Higher Derivatives

WORKSHEET

Higher derivatives

1 Find the first and second derivatives of the following.

a x^2

b x^5

c $3x^4 - 2x^3 - x^2 + 7x - 1$

d $\sin(x)$

e $5\sin(2x)$

f $3x^{-5}$

g $3x - \pi$

h $x^5 - 4x^3 + 3x$

i e^x

j $3e^{2x}$

k $4\cos(3x)$

l $2x(x^2 - 7x + 1)$

m $s(t) = ut + \frac{1}{2}at^2$

n $\dfrac{x^2 - 1}{x + 6}$

2 Given $g(x) = 4x^4 + x^3 - 2x^2 + 6x - 1$, complete the table.

x	-2	-1	0	1	2
$g(x)$					
$g'(x)$					
$g''(x)$					

3 Find the fourth derivative of the following.

a $5x^4$

b $3e^{2x}$

c $3\sin(x)$

d $x^7 - 2x^5 + x^4 - x^2 + 9$

e $-x^{-1}$

f $\frac{1}{6}(2x - 1)^6$

4 Find the value(s) of x for which $f''(x) = 0$, given that:

a $f(x) = x^3 - 6x^2 - x + 10$

b $f(x) = 2x^3 + 5x^2 - 3x + 1$

c $f(x) = \dfrac{x^4}{12} - \dfrac{x^3}{3} - \dfrac{15x^2}{2} + 11x - 7$

d $f(x) = \dfrac{x^3}{3} + x^2 - \dfrac{x}{3} + \dfrac{1}{3}$

e $f(x) = 5x^7$

f $f(x) = \dfrac{x^4}{12} - \dfrac{9x^2}{2} + \dfrac{2x}{3} - 8$

5 The position (in metres) of an object after t seconds is given by $x(t) = t^3 - 4t^2 + 4t$. Find:

a the initial velocity of the object

b the velocity of the object after 1 second

c the velocity of the object after 3 seconds

d the initial acceleration of the object

e the acceleration of the object after 3 seconds

f the position, velocity and acceleration of the object after 2 seconds

g the position, velocity and acceleration of the object after 4 seconds.

Answers

1 **a** $2x$ 2

    b $5x^4$ $20x^3$

    c $12x^3 - 6x^2 - 2x + 7$ $36x^2 - 12x - 2$

    d $\cos(x)$ $-\sin(x)$

    e $10\cos(2x)$ $-20\sin(2x)$

    f $-15x^{-6}$ $90x^{-7}$

    g 3 0

    h $5x^4 - 12x^2 + 3$ $20x^3 - 24x$

    i e^x e^x

    j $6e^{2x}$ $12e^{2x}$

    k $-12\sin(3x)$ $-36\cos(3x)$

    l $6x^2 - 28x + 2$ $12x - 28$

    m $s'(t) = u + at$ $s''(t) = a$

    n $\dfrac{x^2 + 12x + 1}{(x + 6)^2}$ $\dfrac{70}{(x + 6)^3}$

2

x	-2	-1	0	1	2
$g(x)$	35	-6	-1	8	75
$g'(x)$	-102	-3	6	21	138
$g''(x)$	176	38	-4	50	200

3 **a** 120

    b $48e^{2x}$

    c $3\sin(x)$

    d $840x^3 - 240x + 24$

    e $-24x^{-5}$

    f $3840x^2 - 3840x + 960$

4 **a** 2

    b $-\dfrac{5}{6}$

    c $5, -3$

    d -1

    e 0

    f $3, -3$

5 a 4 m/s

  b -1 m/s

  c 7 m/s

  d -8 m/s^2

  e 10 m/s^2

  f 0 m   0 m/s   4 m/s^2

  g 16 m   20 m/s   16 m/s^2