left(1-\left(-\frac{5}{12}\right)\times\left(-\frac{3}{4}\right)\right) \\ &= \left(-\frac{14}{12}\right)\div\left(1-\frac{15}{48}\right) \\ &= -\frac{56}{33}\end{aligned}$$

Question 22

$$\sin x\cos\frac{\pi}{6}+\cos x\sin\frac{\pi}{6}=\sin\left(x+\frac{\pi}{6}\right)$$

$$\sin\left(x+\frac{\pi}{6}\right)=\frac{1}{\sqrt{2}} \qquad 0\le x\le 2\pi$$

$$x+\frac{\pi}{6}=\frac{\pi}{4},\frac{3\pi}{4} \qquad \frac{-\pi}{6}\le x-\frac{\pi}{6}\le\frac{11\pi}{6}$$

$$x=\frac{\pi}{4}-\frac{\pi}{6},\frac{3\pi}{4}-\frac{\pi}{6}$$

$$=\frac{\pi}{12},\frac{7\pi}{12}$$

Question 23

$$\cos x\cos 20°+\sin x\sin 20°=\cos(x-20°)$$

$$\cos(x-20°)=\frac{1}{2} \qquad 0°\le x\le 360°$$

$$(x-20°)=60°,\ 300°$$

$$x=80°,\ 320°$$

Question 24

$$\sin x\cos 70+\cos x\sin 70=0.5 \qquad -180°\le x\le 180°$$

$$\sin(x+70)=0.5 \qquad -110°\le x+70°\le 250°$$

$$x+70=30,150$$

$$x=-40°,80°$$

Question 25

$$\sin(x+30) = \cos x$$
$$\sin x\cos 30 + \cos x \sin 30 = \cos x$$
$$\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x = \cos x$$
$$\frac{\sqrt{3}}{2}\sin x = \frac{1}{2}\cos x$$
$$\sqrt{3}\sin x = \cos x$$
$$\sqrt{3}\tan x = 1$$
$$\tan x = \frac{1}{\sqrt{3}}$$
$$x = 30°, 210°$$

Miscellaneous exercise eight

Question 1

Amplitude: 5

Period: 2π

Question 2

Amplitude: 7

Period: 2π

Question 3

Amplitude: 3

Period: 2π

Question 4

Amplitude: 1

Period: $\frac{2\pi}{2} = \pi$

Question 5

Amplitude: 1

Period: $\frac{2\pi}{3}$

Question 6

Amplitude: 1

Period: $\frac{2\pi}{0.5} = 4\pi$

Question 7

Amplitude: 3

Period: $\frac{2\pi}{4} = \frac{\pi}{2}$

Question 8

Amplitude: 4

Period: $\frac{2\pi}{5}$

Question 9

Amplitude: 2

Period: $\frac{2\pi}{\pi} = 2$

Question 10

θ	$-\frac{3\pi}{4}$	$-\frac{2\pi}{3}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{4\pi}{3}$	$\frac{7\pi}{3}$	$\frac{9\pi}{4}$	11π
$\sin\theta$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	0
$\cos\theta$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	–1
$\tan\theta$	1	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\sqrt{3}$	1	0

Question 11

a $y = 3x - a \qquad m = 3$

$y = x - b \qquad m = 1$

Neither parallel or perpendicular

b $y = 0.5x + c \qquad m = 0.5$

$y = 0.5x + \dfrac{d}{2} \qquad m = 0.5$

Parallel

c $y = 0.5x + \dfrac{e}{2} \qquad m = 0.5$

$y = -2x + 1 \qquad m = -2$

Perpendicular as gradients multiply to -1

Question 12

a $\cos 50 = \dfrac{x^2 + 6.9^2 - 10.2^2}{2 \times 6.9 \times x}$

$x = -4.29, 13.16$

$x > 0 \therefore x = 13.2$

b $\dfrac{\sin 50}{10.2} = \dfrac{\sin \theta}{6.9}$

$\sin \theta = \dfrac{6.9 \sin 50}{10.2}$

$\theta = 31.2°$

Remaining angle $180 - 50 - 31.2 = 98.8°$

$\dfrac{x}{\sin 98.8} = \dfrac{10.2}{\sin 50}$

$x = 13.2 \text{ cm}$

Question 13

Smallest angle is opposite the smallest side

$$\cos\theta = \frac{33^2 + 55^2 - 27^2}{2 \times 33 \times 55}$$

$$\theta = 21°$$

Question 14

Using the null factor

a (2, 0), (3, 0), (–2, 0), (–7, 0)

b (–3, 0), (2, 0), (0, 0), (4, 0)

c (–3, 0), (2, 0), (3, 0)

d (2, 0)

e $2x^2 - 3x + 2 = 0$

$(2x + 1)(x - 2) = 0$

$2x + 1 = 0$ or $x - 2 = 0$

$x = -\frac{1}{2}$ $\quad$ $x = 2$

x-intercepts $(-\frac{1}{2}, 0), (2, 0), (7, 0)$

f $y = (x^2 - x - 30)(4x^2 - 8x - 21)$

$= (x - 6)(x + 50)(2x + 3)(2x - 7)$

$\therefore$ x-intercepts are $(-5, 0), (-\frac{3}{2}, 0), (\frac{7}{2}, 0), (6, 0)$

Question 15

$f_1 : y = k_1 x$ is linear and passes through the origin

$m = \dfrac{4-(-4)}{16} = \dfrac{1}{2}$

$\Rightarrow k = \dfrac{1}{2}$

f_2 is linear

$x + y + k_2 = 0 \quad \Rightarrow \qquad y = -x - k_2$

y-int $(0,10), (0,-k_2)$

$\Rightarrow k_2 = -10$

f_3 is quadratic with turning point at $(0,0)$

$\left.\begin{aligned} 4 &= k_3(1)^2 \\ 16 &= k_3(2)^2 \end{aligned}\right\} \quad k_3 = 4$

f_4 is a quadratic with a turning point at $(0,0)$

$y = (k_4 x)^2 = 4x^2$

$k_4 = \pm 2$

f_5 is quadratic

$y = k_5 x^2 + k_6 x + k_7$

k_7 is the $y-$intercept $\Rightarrow k_7 = 13$

$k_5(x-3)^2 - 5 = k_5 x^2 + k_6 x + 13$

When $x = 0$,

$9k_5 - 5 = 13$

$9k_5 = 18$

$k_5 = 2$

$2(x-3)^2 - 5$

$= 2(x^2 - 6x + 9) - 5$

$= 2x^2 - 12x + 13$

$\Rightarrow k_6 = -12$

f_6 is reciprocal

$xy = k_8$

$(-5) \times (-2) = k_8$

$k_8 = 10$

f_7 is a cubic with one x-intercept

$y = k_9(x + k_{10})^3 + k_{11}$

k_{11} is the vertical shift $\Rightarrow k_{11} = 1$

$y = k_9(x - 3)^2 + 1 \qquad \Rightarrow k_{10} = -3$

$0 = k_9(-1) + 1$

$-1 = -1k_9$

$k_9 = 1$

f_8 is a quadratic

$y = k_{12}(x + k_{13})^2 + k_{14}$

$(-k_{13}, k_{14})$ is the turning point

$k_{13} = -3$

k_{14} is the vertical translation $\Rightarrow k_{14} = -5$

$13 = k_{12}(0 - 3)^2 - 5$

$13 = 9k_{12} - 5$

$9k_{12} = 18$

$k_{12} = 2$

f_9 is another version for the cubic graph

$y = k_{15}x^3 + k_{16}x^2 + k_{17}x + k_{18}$

$k_{18} = 1$

$k_{15}x^3 + k_{16}x^2 + k_{17}x + 1 = 1(x - 3)^2 + 1$

$= x^3 - 9x^2 + 27x + 26$

$k_{15} = -1$

$k_{16} = -9$

$k_{17} = 27$

f_{10} is a sin graph with a mean value of 0

$y = k_{19}\sin k_{20}x$

The sin curve given is $5\sin 2x$

$k_{19}\sin k_{20}x = 5\sin 2x$

$k_{19} = 5$

$k_{20} = 2$

f_{11} is cosine graph with a horizontal translation and a mean value of 0

$y = k_{21}\cos k_{22}(x - k_{23})$

$k_{23} = 45°$

$k_{21} = 5$

$k_{22} = 2$

$f_{12} = k_{24} + k_{25}\sin k_{26}x$

k_{24} is a vertical shift $\Rightarrow k_{24} = 5$

$k_{25} < 0$ due to shape of sin curve

As the amplitude is 4, $k_{25} = -4$

k_{26} is the number of complete cycles in 2π

$\Rightarrow k_{26} = 3$

Question 16

a $2x^3 + x^2 - 22x + 24 = (x - 2)(ax^2 + bx + c)$

$a = 2$

$c = -12$

b $(x - 2)(2x^2 + bx - 12) = 2x^3 + bx^2 - 12x - 4x^2 - 2bx + 24$

$2x^3 + x^2 - 22x + 24 = 2x^3 + (b - 4)x^2 - (12 + 2b)x + 24$

$b - 4 = 1$

$b = 5$

$(x - 2)(2x^2 + 5x - 12) = (x - 2)(2x - 3)(x + 4)$

$\therefore$ x-intercepts are (2, 0), (1.5, 0), (–4, 0)

Question 17

a $y = f(x) + 5$ represents a vertical translation of 5

$\therefore$ max $(-1, 26)$, min$(3, -6)$

b $y = f(x) - 5$ represents a vertical translation of -5

$\therefore$ max $(-1, 16)$, min$(3, -16)$

c $y = -f(x)$ represents a reflection in the y-axis

$\therefore$ max $(1, 21)$, min$(-3, -11)$

d $y = -f(x)$ represents a reflection about the x-axis

$\therefore$ max $(3, 11)$, min $(-1, -21)$

e $y = 3f(x)$ represents a vertical dilation by a factor of 3

$\therefore$ max $(-1, 63)$, min $(3, -33)$

f $y = f(2x)$ represents a horizontal dilation of factor $\frac{1}{2}$

$\therefore$ max $(-\frac{1}{2}, 21)$, min $(1.5, -11)$

Question 18

a x-intercept when $y = 0$

$0 = (x+1)(x-2)(x-5)$

$x = -1, 2, 5$

$\Rightarrow (-1, 0), (2, 0), (5, 0)$

b y-intercept when $x = 0$

$y = (0+1)(0-2)(0-5)$

$x = 10$

$\Rightarrow (10, 0)$

c $y = (x+1)(x-2)(x-5)$

$(1, a)$ is on the curve

$a = (1+1)(1-2)(1-5)$

$a = 8$

d $y = (x+1)(x-2)(x-5)$

$(3, b)$ is on the curve

$b = (3+1)(3-2)(3-5)$

$= -8$

e $y = (x+1)(x-2)(x-5)$

$(4, c)$ is on the curve

$c = (4+1)(4-2)(4-5)$

$= -10$

f $y = (x-3)^2 - 4$

Turning point (3, 4) and is a minimum

g y-intercept when $x = 0$

$y = (0-3)^2 - 4$

$= 5$

$\Rightarrow (0, 5)$

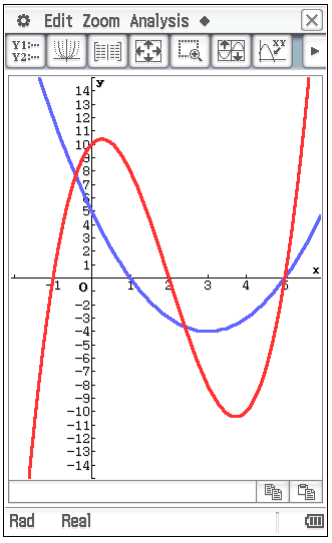
h $y = (x-3)^2 - 4$

$(5, d)$ is on the curve

$d = (5-3)^2 - 4$

$= 0$

i



Points of intersection $x = -0.4, 2.4, 5$

Question 19

$$\cos\theta = \frac{10^2 + 10^2 - 16^2}{2 \times 10 \times 10}$$

$\theta = 1.855$ rads

Area of segment

$$\frac{1}{2} \times 10^2 \times (1.855 - \sin 1.855)$$

$= 44.756$

Shaded area

$\pi \times 10^2 - 44.756$

$= 269.4 \text{ cm}^2$