SADLER MATHEMATICS METHODS UNIT 1

WORKED SOLUTIONS

Chapter 9 Sets and probability

Exercise 9A

Question 1

a $\frac{1}{2}$

b $\frac{1}{2}$

c $\frac{1}{2}$

d $\frac{1}{3}$

e $\frac{2}{3}$

Question 2

a $\frac{1}{2}$

b $\frac{1}{2}$

c $\frac{5}{12}$

d $\frac{1}{18}$

e $\frac{5}{18}$

f $\frac{13}{18}$

Question 3

a 0.3

b 0.7

c 0.5

d 0.8

Question 4

a $\frac{1}{8}$

b $\frac{1}{8}$

c $\frac{3}{8}$

d $\frac{1}{2}$

e $\frac{1}{8}$

f $\frac{1}{4}$

Question 5

Sum of $1+2+3+4+5+6=21$

Number on bottom	1	2	3	4	5	6
Sum of visible numbers	20	19	18	17	16	15

a $\text{P(the total obtained is less than 15)} = 0$

b $\text{P(the total obtained is more than 15)} = \frac{5}{6}$

c $\text{P(the total obtained is divisible by 3)} = \frac{1}{3}$

d $\text{P(the total obtained is divisible by 5)} = \frac{1}{3}$

e $\text{P(the total obtained is divisible by both 3 and 5)} = \frac{1}{6}$

Question 6

a $\frac{1}{10}$

b $\frac{3}{20}$

c $\frac{47}{100}$

d $\frac{12}{25}$

e $\frac{7}{10}$

f $\frac{11}{50}$

Question 7

a 0.327

b 0.672

Question 8

a 7

b 9

c 10

d 3

e $\{8, 9, 10\}$

f $\{1, 3, 5, 7, 9\}$

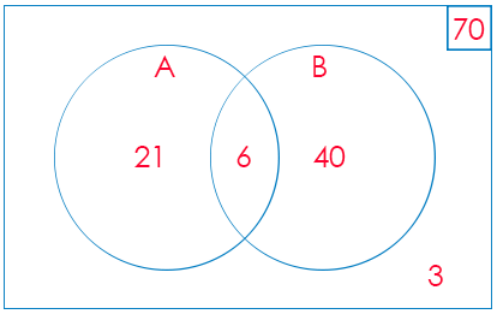
g $\{9\}$

h $\{1, 3, 5, 7, 8, 9, 10\}$

Question 9

a $n(A \cup B) = 67$

b $n(\overline{A \cup B}) = 3$

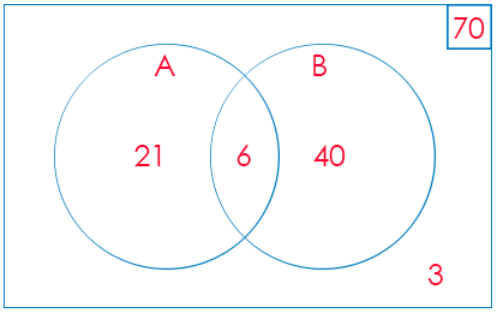


Question 10

$46 - 25 = 21$

$42 + 21 = 63$

$\therefore 80 - 63 = 17$

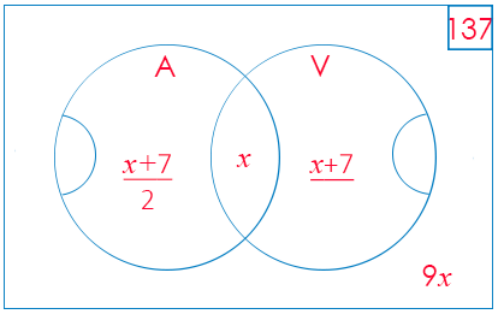


Question 11

$$x + 7 + 5 + 2x + 9 = 72$$
$$3x + 21 = 72$$
$$3x = 51$$
$$x = 17$$

Question 12

$$\frac{x+7}{2} + x + x + 7 + 9x = 137$$
$$\frac{x+7}{2} + 11x = 130$$
$$x + 7 + 22x = 260$$
$$23x = 253$$
$$x = 11$$



Question 13

$$n(\overline{B \cup C}) = x + 4 = 11$$
$$x = 7$$

$$n(U) = x + 4 + x + 1 + 5 + 3x + 3 + 2x - 1 + 4$$
$$= 7x + 16$$

a If $x = 7$, $n(U) = 49 + 16$
$= 65$

b $n(A \cap B) = 4 + x + 1$
$= 4 + 7 + 1$
$= 12$

c $n(A \cap B \cap C) = x + 1$
$= 8$

Question 14

a 0.6

b 0.2

c 0.1

d 0.7

e 0.9

f 0.3

Question 15

a $\frac{10}{19}$

b $\frac{12}{19}$

c $\frac{9}{19}$

d $\frac{10}{19}$

e $\frac{1}{19}$

f $\frac{3}{19}$

g $\frac{13}{19}$

h $\frac{16}{19}$

i $\frac{18}{19}$

Question 16

a $\frac{17}{40}$

b $\frac{13}{40}$

c 0

d $\frac{3}{4}$

e $\frac{1}{4}$

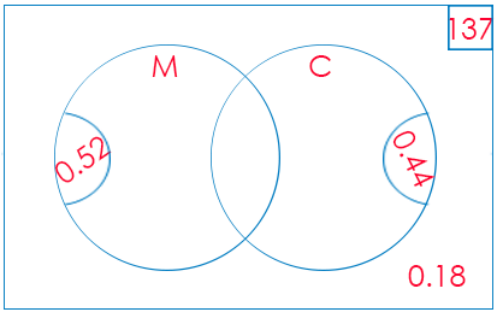
f 1

Question 17

$$\begin{aligned} P(A \cap B) &= P(A) + P(B) - P(A \cup B) \\ &= 0.3 + 0.5 - 0.6 \\ &= 0.2 \end{aligned}$$

Question 18

$$\begin{aligned} P(M \cap C) &= P(M) + P(C) - P(M \cup C) \\ &= 0.52 + 0.44 - 0.82 \\ &= 0.14 \end{aligned}$$

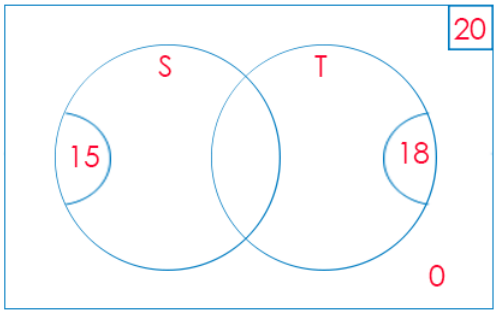


a $P(F \cap C) = 0.3$

b $P(M \cap \overline{C}) = 0.38$

Question 19

$15+18-20=13$



a $P(\text{both})=\frac{13}{20}$

b $P(\text{team})=\frac{13}{15}$

Exercise 9B

Question 1

a $\frac{1}{6}$

b $\frac{1}{5}$

Question 2

a $\frac{1}{4}$

b $\frac{1}{3}$

Question 3

a $\frac{1}{18}$

b $\frac{1}{6}$

Question 4

a $\frac{1}{52}$

b $\frac{1}{20}$

Question 5

a $\frac{28}{100} = \frac{7}{25}$

b $\frac{30}{52} = \frac{15}{26}$

Question 6

a $\frac{60}{100} = \frac{3}{5}$

b $\frac{47}{100}$

c $\frac{70}{100} = \frac{7}{10}$

d $\frac{37}{100}$

e $\frac{40}{100} = \frac{2}{5}$

f $\frac{53}{100}$

g $\frac{37}{47}$

h $\frac{23}{53}$

Question 7

a $$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{P(x = 6 \ \cap \ x > 4)}{P(x > 4)}$$

$$= \frac{1}{6} \div \frac{1}{3}$$

$$= \frac{1}{2}$$

b $$P(A|C) = \frac{P(A \cap C)}{P(C)}$$

$$= \frac{P(x = 6 \cap x \text{ is even})}{P(x \text{ is even})}$$

$$= \frac{1}{6} \div \frac{1}{2}$$

$$= \frac{1}{3}$$

c $$P(B|C) = \frac{P(B \cap C)}{P(C)}$$

$$= \frac{P(x > 4 \cap x \text{ is even})}{P(x \text{ is even})}$$

$$= \frac{1}{6} \div \frac{1}{2}$$

$$= \frac{1}{3}$$

d $$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$$= \frac{P(x > 4 \cap \ x = 6 \ )}{P(x = 6 \ )}$$

$$= \frac{1}{6} \div \frac{1}{6}$$

$$= 1$$

e $$P(C|A) = \frac{P(C \cap A)}{P(A)}$$
$$= \frac{P(x \text{ is even} \cap x = 6)}{P(x = 6)}$$
$$= \frac{1}{6} \div \frac{1}{6}$$
$$= 1$$

f $$P(C|B) = \frac{P(C \cap B)}{P(B)}$$
$$= \frac{P(x \text{ is even} \cap x > 4)}{P(x > 4)}$$
$$= \frac{1}{6} \div \frac{1}{3}$$
$$= \frac{1}{2}$$

The reader should note that although the formal working out is shown above, listing the sample spaces and working from those as shown in the text examples is a much simpler approach.

Question 8

a $0.6 + 0.1 = 0.7$

b $0.2 + 0.1 = 0.3$

c $1 - 0.1 = 0.9$

d $1 - 0.7 = 0.3$

e $1 - 0.3 = 0.7$

f $$P(A|B) = \frac{P(A \cap B)}{P(B)}$$
$$= \frac{0.1}{0.3}$$
$$= \frac{1}{3}$$

g $$P(B|A) = \frac{P(B \cap A)}{P(A)}$$
$$= \frac{0.1}{0.7}$$
$$= \frac{1}{7}$$

h $$P(A|A \cup B) = \frac{P(A \cap (A \cup B))}{P(A \cup B)}$$
$$= \frac{P(A)}{P(A \cup B)}$$
$$= \frac{0.7}{0.9}$$
$$= \frac{7}{9}$$

i $$P(B|A \cap B) = \frac{P(B \cap (A \cap B))}{P(A \cap B)}$$
$$= \frac{0.1}{0.1}$$
$$= 1$$

Question 9

a $\frac{3}{9} = \frac{1}{3}$

b $\frac{5}{9}$

c $\frac{7}{9}$

d $\frac{1}{9}$

e $1 - \frac{1}{3} = \frac{2}{3}$

f $1 - \frac{5}{9} = \frac{4}{9}$

g $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$= \frac{1}{9} \div \frac{5}{9}$

$= \frac{1}{5}$

h $P(B|A) = \frac{P(B \cap A)}{P(A)}$

$= \frac{1}{9} \div \frac{3}{9}$

$= \frac{1}{3}$

i $P(B|A\text{È}B) = \frac{P(B \cap (A \cup B))}{P(A \cup B)}$

$= \frac{P(B)}{P(A \cup B)}$

$= \frac{5}{9} \div \frac{7}{9}$

$= \frac{5}{7}$

Question 10

a $\frac{4}{10} = \frac{2}{5}$

b $\frac{1}{10}$

c $P(A|D) = \frac{P(A \cap D)}{P(D)}$

$= \frac{1}{10} \div \frac{4}{10}$

$= \frac{1}{4}$

d $\frac{6}{10} = \frac{3}{5}$

e $P(E \mid A \text{ or } D \text{ only}) = \frac{P(E \cap A \text{ or D only})}{P(A \text{ or D only})}$

$= \frac{P(E \cap A) + P(E \cap D)}{P(A \text{ or D only})}$

$= \frac{2}{10} \div \frac{6}{10}$

$= \frac{1}{3}$

Question 11

a $\frac{1}{6}$

b $\{1, 3, 5\}$

$P(x = 3 \mid x \text{ is odd}) = \frac{P(x = 3 \cap x \text{ is odd})}{P(x \text{ is odd})}$

$= \frac{1}{6} \div \frac{3}{6}$

$= \frac{1}{3}$

c $\{2,4,6\}$

$$\begin{aligned}\mathrm{P}(x=3 \mid x \text{ is even}) &= \frac{\mathrm{P}(x=3 \cap x \text{ is even})}{\mathrm{P}(x \text{ is even})} \\ &= 0\end{aligned}$$

d $\{3\}$

$$\begin{aligned}\mathrm{P}(x \text{ is even} \mid x=3) &= \frac{\mathrm{P}(x \text{ is even} \cap x=3)}{\mathrm{P}(x=3)} \\ &= 0\end{aligned}$$

e $\{1,2,3\}$

$$\begin{aligned}\mathrm{P}(x=2 \mid x<4) &= \frac{\mathrm{P}(x=2 \cap x<4)}{\mathrm{P}(x<4)} \\ &= \frac{1}{6} \div \frac{3}{6} \\ &= \frac{1}{3}\end{aligned}$$

f $\{1,2,3\}$

$$\begin{aligned}\mathrm{P}(x \neq 2 \mid x<4) &= \frac{\mathrm{P}(x \neq 2 \cap x<4)}{\mathrm{P}(x<4)} \\ &= \frac{2}{6} \div \frac{3}{6} \\ &= \frac{2}{3}\end{aligned}$$

g $\{1,2,3\}$

$$\begin{aligned}\mathrm{P}(x<2 \mid x<4) &= \frac{\mathrm{P}(x<2 \cap x<4)}{\mathrm{P}(x<4)} \\ &= \frac{1}{6} \div \frac{3}{6} \\ &= \frac{1}{3}\end{aligned}$$

h $\{1,2,3\}$

$$\begin{aligned}\mathrm{P}(x \leq 2 \mid x<4) &= \frac{\mathrm{P}(x \leq 2 \cap x<4)}{\mathrm{P}(x<4)} \\ &= \frac{2}{6} \div \frac{3}{6} \\ &= \frac{2}{3}\end{aligned}$$

Question 12

a $\frac{1}{10}$

b $\{2, 4, 6, 8, 10\}$

$$\begin{aligned} \mathrm{P}(x = 8 \mid x \text{ is even}) &= \frac{\mathrm{P}(x = 8 \cap x \text{ is even})}{\mathrm{P}(x \text{ is even})} \\ &= \frac{1}{10} \div \frac{5}{10} \\ &= \frac{1}{5} \end{aligned}$$

c $\{2, 3, 5, 7\}$

$$\begin{aligned} \mathrm{P}(x = 8 \mid x \text{ prime}) &= \frac{\mathrm{P}(x = 8 \cap x \text{ prime})}{\mathrm{P}(x \text{ prime})} \\ &= 0 \end{aligned}$$

d $\{2, 3, 5, 7\}$

$$\begin{aligned} \mathrm{P}(x = 7 \mid x \text{ prime}) &= \frac{\mathrm{P}(x = 7 \cap x \text{ prime})}{\mathrm{P}(x \text{ prime})} \\ &= \frac{1}{10} \div \frac{4}{10} \\ &= \frac{1}{4} \end{aligned}$$

e $\{5, 6, 7, 8, 9, 10\}$

$$\begin{aligned} \mathrm{P}(x = 8 \mid x > 4) &= \frac{\mathrm{P}(x = 8 \cap x > 4)}{\mathrm{P}(x > 4)} 1 \\ &= \frac{1}{10} \div \frac{6}{10} \\ &= \frac{1}{6} \end{aligned}$$

f $\{5, 6, 7, 8, 9, 10\}$

$$\begin{aligned} \mathrm{P}(x \neq 8 \mid x > 4) &= \frac{\mathrm{P}(x \neq 8 \cap x > 4)}{\mathrm{P}(x > 4)} 1 \\ &= \frac{5}{10} \div \frac{6}{10} \\ &= \frac{5}{6} \end{aligned}$$

g $\{6, 7, 8, 9, 10\}$

$$\begin{aligned} \mathrm{P}(x > 8 \mid x \geq 6) &= \frac{\mathrm{P}(x > 8 \cap x \geq 6)}{\mathrm{P}(x \geq 6)} \\ &= \frac{2}{10} \div \frac{5}{10} \\ &= \frac{2}{5} \end{aligned}$$

Question 13

a
$$\begin{aligned} \mathrm{P}(x = 7 \mid x < 10) &= \frac{\mathrm{P}(x = 7 \cap x < 10)}{\mathrm{P}(x < 10)} \\ &= \frac{1}{20} \div \frac{9}{20} \\ &= \frac{1}{9} \end{aligned}$$

b
$$\begin{aligned} \mathrm{P}(x = 7 \mid x > 10) &= \frac{\mathrm{P}(x = 7 \cap x > 10)}{\mathrm{P}(x > 10)} \\ &= 0 \end{aligned}$$

c $\{3, 6, 9, 12, 15, 18\}$

$$\begin{aligned} \mathrm{P}(x = 6 \mid x \text{ is a multiple of } 3) &= \frac{\mathrm{P}(x = 6 \cap x \text{ is a multiple of } 3)}{\mathrm{P}(x \text{ is a multiple of } 3)} \\ &= \frac{1}{20} \div \frac{6}{20} \\ &= \frac{1}{6} \end{aligned}$$

d $\{1, 2, 3, 4, 6, 12\}$

$$\begin{aligned} \mathrm{P}(x = 6 \mid x \text{ is a factor of } 12) &= \frac{\mathrm{P}(x = 6 \cap x \text{ is a factor of } 12)}{\mathrm{P}(x \text{ is a factor of } 12)} \\ &= \frac{1}{20} \div \frac{6}{20} \\ &= \frac{1}{6} \end{aligned}$$

e $\{1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

$$P(x \text{ is even} | x \neq 4 \text{ or } 10) = \frac{P(x \text{ is even} \cap x \neq 4 \text{ or } 10)}{P(x \neq 4 \text{ or } 10)}$$

$$= \frac{8}{20} \div \frac{18}{20}$$

$$= \frac{4}{9}$$

Question 14

a $P(B|A) = P(\text{Total of } 10 \,|\, \text{first roll a } 5)$

$$= \frac{P(\text{Total } 10 \cap \text{first roll a } 5)}{P(\text{first roll a } 5)}$$

$$= \frac{1}{36} \div \frac{6}{36}$$

$$= \frac{1}{6}$$

b $P(A|B) = P(\text{first roll a } 5|)\text{Total of } 10$

$$= \frac{P(\text{first roll a } 5 \cap \text{Total } 10)}{P(\text{Total } 10)}$$

$$= \frac{1}{36} \div \frac{3}{36}$$

$$= \frac{1}{3}$$

c $P(D|C) = P(\text{total of } 6|\text{first is a } 2)$

$$= \frac{P(\text{total of } 6 \cap \text{first is a } 2)}{P(\text{first is a } 2)}$$

$$= \frac{1}{6} \div \frac{6}{36}$$

$$= \frac{1}{6}$$

d $P(C|D) = P(\text{first is a 2}|\text{total of 6})$

$$= \frac{P(\text{first is a 2} \cap \text{total of 6})}{P(\text{total of 6})}$$

$$= \frac{1}{6} \div \frac{5}{36}$$

$$= \frac{1}{5}$$

e $P(F|E) = P(\text{total of 5}|\text{first is odd})$

$$= \frac{P(\text{total of 5} \cap \text{first is odd})}{P(\text{first is odd})}$$

$$= \frac{2}{36} \div \frac{18}{36}$$

$$= \frac{1}{9}$$

f $P(E|F) = P(\text{first is odd}|\text{total of 5})$

$$= \frac{P(\text{first is odd} \cap \text{total of 5})}{P(\text{total of 5})}$$

$$= \frac{2}{36} \div \frac{4}{36}$$

$$= \frac{1}{2}$$

Question 15

a $\frac{1}{20}$

b $\frac{1}{2}$

c $\{2, 3, 5, 7, 11, 13, 15, 17\}$

$$P(\text{prime}) = \frac{8}{20}$$

$$= \frac{2}{5}$$

d $\{3, 6, 9, 12, 1, 5, 18\}$

$$\text{P(multiple of 3)} = \frac{6}{20}$$
$$= \frac{3}{10}$$

e $\{1, 2, 3, 4, 6, 12\}$

$$\text{P(factor of 12)} = \frac{6}{20}$$
$$= \frac{3}{10}$$

f P(5|odd number)

$$= \frac{\text{P}(5 \cap \text{odd number})}{\text{P(odd number)}}$$
$$= \frac{1}{20} \div \frac{10}{20}$$
$$= \frac{1}{10}$$

g $\text{P}(3 \mid x < 6)$

$$= \frac{\text{P}(3 \cap x < 6)}{\text{P}(x < 6)}$$
$$= \frac{1}{20} \div \frac{5}{20}$$
$$= \frac{1}{5}$$

h $\text{P}(15 \mid x > 9)$

$$= \frac{\text{P}(15 \cap x > 9)}{\text{P}(x > 9)} =$$
$$= \frac{1}{20} \div \frac{11}{20}$$
$$= \frac{1}{11}$$

i $P(9 \mid \text{multiple of } 3)$

$$= \frac{P(9 \cap \text{multiple of } 3)}{P(\text{multiple of } 3)}$$

$$= \frac{1}{20} \div \frac{6}{20}$$

$$= \frac{1}{6}$$

j $P(\text{multiple of } 3 | \text{factor of } 12)$

$$= \frac{P(\text{multiple of } 3 \cap \text{factor of } 12)}{P(\text{factor of } 12)}$$

$$= \frac{3}{20} \div \frac{6}{20}$$

$$= \frac{1}{2}$$

k $P(\text{multiple of } 3 | x > 15)$

$$= \frac{P(\text{multiple of } 3 \cap x > 15)}{P(x > 15)}$$

$$= \frac{1}{20} \div \frac{5}{20}$$

$$= \frac{1}{5}$$

l $P(\text{multiple of } 3 | x \geq 15)$

$$= \frac{P(\text{multiple of } 3 \cap x \geq 15)}{P(x \geq 15)}$$

$$= \frac{2}{20} \div \frac{6}{20}$$

$$= \frac{1}{3}$$

Question 16

a $P(A) = P(x > 300)$

$$= \frac{12}{24}$$

$$= \frac{1}{2}$$

b $P(B) = P(x > 400)$

$$= \frac{6}{24}$$

$$= \frac{1}{4}$$

c $P(A|B) = P(x > 300 | x > 400)$

$$= 1$$

All numbers greater than 400 are greater than 300

d $P(B|A) = P(x > 400 | x > 300)$

$$= \frac{P(x > 400)}{P(x > 300)}$$

$$= \frac{1}{4} \div \frac{1}{2}$$

$$= \frac{1}{2}$$

e $P(A|\overline{B}) = P(x > 300 | x \leq 400)$

$$= \frac{P(300 < x \leq 400)}{P(x \leq 400)}$$

$$= \frac{1}{4} \div \frac{3}{4}$$

$$= \frac{1}{3}$$

f $P(\overline{A}|B) = P(x \leq 300 | x > 400)$

$$= 0$$

Question 17

The little thought

Each toss of a coin is independent and therefore what happened has no impact on what will happen.

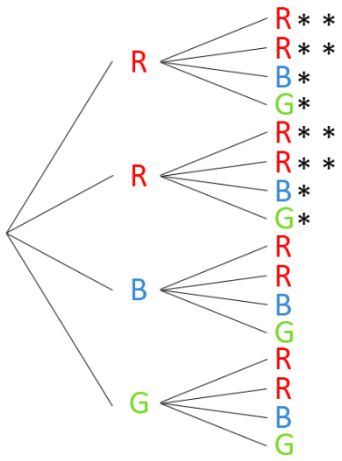
a $\frac{1}{2}$

b $\frac{1}{2}$

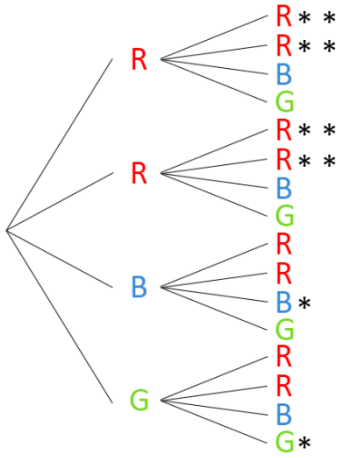
Question 1

a $\text{P(two reds}|\text{first one red)} = \frac{4}{8}$

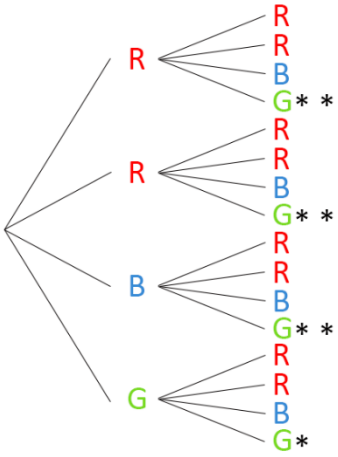
$= \frac{1}{2}$



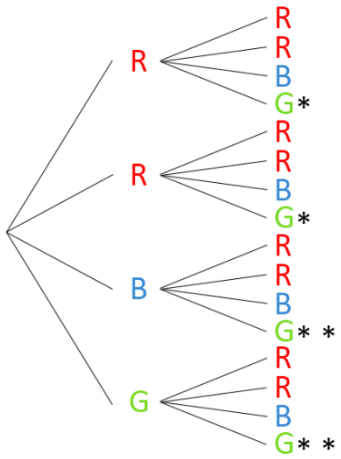
b $\text{P(two reds}|\text{both the same colour)} = \frac{4}{6} = \frac{2}{3}$



c $\text{P(two different}|\text{2nd one green)} = \frac{3}{4}$

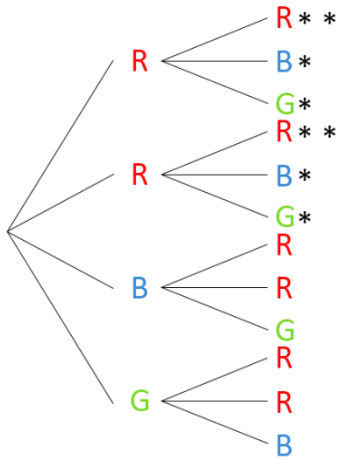


d P(no reds|2nd one green) $= \frac{2}{4} = \frac{1}{2}$

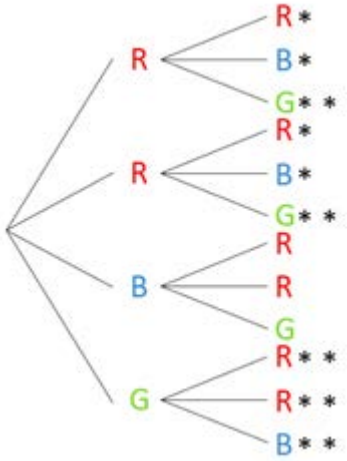


Question 2

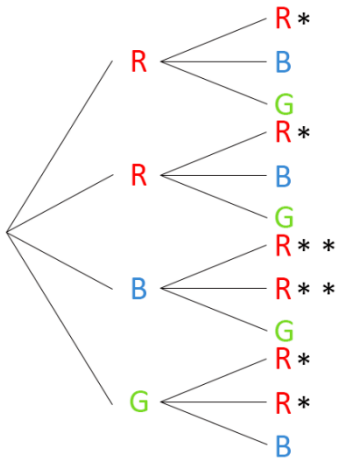
a P(two reds|first red) $= \frac{2}{6} = \frac{1}{3}$



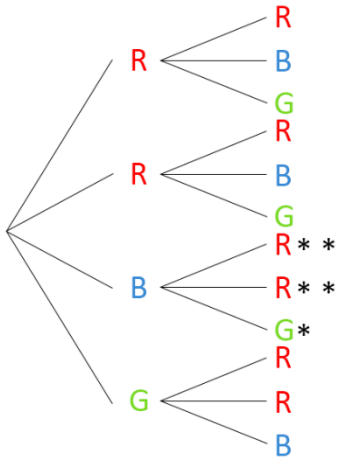
b P(one green|first not blue) $= \frac{5}{9}$



c P(first blue|second red) $= \frac{2}{6} = \frac{1}{3}$

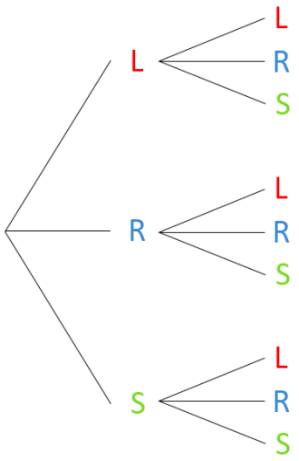


d P(second red|first blue) $= \frac{2}{3}$



Question 3

a P(Laurie wins both rounds) $= \frac{1}{9}$

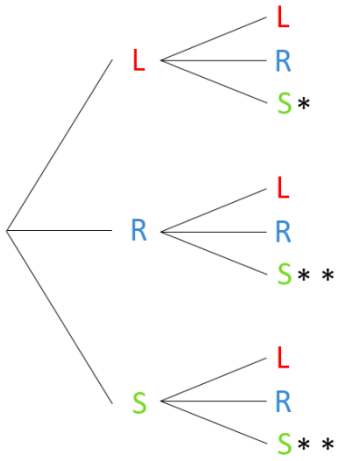


b P(Rob wins at least one round) $= \frac{5}{9}$

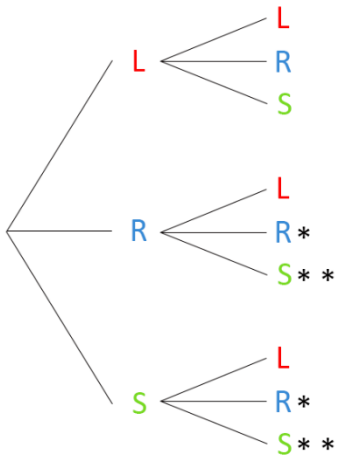
*consider using complementary event:

1 – P(Rob wins no rounds)

c P(Laurie wins neither|Steven wins the second round) $= \frac{2}{3}$



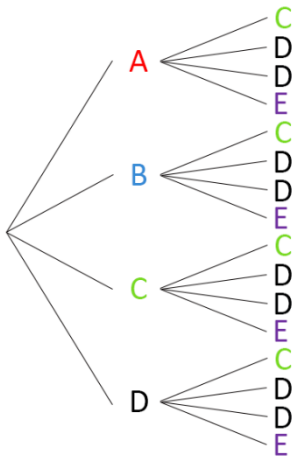
d P(Steven wins the second round|Laurie wins neither) $= \frac{2}{4} = \frac{1}{2}$



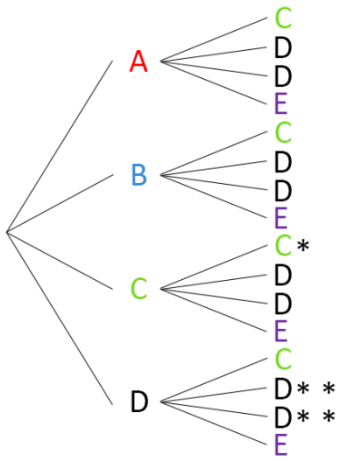
Question 4

a P(one of letters is an A) $= \frac{4}{16} = \frac{1}{4}$

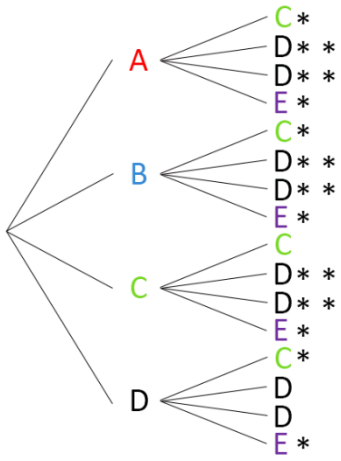
b P(the two letters are the same) $= \frac{3}{16}$



c P(first letter is a D|two letters are the same) $= \frac{2}{3}$



d P(second letter is D|two letters are not the same) $= \frac{6}{13}$



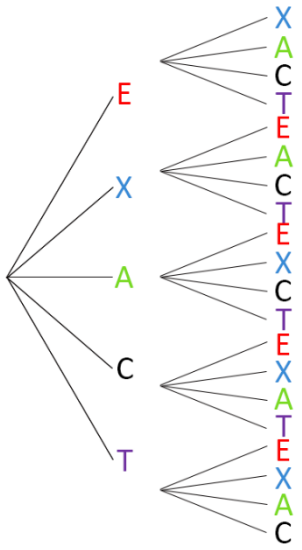
Question 5

a $P(AT) = \frac{1}{20}$

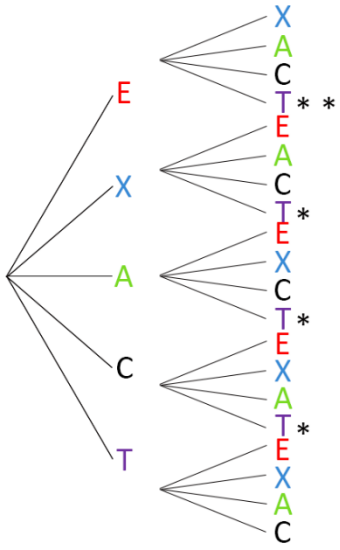
b $P(\text{starts with E}) = \frac{4}{20} = \frac{1}{5}$

c $P(\text{ends with T}) = \frac{4}{20} = \frac{1}{5}$

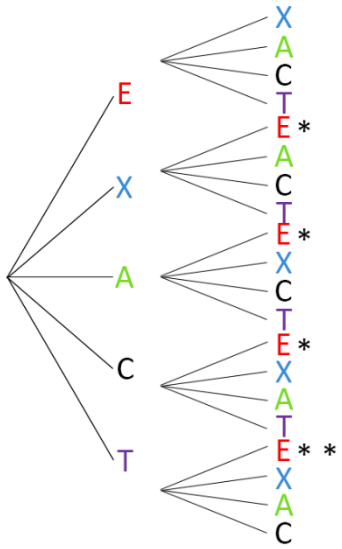
d $P(\text{starts with E and ends with T}) = \frac{1}{20}$



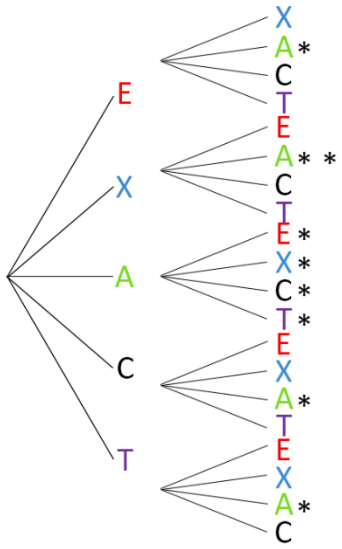
e $P(\text{starts with E}|\text{ends with T}) = \frac{1}{4}$



f P(starts with T|ends with E) $= \frac{1}{4}$



g P(contains an X|contains an A) $= \frac{1}{8}$



Question 1

a $\frac{5}{36}$

b $\frac{1}{36}$

c $\frac{11}{36}$

d $\frac{2}{36} = \frac{1}{18}$

e $\frac{16}{36} = \frac{4}{9}$

		BLUE DIE					
		1	2	3	4	5	6
RED DIE	1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
	2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
	3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
	4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
	5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
	6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

		BLUE DIE					
		1	2	3	4	5	6
RED DIE	1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
	2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
	3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
	4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
	5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
	6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Question 2

a $\frac{1}{2}$

b $\frac{1}{6}$

c $\frac{1}{12}$

d $\frac{7}{12}$

e $\frac{3}{12} = \frac{1}{4}$

f $\frac{1}{2}$

		DIE					
		1	2	3	4	5	6
COIN	Head	H, 1	H, 2	H, 3	H, 4	H, 5	H, 6
	Tail	T, 1	T, 2	T, 3	T, 4	T, 5	T, 6

Question 3

a $\frac{1}{52}$

b $\frac{4}{52} = \frac{1}{13}$

c $\frac{1}{2}$

d $\frac{4}{52} = \frac{1}{13}$

e $\frac{13}{52} = \frac{1}{4}$

f $\frac{4}{52} = \frac{1}{13}$

g $\frac{48}{52} = \frac{12}{13}$

h $\frac{12}{52} = \frac{3}{13}$

i $\frac{2}{52} = \frac{1}{26}$

j $\frac{28}{52} = \frac{7}{13}$

k $\frac{1}{52}$

l $\frac{16}{52} = \frac{4}{13}$

Question 4

a $\frac{1}{2}$

b $\frac{1}{4}$

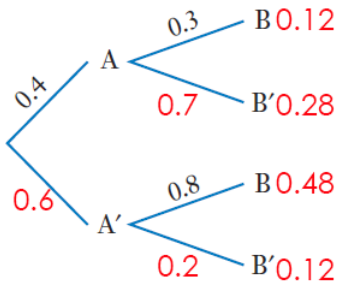
c $\frac{3}{4}$

d $\frac{1}{2}$

Question 1

a $P(A')=0.6$

b $P(A \text{ and } B)$
$=0.12$



c $P(B) = 0.12 + 0.48$
$= 0.6$

d $P(B|A) = \dfrac{P(B \cap A)}{P(A)}$
$= \dfrac{0.12}{0.4}$
$= 0.3$

e $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$
$= \dfrac{0.12}{0.6}$
$= 0.2$

f $P(A|B') = \dfrac{P(A \cap B')}{P(B')}$
$= \dfrac{0.28}{0.4}$
$= 0.7$

Question 2

a $P(A') = 0.4$

b P(neither A or B)=0.2

c $P(B) = 0.06 + 0.2$
$= 0.26$

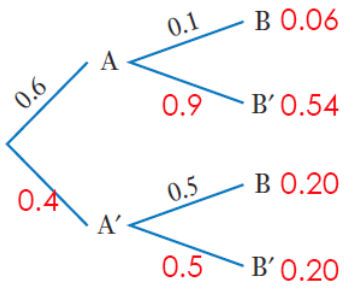
d P(at least one of A or B)

$= 0.06 + 0.54 + 0.2$
$= 0.8$

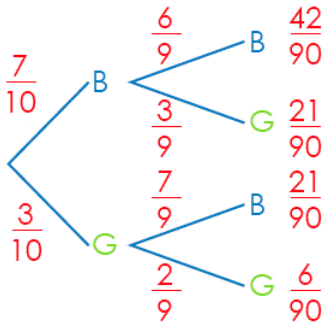
$*1 - P(\text{neither})$
$= 1 - 0.2$
$= 0.8$

e $P(B|A) = \dfrac{P(B \cap A)}{P(A)}$
$= \dfrac{0.06}{0.6}$
$= 0.1$

f $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$
$= \dfrac{0.06}{0.26}$
$= \dfrac{3}{13}$



Question 3



a $\text{P(B then G)} = \frac{21}{90}$

$= \frac{7}{30}$

b $\text{P(BG or GB)} = \frac{21}{90} + \frac{21}{90}$

$= \frac{42}{90}$

$= \frac{7}{15}$

c $\text{P(same colour)} = \frac{42}{90} + \frac{6}{90}$

$= \frac{48}{90}$

$= \frac{8}{15}$

d $\text{P(BB|same colour)} = \frac{\text{P(BB)}}{\text{P(same)}}$

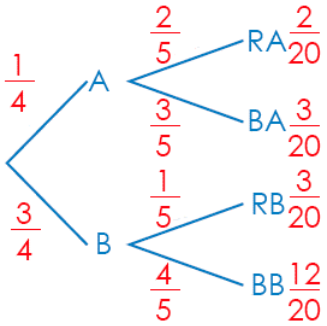
$= \frac{42}{90} \div \frac{48}{90}$

$= \frac{42}{48}$

$= \frac{7}{8}$

Question 4

a $\text{P(marble from B)} = \frac{3}{4}$

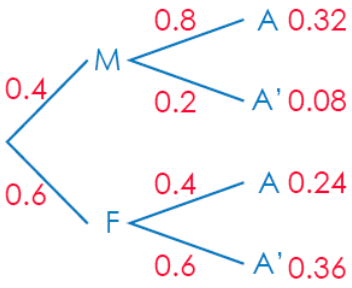


b $$\text{P(blue}|\text{Bag B)} = \frac{\text{P(Blue} \cap \text{Bag B)}}{\text{P(Bag B)}}$$
$$= \frac{12}{20} \div \frac{3}{4}$$
$$= \frac{4}{5}$$

c $$\text{P(marble from Bag A}|\text{blue marble)} = \frac{\text{P(from A} \cap \text{blue)}}{\text{P(blue)}}$$
$$= \frac{3}{20} \div \left(\frac{3}{20} + \frac{12}{20} \right)$$
$$= \frac{3}{20} \div \frac{15}{20}$$
$$= \frac{3}{15}$$
$$= \frac{1}{5}$$

Question 5

a $\text{P(female)} = 0.6$

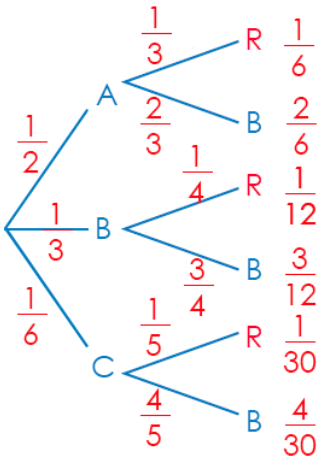


b $$\text{P(studying course A+} = 0.32 + 0.24$$
$$= 0.56$$

c $$\text{P(male}|\text{studying course A)} = \frac{\text{P(male studying course A)}}{\text{P(studying course A)}}$$
$$= 0.32 \div (0.32 + 0.24)$$
$$= \frac{4}{7}$$

d $$\text{P(female}|\text{not studying course A)} = \frac{\text{P(female} \cap \text{not studying course A)}}{\text{P(not studying course A)}}$$
$$= 0.36 \div (0.08 + 0.36)$$
$$= \frac{9}{11}$$

Question 6



a $P(\text{Bag A}) = \frac{1}{2}$

b $P(\text{Bag C}) = \frac{1}{6}$

c $P(\text{red}) = \frac{1}{6} + \frac{1}{12} + \frac{1}{30}$

$= \frac{17}{60}$

d $P(\text{blue}) = 1 - P(\text{red})$

$= 1 - \frac{17}{60}$

$= \frac{43}{60}$

e $P(\text{red and from Bag A}) = \frac{1}{6}$

f $P(\text{blue or from Bag B}) = \frac{43}{60} + \frac{1}{12}$

$= \frac{4}{5}$

g $P(\text{from Bag A}|\text{red}) = \frac{P(\text{BagA} \cap \text{red})}{P(\text{red})}$

$= \frac{1}{6} \div \frac{17}{60}$

$= \frac{10}{17}$

h $P(\text{from Bag B}|\text{blue}) = \frac{P(\text{from bag B} \cap \text{blue})}{P(\text{blue})}$

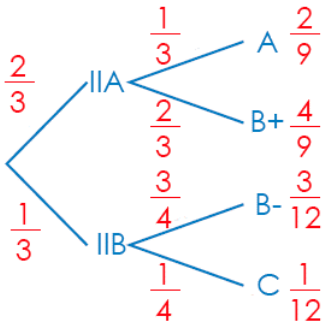
$= \frac{3}{12} \div \frac{43}{60}$

$= \frac{15}{43}$

Question 7

a $\text{P(grade B)} = \frac{4}{9} + \frac{3}{12}$

$= \frac{25}{36}$



b $\text{P(B+|B)} = \frac{\text{P(B+} \cap \text{B)}}{\text{P(B)}}$

$= \frac{4}{9} \div \frac{25}{36}$

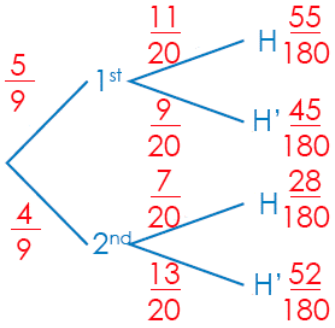
$= \frac{16}{25}$

Question 8

a $\text{P(second year student)} = \frac{4}{9}$

b $\text{P(second year or at home)} = \frac{4}{9} + \frac{55}{180}$

$= \frac{3}{4}$



c $\text{P(first year|at home)} = \frac{\text{P(first year } \cap \text{ at home)}}{\text{P(at home)}}$

$= \frac{55}{180} \div \left(\frac{55}{180} + \frac{28}{180} \right)$

$= \frac{55}{180} \div \frac{83}{180}$

$= \frac{55}{83}$

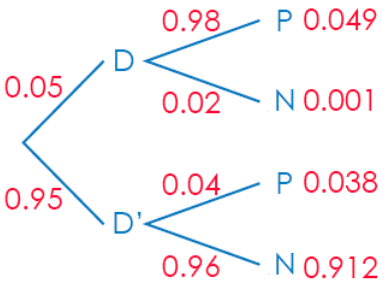
Question 9

a $\text{P(no disease and negative result)} = 0.912$

b $\text{P(no disease and positive result)}\ 0.038$

c $\text{P(incorrect result)} = 0.001 + 0.038$
$= 0.039$

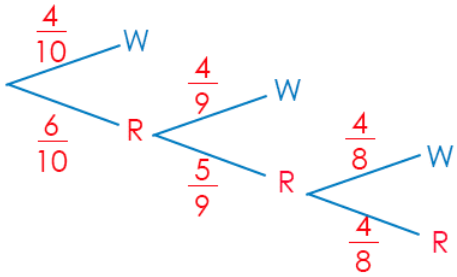
d $\text{P(disease}|\text{positive result)} = \dfrac{\text{P(disease} \cap \text{positive result)}}{\text{P(positive result)}}$
$= 0.049 \div (0.049 + 0.038)$
$= 0.563$



Question 10

a $\text{P(1st disc not white)} = \dfrac{3}{5}$

b $\text{P(3 red)} = \dfrac{3}{5} \times \dfrac{5}{9} \times \dfrac{4}{8}$
$= \dfrac{1}{6}$



c $\text{P(2 discs selected)} = \text{P(Red then white)}$
$= \dfrac{6}{10} \times \dfrac{4}{9}$
$= \dfrac{4}{15}$

d $\text{P(2 red and 1 white)} = \dfrac{6}{10} \times \dfrac{5}{9} \times \dfrac{4}{8}$
$= \dfrac{1}{6}$

e $\text{P(2 red and 1 white}|\text{more than one is selected)} = \dfrac{\text{P(RRW)}}{\text{P(2 or 3 discs selected)}}$
$= \dfrac{\text{P(RRW)}}{\text{P(RW or RRW or RRR)}}$
$= \dfrac{1}{6} \div \left(\dfrac{4}{15} + \dfrac{1}{6} + \dfrac{1}{6}\right)$
$= \dfrac{5}{18}$

Exercise 9F

Question 1

a $\text{P}(2 \text{ reds}) = \frac{6}{10} \times \frac{5}{9}$

$= \frac{1}{3}$

b $\text{P}(\text{RR or BB}) = \frac{1}{3} + \frac{4}{10} \times \frac{3}{9}$

$= \frac{7}{15}$

c $\text{P}(\text{no blue}) = \text{P}(2 \text{ reds})$

$= \frac{1}{3}$

d $\text{P}(\text{at least 1 blue}) = 1 - \text{P}(\text{no blue})$

$= \frac{2}{3}$

Question 2

$\text{P}(\text{faulty}) = \frac{4}{100}$

a $\text{P}(4 \text{ non faulty}) = \frac{96}{100} \times \frac{95}{99} \times \frac{94}{98} \times \frac{93}{97}$

$= 0.8472$

b $\text{P}(\text{at least one is faulty}) = 1 - \text{P}(\text{none faulty})$

$= 1 - 0.8472$

$= 0.1528$

Question 3

$$\text{P(faulty)} = \frac{3}{200}$$

a $$\text{P(none faulty)} = \frac{197}{200} \times \frac{196}{199} \times \frac{195}{198} \times \frac{194}{197} \times \frac{193}{196}$$
$$= 0.9265$$

b $$\text{P(at least one is faulty)} = 1 - \text{P(none are faulty)}$$
$$= 1 - 0.9265$$
$$= 0.0735$$

Question 4

a $$\text{P(3 reds)} = \frac{4}{10} \times \frac{3}{9} \times \frac{2}{8}$$
$$= \frac{1}{30}$$

b $$\text{P(3 reds)} = \text{P(3 reds or 3 blue or 3 green)}$$
$$= \frac{1}{30} + \frac{3}{10} \times \frac{2}{9} \times \frac{1}{8} + \frac{3}{10} \times \frac{2}{9} \times \frac{1}{8}$$
$$= \frac{1}{20}$$

c $$\text{P(no reds)} = \frac{6}{10} \times \frac{5}{9} \times \frac{4}{8}$$
$$= \frac{1}{6}$$

d $$\text{P(at least 1 red)} = 1 - \text{P(no red)}$$
$$= 1 - \frac{1}{6}$$
$$= \frac{5}{6}$$

Question 5

a $\text{P}(3 \text{ red}) = \frac{4}{10} \times \frac{4}{10} \times \frac{4}{10}$

$= 0.064$

b $\text{P}(3 \text{ the same}) = \left(\frac{4}{10}\right)^3 + \left(\frac{3}{10}\right)^3 + \left(\frac{3}{10}\right)^3$

$= 0.118$

c $\text{P}(\text{no red}) = \left(\frac{6}{10}\right)^3$

$= 0.216$

d $\text{P}(\text{at least 1 red}) = 1 - \text{P}(\text{no reds})$

$= 1 - 0.216$

$= 0.784$

Question 6

a $\text{P}(\text{Y8 male}) = 0.24 \times 0.52$

$= 0.1248$

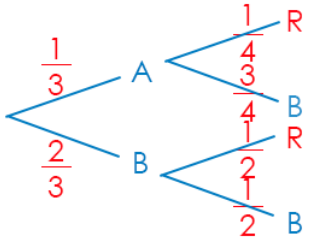
b $\text{P}(\text{Y8 and female}) = 0.24 \times 0.48$

$= 0.1152$

Question 7

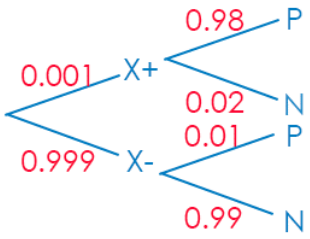
$\text{P}(\text{red}) = \frac{1}{3} \times \frac{1}{4} + \frac{2}{3} \times \frac{1}{2}$

$= \frac{5}{12}$



Question 8

$$\begin{aligned} P(\text{positive result}) &= 0.001 \times 0.98 \times 0.999 \times 0.01 \\ &= 0.01097 \end{aligned}$$



Question 9

a $P(A) = \frac{1}{2}$

b $P(B) = \frac{2}{3}$

c $$\begin{aligned} P(A \cap B) &= \frac{1}{2} \times \frac{2}{3} \\ &= \frac{1}{3} \end{aligned}$$

d $$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= \frac{1}{2} + \frac{2}{3} - \frac{1}{3} \\ &= \frac{5}{6} \end{aligned}$$

Question 10

a $P(A) = \frac{1}{3}$

b $P(B) = \frac{1}{2}$

c $P(A \cap B) = \frac{1}{3} \times \frac{1}{2}$

$= \frac{1}{6}$

d $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{3} + \frac{1}{2} - \frac{1}{6}$

$= \frac{2}{3}$

Question 11

a $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.4 + 0.5 - 0.1$

$= 0.8$

b $P(\overline{A \cup B}) = 1 - 0.8$

$= 0.2$

c $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$= \frac{0.1}{0.5}$

$= 0.2$

d $P(B|A) = \frac{P(B \cap A)}{P(A)}$

$= \frac{0.1}{0.4}$

$= 0.25$

Question 12

a $$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= 0.5 + 0.8 - 0.9 \\ &= 0.4 \end{aligned}$$

b $$\begin{aligned} P(A|B) &= \frac{P(A \cap B)}{P(B)} \\ &= \frac{0.4}{0.8} \\ &= 0.5 \end{aligned}$$

c $$\begin{aligned} P(B|A) &= \frac{P(B \cap A)}{P(A)} \\ &= \frac{0.4}{0.5} \\ &= 0.8 \end{aligned}$$

Question 13

a $$\begin{aligned} P(\text{all fail}) &= 0.02 \times 0.2 \times 0.15 \times 0.01 \times 0.005 \\ &= 0.00000003 \end{aligned}$$

b $$\begin{aligned} P(\text{none fail}) &= 0.98 \times 0.8 \times 0.85 \times 0.99 \times 0.995 \\ &\approx 0.66 \text{ (2dp)} \end{aligned}$$

c $$\begin{aligned} P(\text{at least 1 will fail}) &= 1 - P(\text{none fail}) \\ &= 1 - 0.66 \\ &= 0.34 \end{aligned}$$

Question 14

a $\text{P(both correct)} = \frac{1}{6} \times \frac{1}{6}$

$= \frac{1}{36}$

b $\text{P(neither correct)} = \frac{5}{6} \times \frac{5}{6}$

$= \frac{25}{36}$

c $\text{P(at least one is correct)} = 1 - \text{P(none correct)}$

$= 1 - \frac{25}{36}$

$= \frac{11}{36}$

Question 15

a $\text{P(X and Y both defective)} = 0.005 \times 0.01$

$= 0.00005$

b $\text{P(neither defective)} = 0.995 \times 0.99$

$= 0.98505$

c $\text{P(at least 1 defective)} = 1 - \text{P(neither defective)}$

$= 1 - 0.98505$

$= 0.01495$

Question 16

a $\text{P(all 3 defective)} = 0.005 \times 0.01 \times 0.002$

$= 0.0000001$

b $\text{P(none are defective)} = 0.995 \times 0.99 \times 0.998$

≈ 0.983 (3dp)

c $\text{P(at least one defective)} = 1 - \text{P(none defective)}$

$= 1 - 0.983$

$= 0.017$

Question 17

a $\mathrm{P}(\text{bag B}) = \frac{1}{2}$

b $\mathrm{P}(\text{yellow from B}) = \frac{1}{2} \times \frac{4}{5}$

$= \frac{2}{5}$

c $\mathrm{P}(\text{yellow}) = \frac{1}{2} \times \frac{2}{5} + \frac{1}{2} \times \frac{4}{5}$

$= \frac{3}{5}$

d $\mathrm{P}(\text{yellow or bag B}) = \mathrm{P}(\text{yellow}) + \mathrm{P}(\text{bag B}) - \mathrm{P}(\text{yellow} \cap \text{bag B})$

$= \frac{3}{5} + \frac{1}{2} - \frac{2}{5}$

$= \frac{7}{10}$

Question 18

a $\mathrm{P}(\text{bag B} \cap \text{yellow}) = \frac{2}{3} \times \frac{4}{5}$

$= \frac{8}{15}$

b $\mathrm{P}(\text{yellow or bag B}) = \mathrm{P}(\text{yellow}) + \mathrm{P}(\text{bag B}) - \mathrm{P}(\text{bag B and yellow})$

$= \left(\frac{1}{3} \times \frac{2}{5} + \frac{2}{3} \times \frac{4}{5}\right) + \frac{2}{3} - \frac{8}{15}$

$= \frac{4}{5}$

Question 19

a $P(A \cup B) = P(A) + P(B) - P(A \cup B)$

$= 0.65 + 0.34 - 0.86$

$= 0.13$

b $P(B|A) = \dfrac{P(B \cap A)}{P(A)}$

$= \dfrac{0.13}{0.65}$

$= 0.2$

Question 20

$P(B|A) = \dfrac{P(B \cap A)}{P(A)}$

$0.2 = \dfrac{0.1}{P(A)}$

$P(A) = 0.5$

$P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

$0.25 = \dfrac{0.1}{P(B)}$

$P(B) = 0.4$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.5 + 0.4 - 0.1$

$= 0.8$

Question 21

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$$\frac{1}{4} = \frac{3}{22} \div P(A)$$

$$P(A) = \frac{3}{22} \div \frac{1}{4}$$

$$= \frac{6}{11}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\frac{2}{5} = \frac{3}{22} \div P(B)$$

$$P(B) = \frac{3}{22} \div \frac{2}{5}$$

$$= \frac{15}{44}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{6}{11} + \frac{15}{44} - \frac{3}{22}$$

$$= \frac{3}{4}$$

Exercise 9G

Question 1

$P(A|B) = \frac{1}{2}$, $P(A) = 0.55$

or

$P(A) = 0.55$, $P(B) = 0.5$

If independent $P(A) \times P(B) = P(A \cap B)$

$0.55 \times 0.5 = 0.275 \neq 0.25$

$\therefore$ not independent

Question 2

$P(B|A) = \frac{3}{4}$, $P(B) = 0.75$ $\therefore$ independent

or

If independent $P(A) \times P(B) = P(A \cap B)$

$0.4 \times 0.75 = 0.3$

$\therefore$ independent

Question 3

$P(A|B) = \frac{0.1}{0.4} = \frac{1}{4}$, $P(A) = 0.25$ $\therefore$ independent

Question 4

$P(A) = 0.2$, $P(B) = 0.2$

$P(A) \times P(B) = 0.2 \times 0.2$

$= 0.04$

$P(A \cap B) = 0.1 \neq 0.04$

$\therefore$ not independent

Question 5

$P(A \cap B) = 0 \therefore A \text{ and } B \text{ are mutually exclusive}$

Question 6

$$P(A \cup B) = 1 - 0.2$$
$$= 0.8$$
$$0.8 = 0.3 + 0.5 + P(A \cap B)$$
$$P(A \cap B) = 0 \therefore A \text{ and } B \text{ are mutually exclusive}$$

Question 7

$$P(A \cup B) = 1 - 0.1$$
$$= 0.9$$
$$P(A \cup B) = 0.25 + 0.55 + P(A \cap B)$$
$$P(A \cap B) = 0.9 - (0.25 + 0.55)$$
$$= 0.1$$
$\therefore A \text{ and } B \text{ are not mutually exclusive}$

Question 8

$P(A \cap B) \neq 0 \therefore A \text{ and } B \text{ are not mutually exclusive}$

Question 9

a Mutually exclusive, cannot obtain 3 and 4 at the same time.

b Not mutually exclusive, 2 is even and less than 5, so it is possible for both events to occur together.

c Not mutually exclusive, 2 is both prime and even so it is possible for both events to occur together.

d Mutually exclusive, it is not possible to have a number less than 3 and greater than 5.

e Not mutually exclusive, 2 is both less than 3 and less than 5, so it is possible for both events to occur together.

Question 10

a If marbles are replaced, each draw is independent.

b If marble is not replaced, the result of the first draw affects the second, so they are dependent.

Question 11

$P(T) = \frac{1}{2} \qquad P(T|A) = \frac{1}{2}$

$\therefore$ independent

$P(6) = \frac{1}{6} \qquad P(6 \mid T) = \frac{1}{6}$

$\therefore$ independent

$P(T \cap 6) = \frac{1}{12},\ P(T) = \frac{1}{2},\ P(6) = \frac{1}{6}$

$\frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$

$\therefore P(T) \times P(6) = P(T \cap 6),$ events are independent

Question 12

a $P(A|B) = P(A) = 0.2$

b $P(B|A) = P(B) = 0.25$

c $P(A \cap B) = P(A) \times P(B)$
$= 0.2 \times 0.25$
$= 0.05$

d $P(A \cup B) = P(A)+P(B)-P(A \cap B)$
$= 0.2 + 0.25 - 0.05$
$= 0.4$
$P(\overline{A \cup B}) = 1 - 0.4$
$= 0.6$

Question 13

a $P(A \cap B) = 0$

b $P(B|A) = \dfrac{P(A \cap B)}{P(A)}$
$= \dfrac{0}{0.2}$
$= 0$

c $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$
$= \dfrac{0}{0.3}$
$= 0$

d $P(A \cup B) = P(A) + P(B)$
$= 0.2 + 0.3$
$= 0.5$

Question 14

$$\begin{aligned} P(A \cup B) &= 1 - 0.25 \\ &= 0.75 \\ P(A \cap B) &= P(A)+P(B)-P(A \cup B) \\ &= 0.25 + 0.5 - 0.75 \\ &= 0 \end{aligned}$$

$\therefore$ A and B are mutually exclusive

Question 15

a $$\begin{aligned} P(A \cap B) &= P(A) \times P(B) \\ &= 0.5 \times 0.6 \\ &= 0.3 \end{aligned}$$

b $$\begin{aligned} P(A \cup B) &= P(A) + P(B)-P(A \cap B) \\ &= 0.5 + 0.6 - 0.3 \\ &= 0.8 \end{aligned}$$

c $P(B|A) = P(B) = 0.6$

d $P(A|B) = P(A) = 0.5$

Question 16

$$\begin{aligned} P(A \cap B) &= P(A)\ P(B) \\ &= 0.25\ P(B) \\ P(A \cup B) &= P(A) + P(B)-P(A \cap B) \\ 0.85 &= 0.25 + P(B) - 0.25\ P(B) \\ 0.6 &= 0.75\ P(B) \\ P(B) &= \frac{0.6}{0.75} \\ &= 0.8 \end{aligned}$$

Question 17

$$\begin{aligned} P(A \cap B) &= P(A)\ P(B) \\ &= 0.25\ P(B) \\ P(A \cap B) &= P(A) + P(B) - P(A \cup B) \\ 0.25\ P(B) &= 0.25 + P(B) - 0.4 \\ 0.15 &= 0.75\ P(B) \\ P(B) &= \frac{0.15}{0.75} \\ &= 0.2 \end{aligned}$$

Question 18

a If A and B are mutually exclusive then $P(A \cap B) = 0$

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) \\ &= 0.2 + 0.4 \\ &= 0.6 \\ P(\overline{A \cup B}) &= 1 - 0.6 \\ &= 0.4 \end{aligned}$$

b If independent $P(A \cap B) = P(A) \times P(B)$

$$\begin{aligned} &= 0.2 \times 0.4 \\ &= 0.08 \\ P(A \cap B) &= P(A) + P(B) - P(A \cup B) \\ &= 0.2 + 0.4 - 0.08 \\ &= 0.52 \end{aligned}$$

$$\begin{aligned} P(\overline{A \cup B}) &= 1 - 0.52 \\ &= 0.48 \end{aligned}$$

Question 19

a If mutually exclusive, $P(A \cap B) = 0$

$$\therefore P(A \cup B) = P(A) + P(B)$$
$$= 0.2 + 0.5$$
$$= 0.7$$
$$P(\overline{A \cup B}) = 1 - 0.7$$
$$= 0.3$$

b
$$P(A \cap B) = P(A) \times P(B)$$
$$= 0.2 \times 0.5$$
$$= 0.10$$
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 0.2 + 0.5 - 0.1$$
$$= 0.6$$

$$P(\overline{A \cup B}) = 1 - 0.6$$
$$= 0.4$$

Question 20

a $x = 0$, A and B cannot occur together.

b
$$P(A|B) = P(A)$$
$$\frac{x}{x+5} = \frac{6+x}{x+13}$$
$$x^2 + 13x = x^2 + 11x + 30$$
$$2x = 30$$
$$x = 15$$

Question 21

a $P(\text{the student is an Engineering student}) = \frac{507}{1448} = 0.35$

b $P(\text{the student is an Engineering student} | \text{the student is male}) = \frac{407}{910} = 0.45$

c $P(\text{the student is an Engineering student} | \text{the student is female}) = \frac{100}{538} = 0.19$

d Results suggest that gender and whether or not a person chooses engineering are not independent. Significant differences between the ratio of engineering students who are male and female compared to a ratio of 35% of students choosing engineering.

Question 22

a $P(\text{the student is on the honours course}) = \frac{2522}{7252} = 0.35$

b $P(\text{the student is on the honours course} | \text{the student is male}) = \frac{982}{2898} = 0.34$

c $P(\text{the student is on the honours course} | \text{the student is female}) = \frac{1540}{4354} = 0.35$

d Proportion of honour students overall is roughly the same as honour students given either gender, suggesting gender and whether a student is an honour student or not are independent.

Question 23

If $P(X) = P(X|Y)$ then X & Y are independent and $P(X \cap Y) = P(X) \times P(Y)$

$$P(X) = a + b$$

$$P(X|Y) = \frac{b}{b+c}$$

$$\therefore a + b = \frac{b}{b+c} \Rightarrow b + c = \frac{b}{a+b}$$

From the diagram

$$P(Y) = b + c$$

$$= \frac{b}{a+b}$$

$$= P(Y|X)$$

$$P(X \cap Y) = b$$

$$P(X) = (a + b)$$

$$P(Y) = (b + c)$$

We know $a + b = \frac{b}{b+c}$

$$(a + b)(b + c) = b$$

$$P(X)P(Y) = P(X \cap Y)$$

Question 1

a $P(A \cap B) = P(A) + P(B) - P(A \cup B)$
$= 0.7 + 0.6 - 0.8$
$= 0.5$

b $P(\overline{A \cap B}) = 1 - 0.5$
$= 0.5$

c $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$
$= \dfrac{0.5}{0.6}$
$= \dfrac{5}{6}$

d $P(\overline{A}\,|B) = \dfrac{0.1}{0.6}$
$= \dfrac{1}{6}$

e $P(A|\,\overline{B}) = \dfrac{0.2}{0.4}$
$= 0.5$

Question 2

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$
$= 0.45 + 0.02 - 0.56$
$= 0.09$

$P(A) \times P(B) = 0.2 \times 0.45$
$= 0.09$
$= P(A \cap B)$

Hence, A and B are independent

Question 3

Completing the square:

$$2x^2 - x - 36 = 0$$
$$2(x^2 - \frac{1}{2}x - 18) = 0$$
$$2((x - \frac{1}{4})^2 - \frac{1}{16} - 18) = 0$$
$$2((x - \frac{1}{4})^2 - 18\frac{1}{16}) = 0$$
$$(x - \frac{1}{4})^2 = 18\frac{1}{16}$$
$$= \frac{289}{16}$$
$$x - \frac{1}{4} = \pm\frac{17}{4}$$
$$x = \pm\frac{17}{4} + \frac{1}{4}$$
$$= -\frac{16}{4}, \frac{18}{4}$$
$$= -4, 4\frac{1}{2}$$

Quadratic formula:

$2x^2 - x - 36 = 0$

$a = 2, b = -1, c = -36$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{1 \pm \sqrt{1 - 4(2)(-36)}}{4}$$
$$= \frac{1 \pm \sqrt{1 + 288}}{4}$$
$$= \frac{1 \pm \sqrt{289}}{4}$$
$$= \frac{1 \pm 17}{4}$$
$$= -\frac{16}{4}, \frac{18}{4}$$
$$= -4, 4\frac{1}{2}$$

Factorisation:

$$2x^2 - x - 36 = 0$$
$$(2x-9)(x+4) = 0$$
$$2x - 9 = 0 \quad \text{or} \quad x + 4 = 0$$
$$2x = 9 \qquad\qquad x = -4$$
$$x = 4\frac{1}{2}$$

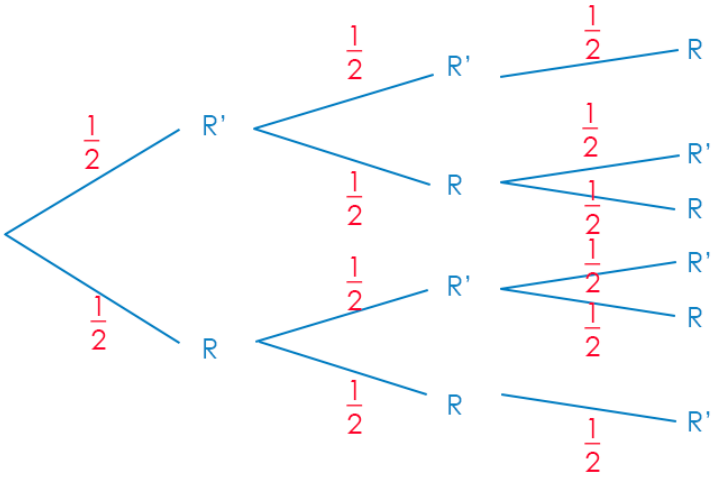
Question 4

a $\mathrm{P}(**\mathrm{R}) = 1$

b P(R, G or Y, R)

$$\mathrm{P}(\mathrm{R},\mathrm{R}',\mathrm{R}) = 1 \times \frac{2}{3} \times \frac{1}{2}$$
$$= \frac{1}{3}$$

c $\mathrm{P}(\text{3rd red}) = \frac{3}{6}$
$$= \frac{1}{2}$$



Question 5

a Period: 180°
Amp: 4

b Period: 120°
Amp: 3

Question 6

When $\sin\theta = -0.53$, the angle is 32° below the x-axis
$x° = -148°, -32°, 212°, 328°$

Question 7

$C\ (x, y)$

$(5, -2) = \left(\frac{x+3}{2}, \frac{-5+y}{2} \right)$

$\frac{x+3}{2} = 5 \Rightarrow x = 7$

$\frac{y-5}{2} = -2 \Rightarrow y = 1$

$\therefore C\ (7, 1)$

Question 8

a $\text{P(female)} = \frac{55}{93}$

b $\text{P(male who walks)} = \frac{7}{93}$

c $\text{P(male given they walk to work)} = \frac{7}{22}$

d $\text{P(someone who walks to work given they are male)} = \frac{7}{38}$

Question 9

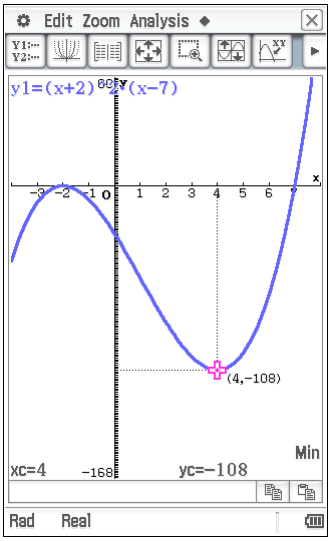
a y-int, $x = 0$

$y = (0+2)^2(0-7)$

$= 4(-7)$

$= -28$

$\therefore \text{B}(0, -28)$



b x-int, $y = 0$

$0 = (x+2)^2(x-7)$

$x + 2 = 0$ or $x - 7 = 0$

$x = -2 \quad x = 7$

$\therefore$ A $(-2, 0)$, D $(7, 0)$

c $a = 4, b = -108$

d $-108 < p < 0$

Question 10

$\sin\theta = 0.24, \ \theta = \dfrac{\pi}{13}$

$(6 + 25\sin\theta)(1 - 2\cos\theta) = 0$

$6 + 25\sin\theta = 0$ or $1 - 2\cos\theta = 0$

$\sin\theta = \dfrac{-6}{25} \qquad 2\cos\theta = 1$

$\theta = \pi + \dfrac{\pi}{13} \qquad \theta = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$

$= \dfrac{14\pi}{13}, \dfrac{25\pi}{13}$

$\therefore \theta = \dfrac{\pi}{3}, \dfrac{14\pi}{13}, \dfrac{5\pi}{3}, \dfrac{25\pi}{13}$

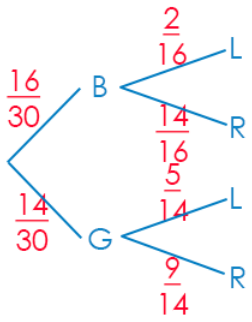
Question 11

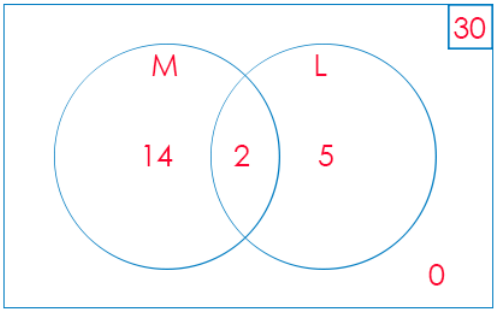
a $P(\text{left handed boy}) = \frac{1}{15}$

b $P(\text{right handed girl}) = \frac{3}{10}$

c $P(\text{left handed given that the chosen student is a girl}) = \frac{5}{14}$

d $P(\text{a girl given that the chosen student is left handed}) = \frac{5}{7}$



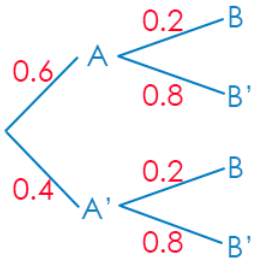


Question 12

a $P(\overline{A} \cap \overline{B}) = 0.32$

b $P(B) = 0.2$

c $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.6 + 0.2 - 0.12$
$= 0.68$



d $P(B|A) = 0.2$

e $P(A|B) = \frac{P(A \cap B)}{P(B)}$
$= \frac{0.12}{0.2}$
$= 0.6$

f Yes, A and B are independent as $P(B|A) = P(B) = 0.2$