Chapter 3

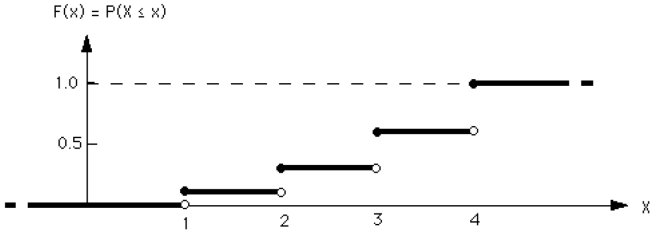
Random Variables

Discrete random variables: probability mass functions, cumulative distribution functions, expectation and variance

1. (a) Yes.

$$E(X) = 3$$

The graph of the CDF is given by



(b) No.

(c) Yes. $E(X) = 36/14$.

(d) No.

(e) Yes. $E(X) = 7/4$.

2. $E(X) = 3$ and $E(X^2 - 2X) = 6$.

3. (a) False.

   (b) False.

4. 0.65.

5. $\mathbb{E}(X) = 0$.

6. 2.

7. (a) $c = 0.36$

   (b) $E(X) = 1.78$, $\text{Var}(X) = 1.0516$.

8. (a) $E(X) = 2.1$, $\text{Var}(X) = 1.09$

   (b) $E(X) = 1.1$, $\text{Var}(X) = 1.09$

   (c) $E(X) = 0$, $\text{Var}(X) = 2.1$.

   (d) $E(X) = 0$, $\text{Var}(X) = 3.2$.

9. (a) $\mathbb{E}(D) = 4.04$, $\sigma_D = 1.8811$

   (b) The probability distribution of X can be expressed in the form of the p.m.f. table below:

x	0	1	2	3	4	5
$p_X(x)$	0.36	0.30	0.16	0.08	0.05	0.05

   (c) The distribution of Y is given by the p.m.f. table below:

y	0	1	2	3
$p_Y(y)$	0.80	0.10	0.05	0.05

   (d) $E(X) = 1.31$, $E(Y) = 0.35$.

Bernoulli and Binomial Distributions

10. (a) $X \sim \text{Bin}(50, 0.10)$

    (b)

    $$\text{P}(X = 7) = 0.1076$$

    (c) $E(X) = 5$, $Var(X)4.5$.

11. (a) $\text{P}(X = 5) = 0.2461$ (4 d.p.)

    (b) $\text{P}(X > 7) = 0.0547$ (4 d.p., Tables).

    (c) $\text{P}(3 \leq X \leq 8) = 0.9346$ (4 d.p., from Tables).

12. (a)

    $$P(X \leq 11) = 0.7175$$

    In the long run, 71.75% of the packets are replaced (surely the business will not survive!).

    (b)

    $$P(X \leq 9) = 0.1109$$

    In the long run, 11.09% of the packets are replaced (much lower, but still very high for a business!).

13. (a) $n = 500$, p could be considered to be fixed as we are only dealing with "blue chip shares" (p will be different from day to day, but that's okay) and the performance of one company to another is probably reasonably independent, though within a particular sector this may not be true (eg if the gold price has fallen then perhaps all gold selling companies will fall in value). I guess

it depends how strong we consider this market sector influence to be as to whether the Binomial random variable is suitable.

(b) No.

(c) If the strength of the earthquakes can be assumed to be independent, and the probability of a reading above 6.5 is fixed for the first 10 earthquakes, then the binomial distribution is reasonable. However, these assumptions are most likely not true.

(d) $n = 31$ but independence between days is unreasonable, so the binomial distribution is not an appropriate model.

14. $\mathrm{P}(X \leq 98) = 0.1880$

Further problems

15. 4

16. (a) Show that $\mathrm{P}(X = 4) = 0.1680$.

$$\begin{aligned}\mathrm{P}(X = 4) &= 1 - \mathrm{P}(X \leq 3) - \mathrm{P}(X = 5) \\ &= 1 - (0.0498 + 0.1494 + 0.2240 + 0.2240) - 0.1848 = 0.1680.\end{aligned}$$

(b) $\mathrm{P}(X < 4) = 0.6472$.

(c) $\mathrm{P}(Y \geq 1) = 0.99807$.

(d) 2.6806, 1.2331.

(e) $\mathrm{P}(X \leq 4) = 0.8152$.

(f) $\mathrm{P}(X > 4 \mid X \geq 3) = 0.3204$.

17. (a) $\mathrm{P}(X \geq 5) = 0.1794$

(b)

(c)

18. 0.45.

19. $\mathrm{Var}(X) = 0.4$.

20. 2.

21. 4

22. (a) $\mathrm{P}(X = 2) = 0.25$.

(b) $\mathrm{P}(Y \geq 1) = 0.7627$.

(c) $\mathbb{E}(Y)0.9375$, $\sigma_Y == 0.9682$.

(d) Interpret your answers.

The probability of at least one child with sickle cell anaemia is very high. On average 1.25 children from such families will have sickle cell anaemia.

Continuous Distributions

23. (a) Mean = $\frac{-5+5}{2} = 0$ pico amps, standard deviation = $(5 - (-5))/\sqrt{12} = 2.89$ pico amps.

(b) $\mathrm{P}(|U| > 3) = 0.4$

24. (a) The function $f_X(x) \geq 0$, and the total area under the graph of $f_X(x)$ is

$$\tfrac{1}{2} \times 4 \times \tfrac{1}{2} = 1,$$

so $f_X(x)$ is a pdf.

(b) $\mathrm{P}(|X| < 1) = 0.75$.