SADLER MATHEMATICS METHODS UNIT 3

WORKED SOLUTIONS

Chapter 8 Discrete random variables

Exercise 8A

Question 1

a Continuous

b Discrete

c Continuous

d Discrete

e Discrete

f Continuous

g Continuous

Question 2

No (Probabilities do not add to 1)

Question 3

No (Probabilities do not add to 1)

Question 4

No (Probability cannot be negative)

Question 5

Yes

Question 6

$$\begin{aligned} k &= 1 - 0.6 \\ &= 0.4 \end{aligned}$$

Question 7

$$\begin{aligned} k &= 1 - 0.95 \\ &= 0.05 \end{aligned}$$

Question 8

$$\begin{aligned} 10k &= 1 \\ k &= 0.1 \end{aligned}$$

Question 9

$$\begin{aligned} 2.25k + 2.8 &= 1 \\ 2.25k &= -1.8 \\ k &= -0.8 \end{aligned}$$

Question 10

x	0	1	2
$\mathbf{P}(X = x)$	0.25	0.5	0.25

(Construct a two-way table or tree diagram if needed and count the number of tails obtained.)

Question 11

a $\mathrm{P}(X = 0) = 0.2$

b $\mathrm{P}(X \geq 1) = 0.8$

c $\mathrm{P}(2 < X \leq 4) = 0.2$

d $\mathrm{P}(X = 1 \mid X \geq 1) = \dfrac{0.4}{0.8} = 0.5$

e $\mathrm{P}(X > 4 \mid X \geq 2) = \dfrac{0.1}{0.4} = 0.25$

f $\mathrm{P}(X \leq 4 \mid X \geq 2) = \dfrac{0.3}{0.4} = 0.75$

Question 12

a $\mathrm{P}(X > 2) = 0.4$

b $\mathrm{P}(X \geq 3) = 0.4$

c $\mathrm{P}(1 < X < 4) = 0.5$

d $\mathrm{P}(X = 3 \mid X > 2) = \dfrac{0.2}{0.4} = 0.5$

e $\mathrm{P}(X = 5 \mid X \geq 3) = \dfrac{0.1}{0.4} = 0.25$

f $\mathrm{P}(X < 4 \mid X \geq 3) = \dfrac{0.2}{0.4} = 0.5$

Question 13

x	0	1	2	3	4	5	6	7	8	9	10
$\mathbf{P}(X = x)$	0.005	0.015	0.055	0.175	0.375	0.625	0.825	0.945	0.985	0.995	1

Question 14

x	0	1	2	3	4	5
$\mathbf{P}(X = x)$	0.04	0.16	0.3	0.3	0.16	0.04

Question 15

$P(\text{No heads}) = 0.6 \times 0.6 = 0.36$

$P(2 \text{ heads}) = 0.4 \times 0.4 = 0.16$

$P(1 \text{ head}) = 1 - 0.3 - 0.16 = 0.48$

x	0	1	2
$\mathbf{P}(X = x)$	0.16	0.48	0.36

Question 16

$P(\text{ no heads}) = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$

$P(1 \text{ head}) = \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2 \times 3 = \frac{4}{9}$

$P(2 \text{ heads}) = \left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right) \times 3 = \frac{2}{9}$

$P(3 \text{ heads}) = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$

x	0	1	2	3
$\mathbf{P}(X = x)$	$\frac{8}{27}$	$\frac{4}{9}$	$\frac{2}{9}$	$\frac{1}{27}$

Question 17

$$P(\text{one red}) = \underset{RBB}{\frac{3}{5}\times\frac{2}{4}\times\frac{1}{3}} + \underset{BRB}{\frac{2}{5}\times\frac{3}{4}\times\frac{1}{3}} + \underset{BBR}{\frac{2}{5}\times\frac{1}{4}\times\frac{3}{3}}$$
$$= 0.3$$

$$P(\text{two red}) = \underset{RRB}{\frac{3}{5}\times\frac{2}{4}\times\frac{2}{3}} + \underset{BRR}{\frac{2}{5}\times\frac{3}{4}\times\frac{2}{3}} + \underset{RBR}{\frac{3}{5}\times\frac{2}{4}\times\frac{2}{3}}$$
$$= 0.6$$

$$P(\text{all red}) = \underset{RRR}{\frac{3}{5}\times\frac{2}{4}\times\frac{1}{3}}$$
$$= 0.1$$

x	1	2	3
$\mathbf{P(X = x)}$	0.3	0.6	0.1

Question 18

$$k + 2k + 3k + 4k + 5k = 1$$
$$15k = 1$$
$$k = \frac{1}{15}$$

x	1	2	3	4	5
$\mathbf{P(X = x)}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{4}{15}$	$\frac{1}{3}$

a $\mathrm{P}(X = \text{even}) = \frac{6}{15} = \frac{2}{5}$

b $\mathrm{P}(X < 2) = \frac{1}{15}$

c $\mathrm{P}(X > 2) = \frac{12}{15} = \frac{4}{5}$

Question 19

$$\begin{aligned} k(5-1)+k(5-2)+k(5-3)+k(5-4) &= 1 \\ 4k+3k+2k+k &= 1 \\ 10k &= 1 \\ k &= 0.1 \end{aligned}$$

x	1	2	3	4
$\mathbf{P}(X = x)$	0.4	0.3	0.2	0.1

a $\mathrm{P}(X = \text{even}) = 0.4$

b $\mathrm{P}(X \leq 2) = 0.7$

c $\mathrm{P}(X \geq 2) = 0.6$

Question 20

a $1-0.2-0.4-0.1=0.3$

b $\text{P}(2 \text{ then } 4)=0.2\times0.3=0.12$

c $\text{P}(2 \text{ and } 4 \text{ in any order})=0.2\times0.3\times2=0.24$

d Total of 6 : 3 and 3 or 2 and 4 or 4 and 2

$\text{P}(3 \text{ then } 3)=0.1\times0.1=0.01$

$\text{P}(\text{total of } 6)=0.01+0.24$

$=0.25$

e $\text{P}(2 \text{ then } 4 | \text{total of } 6)=\dfrac{\text{P}(2 \text{ then } 4)}{\text{P}(\text{total of } 6)}$

$=\dfrac{0.12}{0.25}$

$=0.48$

f $\text{P}(4,3,2)=0.3\times0.1\times0.4=0.012$

g $\text{P}(4,\ 3,\ 2 \text{ in any order})=0.3\times0.1\times0.4\times3!=0.072$

h Total of 10 in 3 spins : 4, 4, 2 or 4, 3, 3

$\text{P}(4,\ 4,\ 2 \text{ in any order})=0.3\times0.3\times0.4\times\dfrac{3!}{2!}=0.108$

$\text{P}(4,\ 3,\ 3 \text{ in any order})=0.3\times0.1\times0.1\times\dfrac{3!}{2!}=0.009$

$\text{P}(\text{total of } 10)=0.108+0.009=0.117$

i $\text{P}(1,\ 1,\ 1 \text{ or } 2,\ 2,\ 2 \text{ or } 3,\ 3,\ 3 \text{ or } 4,\ 4,\ 4)=0.2^3+0.4^3+0.1^3+0.3^3$

$=0.1$

Question 21

$$\begin{aligned}
\mathrm{P}(X=0) &= \frac{45\times44\times43\times42}{50\times49\times48\times47} \\
&= 0.64696 \\
\mathrm{P}(X=1) &= \frac{45\times44\times43\times5}{50\times49\times48\times47}\times\binom{4}{1} \\
&= 0.30808 \\
\mathrm{P}(X=2) &= \frac{45\times44\times5\times4}{50\times49\times48\times47}\times\binom{4}{2} \\
&= 0.04299 \\
\mathrm{P}(X=3) &= \frac{45\times5\times4\times3}{50\times49\times48\times47}\times\binom{4}{3} \\
&= 0.00195 \\
\mathrm{P}(X=4) &= \frac{5\times4\times3\times2}{50\times49\times48\times47} \\
&= 0.00002
\end{aligned}$$

x	0	1	2	3	4
$\mathbf{P}(X = x)$	0.64696	0.30808	0.04299	0.00195	0.00002

Exercise 8B

Question 1

$k = 1 - (0.35 \times 2 + 0.15 + 0.05)$

$= 0.1$

$\mathrm{E}(X) = 1 \times 0.1 + 2 \times 0.35 + 3 \times 0.35 + 4 \times 0.15 + 5 \times 0.05$

$= 2.7$

Question 2

$3k + 0.4 = 1$

$2k = 0.6$

$k = 0.2$

$\mathrm{E}(X) = 0 \times 0.1 + 5 \times 0.1 + 10 \times 0.1 + 15 \times 0.1 + 20 \times 0.2 + 25 \times 0.4$

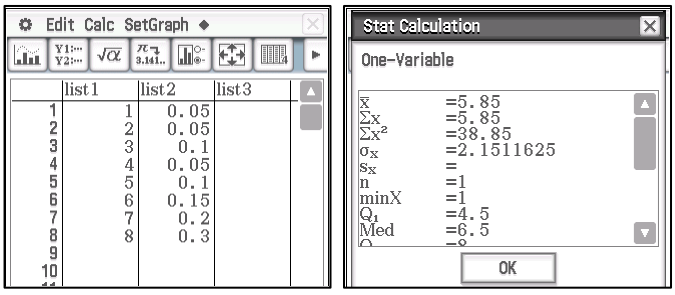
$= 17$

Question 3

$20k = 1$

$k = 0.05$

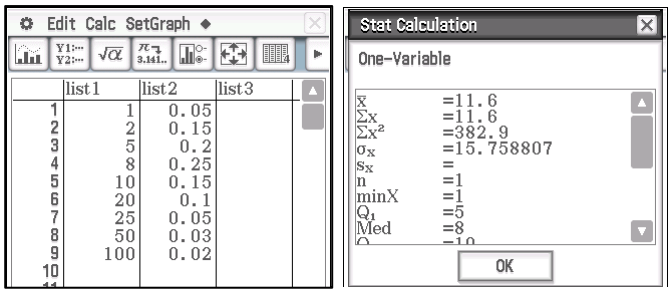
$\mathrm{E}(X) = 5.85$



Question 4

$k = 0.2$

$\mathrm{E}(X) = 11.6$



Question 5

$0.3+p+0.2+q+0.1=1$

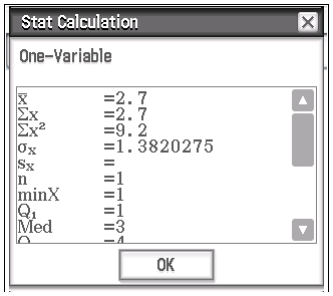
$$p+q=0.4 \quad \rightarrow \text{Equation}1$$

$0.3+2p+0.6+4q+0.5=2.7$

$$2p+4q=1.3 \quad \rightarrow \text{Equation } 2$$

Solving simultaneously

$p=0.15, q=0.25$



$\text{Var}(X)=1.3820275^2$

$=1.91$

Question 6

$p+q=0.5$

$$0+\frac{1}{36}+2\times\frac{1}{18}+3\times\frac{1}{18}+4\times\frac{1}{12}+5\times\frac{1}{12}+6\times\frac{1}{6}+7p+8q=\frac{52}{9}$$

$$7p+8q=\frac{67}{18}$$

Solving simultaneously

$$p=\frac{5}{18}, q=\frac{2}{9}$$

Question 7

$\text{E}(X)=10,\ \text{SD}(X)=1.5$

a If scores are increased by 5, then $\text{E}(X)=15$.

b Increasing all score by 5 does not alter the spread.

$\Rightarrow \text{SD}(X)=1.5$

c $\text{E}(3X-4)=3\times10-4=26$

d $\text{SD}(3X-4)=3\times1.5=4.5$

Question 8

a

x	10	20	30	40	50
$\mathrm{P}(X = x)$	0.3	0.2	0.2	0.2	0.1

$\mathrm{E}(X) = 26$

$\mathrm{Var}(X) = (13.56466)^2 = 184$

b $\mathrm{E}(X + 3) = 29$

c $\mathrm{E}(2X) = 52$

d $\mathrm{E}(2X + 3) = 55$

e $\mathrm{Var}(X + 3) = 184$

f $\mathrm{Var}(2X) = 4 \times 184$

$= 736$

g $\mathrm{Var}(2X + 3) = 736$

Question 9

x	1	2	3	4	5
$\mathrm{P}(X = x)$	0.2	0.2	0.2	0.2	0.2

$\mathrm{E}(X) = 3$

$\mathrm{Var}(X) = 2$

Question 10

x	0	15	30
$\mathrm{P}(X = x)$	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{1}{6}$

$$\mathrm{E}(X) = 0 \times \frac{2}{3} + 15 \times \frac{1}{6} + 30 \times \frac{1}{6}$$
$$= 7.5$$

They should charge \$8 per game.

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Question 11

Let \$ X be the amount of money given back on a single play

x	0	5	10	c
$\mathbf{P}(X = x)$	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{4}$

$$\mathrm{E}(X) = 0 + \frac{5}{8} + \frac{10}{8} + \frac{1}{4}c$$

$$\text{Breakeven} \rightarrow \text{Cost} = \mathrm{E}(X)$$

$$c = \frac{15}{8} + \frac{1}{4}c$$

$$\frac{3}{4}c = \frac{15}{8}$$

$$c = 2.5$$

$\therefore$ Cost should be at least \$2.50 per game.

Question 12

a Mean value = expected value

$$\frac{1}{8}(1 + \ldots + 8) = \frac{36}{8} = 4.5$$

$$\therefore \mathrm{E}(X) = 4.5$$

b $$\text{Mean value of } Y = \frac{1}{8}(1 + 4 + 9 + \ldots + 49 + 64)$$

$$= \frac{1}{8} \times 204$$

$$\mathrm{E}(Y) = 25.5$$

c $$\text{Mean value of } Z = \frac{1}{8}\left(1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{7} + \frac{1}{8}\right)$$

$$= \frac{1}{8}\left(\frac{761}{280}\right)$$

$$\mathrm{E}(Z) = \frac{761}{2240}$$

Question 13

Let $\$X$ represent the prize money

+	0	1	1	3	5
0	0	1	1	3	5
1	1	2	2	4	6
1	1	2	2	4	6
3	3	4	4	6	8
5	5	6	6	8	10

x	0	1	2	3	4	5	6	8	10
$\mathbf{P}(X = x)$	0.04	0.16	0.16	0.08	0.16	0.08	0.2	0.08	0.04

a $\mathrm{P}(X > 6) = 0.12$

b $x = 4$

c

$100 \times 5 = \$500$

$\mathrm{E}(X) = 4$ (prize money per game)

$100 \times 4 = \$400$

After 100 games, the organisers would expect to be "up" by \$100.

Question 14

$\mathrm{E}(X) = 1.85$ cars per fortnight

Scheme 1:

$1.85 \times 250 + 500 = \$962.50$

Scheme 2:

$1.85 \times 475 = \$878.75$

$\therefore$ Scheme 1 is a better choice as the expected fortnightly earnings is higher.

Question 15

a $\mathrm{E}(X) = \$1340$

b $\mathrm{E}(Y) = \$1270$

c Scheme A does have a higher expected value but it also has a 50% chance of losing some of the investment while Scheme B has a 0.1 chance of losing less money.
I would advise Scheme B.

Miscellaneous exercise eight

Question 1

a $N \approx \dfrac{100\ 000}{1+499e^{0}}$

$= 200$

b $N \approx \dfrac{100\ 000}{1+499e^{-0.08\times 5}}$

$= 298$

c $N \approx \dfrac{100\ 000}{1+499e^{-0.08\times 10}}$

$= 444.02 \approx 444$

d $1+499e^{-0.08t} = 1+\dfrac{499}{e^{0.08t}}$

As $t \to \infty,\ e^{0.08t} \to \infty$ and $\dfrac{499}{e^{t}} \to 0. \therefore N$ approaches 100 000.

Question 2

a $-\dfrac{6}{x^2}$

b $6\times -\dfrac{1}{2}x^{-\frac{3}{2}} = -\dfrac{3}{\sqrt{x^3}}$

c $10x - e^{x}$

d $e^{3x^2}\times 6x = 6xe^{3x^2}$

e $e^{3x^2+1}\times 6x = 6xe^{3x^2+1}$

f $(2x-3)5(2x+1)^4\times 2 + (2x+1)^5\times 2$

$= 2(2x+1)^4\left[5(2x-3)+(2x+1)\right]$

$= 2(2x+1)^4\left[10x-15+2x+1\right]$

$= 2(2x+1)^4(12x-14)$

$= 4(2x+1)^4(6x-7)$

g $10\cos x$

h $\cos 10x\times 10 = 10\cos 10x$

Question 3

$3x^2 - 5$

Question 4

a X is not a uniform discrete random variable as each of the probabilities are different.

b X is a discrete random variable as the possible values of X are discrete values and the probability of each one is the same, $\frac{1}{6}$.

c X is not a uniform discrete random variable because the variable involved (height) is continuous, not discrete.

Question 5

$$\frac{5x^2}{x-1} = 5x + 3$$
$$5x^2 = (5x+3)(x-1)$$
$$= 5x^2 - 2x - 3$$
$$2x = -3$$
$$x = 1.5$$
$$y = 5(-1.5) + 3$$
$$= -4.5$$

$\therefore$ Point of intersection $(-1.5, -4.5)$

$$\frac{d}{dx}\left(\frac{5x^2}{x-1}\right) = \frac{(x-1)10x - 5x^2 \times 1}{(x-1)^2}$$
$$= \frac{10x^2 - 10x - 5x^2}{(x-1)^2}$$
$$= \frac{5x^2 - 10}{(x-1)^2}$$
$$= \frac{5x(x-2)}{(x-1)^2}$$

When $x = -1.5$,
$$\frac{dy}{dx} = \frac{5(-1.5)(-3.5)}{(-2.5)^2}$$
$$= 4.2$$

Question 6

$$\frac{dy}{dx} = x^2 \times e^{2x} \times 2 + e^{2x} \times 2x$$
$$= 2xe^{2x}(x+1)$$

When $x = 1$,

$$\frac{dy}{dx} = 2(1) \times e^2 \times 2$$
$$= 4e^2$$

Question 7

$$\frac{dy}{dx} = \frac{(x-2)2x - x^2 \times 1}{(x-2)^2}$$
$$= \frac{2x^2 - 4x - x^2}{(x-2)^2}$$
$$= \frac{x^2 - 4x}{(x-2)^2}$$
$$= \frac{x(x-4)}{(x-2)^2}$$

When $x = 3$,

$$\frac{dy}{dx} = \frac{3 \times (-1)}{12}$$
$$= -3$$

$\therefore$ Gradient of the normal is $\frac{1}{3}$.

Equation of the normal is of the form $y = \frac{1}{3}x + c$

using (3, 9)

$$9 = \frac{1}{3}(3) + c$$
$$c = 8$$

$\therefore$ Equation of normal is $y = \frac{1}{3}x + 8$

$$3y = x + 24$$

Question 8

a $\int_0^2 10x^4 dx$

$$= \left[\frac{10x^5}{5}\right]_0^2$$
$$= \left[2x^5\right]_0^2$$
$$= 2 \times 2^5 - 0$$
$$= 64$$

b $\int_2^4 2dx$

$$= \left[2x\right]_2^4$$
$$= \left[8 - 4\right]$$
$$= 4$$

c $\int_2^3 (2 + 6x)dx$

$$= \left[2x + 3x^2\right]_2^3$$
$$= (6 + 3 \times 9) - (4 + 3 \times 4)$$
$$= 33 - 16$$
$$= 17$$

Question 9

$\int_0^{\frac{\pi}{2}} \sin x dx$

$= \left[-\cos x\right]_0^{\frac{\pi}{2}}$

$= -\cos\frac{\pi}{2} - (-\cos 0)$

$= 0 - (-1)$

$= 1$

$\therefore$ Area $= 1$ unit^2

Question 10

a As $t \to \infty$, $\dfrac{e}{e^{0.13t}} \to 0$

$\therefore V \to 75$

$\therefore$ Terminal velocity = 75 m/s

b **i** $a = \dfrac{dV}{dt}$

$$\frac{dV}{dt} = 75(-(-0.13)e^{-0.13t})$$
$$= 75(0.13e^{-0.13t})$$
$$= \frac{39}{4}e^{-0.13t}$$

When $t = 5$,

$$a = \frac{39}{4}e^{-0.13\times 5}$$
$$= 5.09 \text{ m/s}^2$$

ii When $t = 20$,

$$a = \frac{39}{4}e^{-0.13\times 20}$$
$$= 0.72 \text{ m/s}^2$$

Question 11

$$f''(x) = 20(3-x)^3 + 6x - 6$$
$$f'(x) = -5(3-x)^4 + 3x^2 - 6x + c$$
$$f'(1) = -5(2)^4 + 3 - 6 + c = -83$$
$$c = 0$$
$$\therefore f'(x) = -5(3-x)^4 + 3x^2 - 6x$$

$$f(x) = (3-x)^5 + x^3 - 3x^2 + c$$
$$f(1) = 2^5 + 1^3 - 3 + c = 28$$
$$c + 30 = 28$$
$$c = -2$$
$$\therefore f(x) = (3-x)^5 + x^3 - 3x^2 - 2$$

Question 12

$$y = \frac{x^3 \times x^4}{x^2}$$

$$\frac{dy}{dx} = \frac{x^2(x^3 \times 4x^3 + x^4 \times 3x^2) - x^3 \times x^4 \times 2x}{x^4}$$

$$= \frac{x^2(4x^6 + 3x^6) - 2x^8}{x^4}$$

$$= \frac{7x^8 - 2x^8}{x^4}$$

$$= \frac{5x^8}{x^4}$$

$$= 5x^4$$

Question 13

$$\frac{dy}{dx} = \frac{2x \times e^x - e^x \times 2}{4x^2}$$

$$0 = \frac{e^x(x-1)}{2x^2}$$

$e^x \neq 0,\ x = 1$

When $x = 1$,

$$y = \frac{e^1}{2}$$

$\therefore (1,\ 0.5e)$

Question 14

$$\frac{dy}{dx} = e^x(-\sin x) + \cos x e^x$$

$$0 = e^x(\cos x - \sin x)$$

$e^x \neq 0,\ \cos x - \sin x = 0$

$$\cos x = \sin x$$

$$\tan x = 1$$

$$\therefore x = -\frac{7\pi}{4}, -\frac{3\pi}{4}, \frac{\pi}{4}, \frac{5\pi}{4}$$

Question 15

a $\int 4x\,dx$

$= \frac{4x^2}{2} + c$

$= 2x^2 + c$

b $\int 6e^{2x} dx$

$= 3\int 2e^{2x} dx$

$= 3e^{2x} + c$

c $\int \frac{d}{dx}(x^2 + e^x) dx$

$= x^2 + e^x + c$

d $\int \frac{d}{dx}(x^2 e^x) dx$

$= x^2 e^x + c$

Question 16

a $k + 2k + 4k + k + 4k = 1$

$12k = 1$

$k = \frac{1}{12}$

b $\mathrm{P}(X = 3) = 4k$

$= 4 \times \frac{1}{12}$

$= \frac{1}{3}$

c $\mathrm{P}(X > 3) = \mathrm{P}(X = 4) + \mathrm{P}(X = 5)$

$= \frac{1}{12} + \frac{1}{3}$

$= \frac{5}{12}$

d $\mathrm{P}(X \geq 3) = \mathrm{P}(X = 3) + \mathrm{P}(X > 3)$

$= \frac{1}{3} + \frac{5}{12}$

$= \frac{3}{4}$

e $\mathrm{P}(X = 3 \mid X > 3) = 0$

If $X > 3$, it cannot be equal to 3.

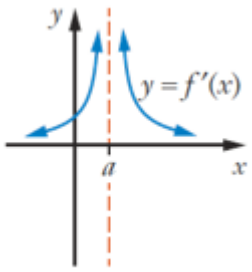
f $$\mathrm{P}(X = 3 \mid X \geq 3) = \frac{\mathrm{P}(X = 3)}{\mathrm{P}(X \geq 3)}$$
$$= \frac{1}{3} \div \frac{3}{4}$$
$$= \frac{4}{9}$$

g $$\mathrm{E}(X) = \sum x_i \mathrm{P}_i$$
$$= 1 \times \frac{1}{12} + 2 \times \frac{2}{12} + 3 \times \frac{4}{12} + 4 \times \frac{1}{12} + 5 \times \frac{4}{12}$$
$$= \frac{1 + 4 + 12 + 4 + 20}{12}$$
$$= \frac{41}{12}$$

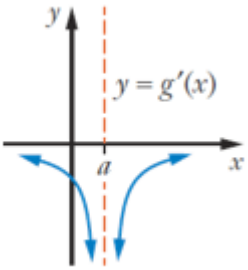
h $\mathrm{SD}(X) = 1.32$

Question 17

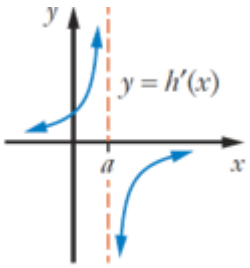
a



b



c



Question 18

a $\int_0^{\frac{5\pi}{6}} (0.5 - \sin x) dx$

$= \left[0.5x + \cos x\right]_0^{\frac{5\pi}{6}}$

$= \left[\frac{1}{2} \times \frac{5\pi}{6} + \cos\frac{5\pi}{6}\right] - \left[0 + \cos 0\right]$

$= \frac{5\pi}{12} - \frac{\sqrt{3}}{2} - 1$

$= \frac{5\pi - 6\sqrt{3} - 12}{12}$

b As $\int_0^{\frac{5\pi}{6}} (0.5 - \sin x)\, dx < 0$

$\left|\int_0^{\frac{5\pi}{6}} (0.5 - \sin x) dx\right| = -1 \times \int_0^{\frac{5\pi}{6}} (0.5 - \sin x) dx$

$= -\left[\frac{5\pi - 6\sqrt{3} - 12}{12}\right]$

$= \frac{12 + 6\sqrt{3} - 5\pi}{12}$

c $0.5 - \sin x = 0$

$$\sin x = \frac{1}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\therefore \int_0^{\frac{\pi}{6}} (0.5 - \sin x)dx - \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (0.5 - \sin x)dx$$

$$= \left[\frac{1}{2}x + \cos x\right]_0^{\frac{\pi}{6}} - \left[\frac{1}{2}x + \cos x\right]_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$$

$$= \left(\frac{1}{2} \times \frac{\pi}{6} + \cos\frac{\pi}{6}\right) - \left(\frac{1}{2} \times 0 + \cos 0\right) - \left(\left(\frac{1}{2} \times \frac{5\pi}{6} + \cos\frac{5\pi}{6}\right) - \left(\frac{1}{2} \times \frac{\pi}{6} + \cos\frac{\pi}{6}\right)\right)$$

$$= \frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1 - \left(\left(\frac{5\pi}{12} - \frac{\sqrt{3}}{2}\right) - \left(\frac{\pi}{12} + \frac{\sqrt{3}}{2}\right)\right)$$

$$= \frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1 - \frac{5\pi}{12} + \frac{\sqrt{3}}{2} + \frac{\pi}{12} + \frac{\sqrt{3}}{2}$$

$$= -\frac{3\pi}{12} - 1 + \frac{3\sqrt{3}}{2}$$

$$= \frac{6\sqrt{3} - \pi - 4}{4} \text{ units}^2$$