Generators and transformers.

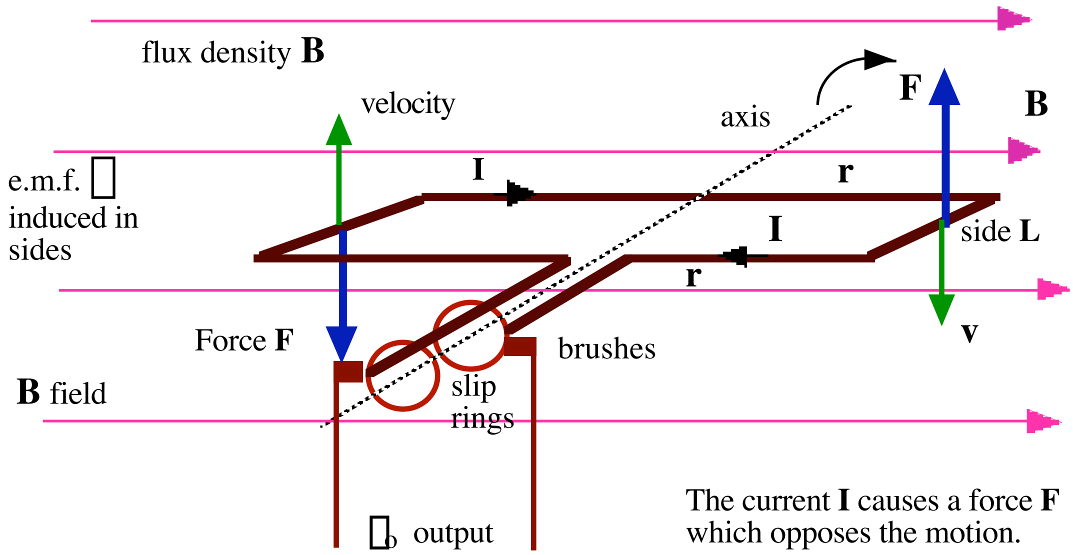
9.1 The structure of a simple a.c. generator.

The a.c. generator is very similar to the electric motor, but alternating current generators do not have a split ring commutator, instead they have two slip rings. It does not have multiple coils as a.c. is required [except for three phase power], but it does have a iron core to increase the flux density. The magnetic field is not radial, since this would not give a sinusoidal e.m.f.

9.2 Theory

As the coil of **N** turns rotates in a clockwise direction, an e.m.f. is induced in the two sides, each of length **L**.

Fig.9[i] A simple a.c. generator



The e.m.f. in **each side** is given by

ε **= v L B sin** θ where **v** is the velocity and θ is the angle between **v** and the field **B**.

. **v sin** θ is the component of the velocity perpendicular to the field.

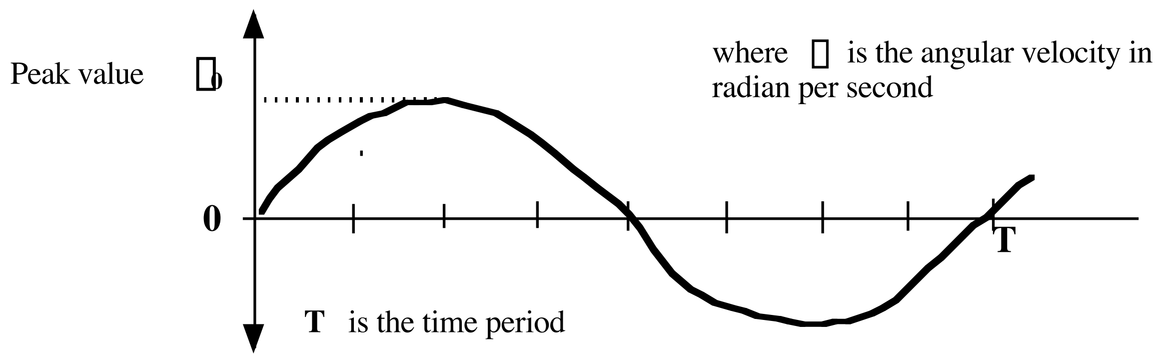
θ is 90^0 as shown above and the e.m.f. is maximum. [sin 90^0 = 1.00]

The coil rotates at constant angular velocity ω such that θ **=** ω **t** and the angle θ varies uniformly with time. Therefore if the induced e.m.f. varies sinusoidally with time and $\theta = 0$ when the coil is vertical, then the e.m.f. is given by the expression:-

ε **= 2 v L B N sin** ω**t** for both sides of the coil and **N** turns

Therefore ε **=** ε_0 **sin** ω**t** where ε_0 is the maximum e.m.f.

Fig.9[ii] Graph of e.m.f. induced against time



$T = {}^{1}/_{f}$ $\{ = {}^{2\pi}/_{\omega} \}$ $v = r\omega$ circular motion

and area of coil **A = 2 r L** **f** is the frequency .

The peak voltage ε_0 is proportional to speed of rotation or frequency.

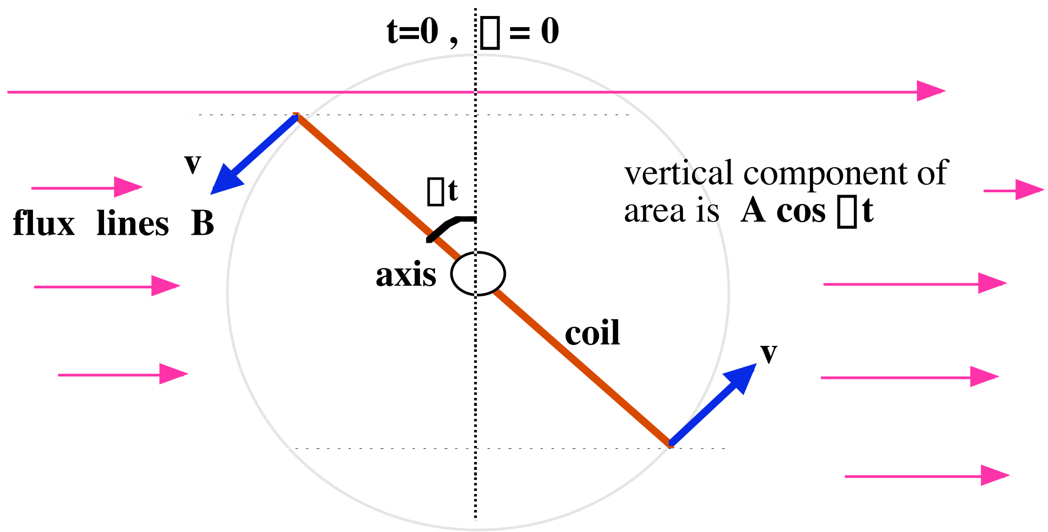
Thus $\varepsilon = \varepsilon_0 \sin \omega t$ where $\varepsilon_0 = B\,A\,N\,\omega$

$\varepsilon_0 = B\,A\,N\,2\pi f$

Therefore peak or maximum e.m.f. $\varepsilon_0 \propto$ frequency [speed] **f**

Calculus students may calculate the rate of change of flux through the coil of area **A**.
Angular velocity is 'ω' in radian per second

Fig.9[v] Rotating coil in a uniform field.



$\phi = B\,A\cos\omega t$ and therefore the rate of change of flux is

$\varepsilon = -N\,{}^{d\phi}/_{dt} = +B\,A\,N\,\omega \sin \omega t$

n.b. Students need only know

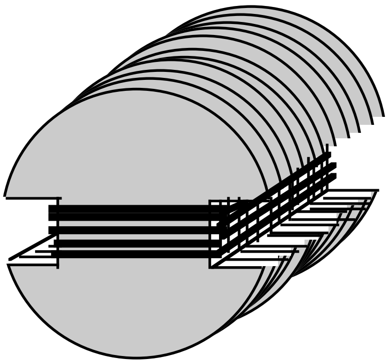
- [i] **that the induced e.m.f. is sinusoidal w.r.t. time.**
- [ii] **the e.m.f. is proportional to the frequency or speed of rotation**
- [iii] **the e.m.f. is proportional to the field B, area A, no. of turns N.**
- [iv] **the emf is maximum when the plane of the coil is parallel to the magnetic field and zero when the plane of the coil is perpendicular to the field**

9.3 Laminations.

The iron core is also rotating in the magnetic field and therefore there will be an e.m.f. induced in the core. This will be in the same direction as the current in the coils. These current are called eddy currents and they can be quite large. Lenz's law tells us that they will cause a force opposing the rotation of the coil.

They will also cause the core to get hot wasting energy. Eddy currents are kept to a minimum by laminating the iron core. It is effectively made up of iron discs separated by insulating glue. This allows the magnetic flux to pass radially but stops the eddy currents flowing around the perimeter. Motors and transformers also have iron cores that are laminated for the same reason.

Fig.9[iii] A laminated iron core



9.4 Root Mean Square [r.m.s.] current and voltage.

[i] Definition.

The root mean square e.m.f. [ε_{rms}] is the effective value of the e.m.f. It can be shown to be equal to the peak value of the e.m.f. times the square root of two. $\varepsilon_o = \varepsilon_{rms} \times 2^{1/2}$

The same applies to the a.c. current and this should be obvious from the formula for power in a resistor, where if the current varies we need to use the square root of the mean value of the square of the current.

$\mathbf{P = R\ [I_{rms}]^2}$ $\varepsilon_o = \varepsilon_{rms} \times 2^{1/2}$

The following proof is definitely not needed for the exam.

[ii] Proof

If we consider the effect of the current over one cycle we must use the calculus and integrate. The instantaneous power [or the energy dissipated per second] is **VI** or in a resistor **RI^2** . Of course **V** and **I** are variables and we need an average value that gives the correct power over a period of time. If we consider a very small interval of time **dt,** over which the value of the current **I** can be considered constant, then the energy dissipated **dE** is given by

$\mathbf{dE = RI^2\ dt}$ which is reasonable since **dt** is as small as we wish.

$$I = I_0 \sin \omega t \qquad \text{Energy per cycle} = \int_0^T RI^2 dt$$

$$\text{Energy} = \int_0^T R I_0^2 \sin^2 \omega t \qquad T = 2\pi/\omega$$

$$\text{Energy} = \frac{R I_o^2}{4} \left[2t - \sin 2\omega t \right]_0^T = \frac{R I_o^2 \pi}{\omega}$$

$$\text{Average power} = \frac{\text{Energy}}{T} = \frac{R I_0^2}{2}$$

Therefore if the effective current is I_{rms}

The mean power is thus given by $P = R I_{rms}^2 = \frac{R I_o^2}{2}$

$$\text{and } I_{rms} = \frac{I_o}{\sqrt{2}}$$

e.g. The peak value of the mains voltage, which has a r.m.s. value of 240V is therefore

$\square_o$ = 240 x 1.414 = 340V

9.5 A simple d.c. generator.

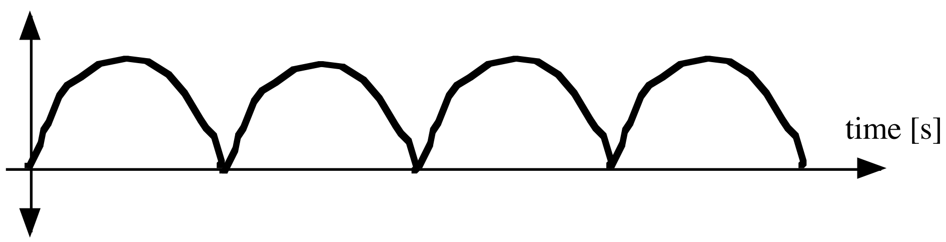
A simple d.c. motor becomes a d.c. generator [dynamo] when rotated by an external agent. The output is as shown below for a **uniform field**. Since the split ring commutator reverses the a.c.emf induced in the coil , a one way or d.c. output is obtained.

If **multiple coils** and a **radial field** are used then the output is nearly constant. This output can be smoothed by using a capacitor and a 'choke' giving a very steady voltage.

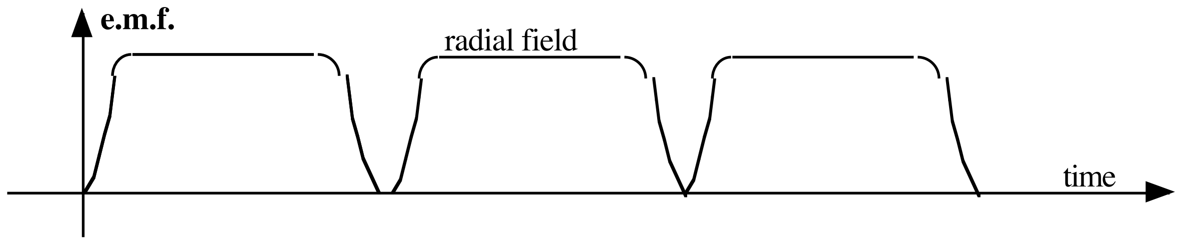
Fig.9[iv] Graph of output for a d.c. simple generator.

Output for a single coil in a uniform field

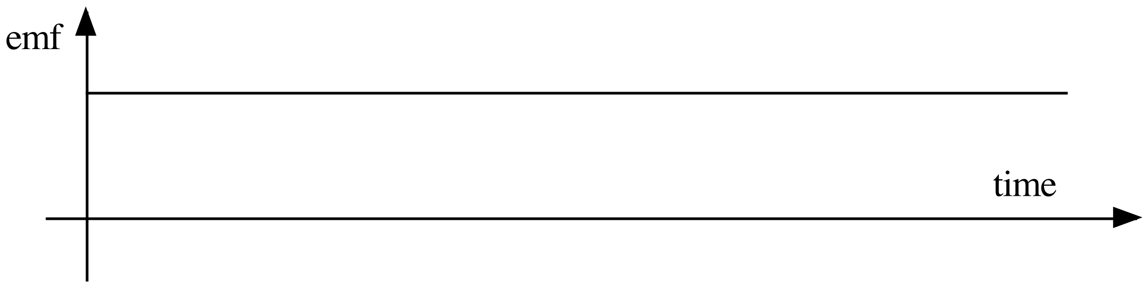
output [V]



Output of a single coil with a radial field



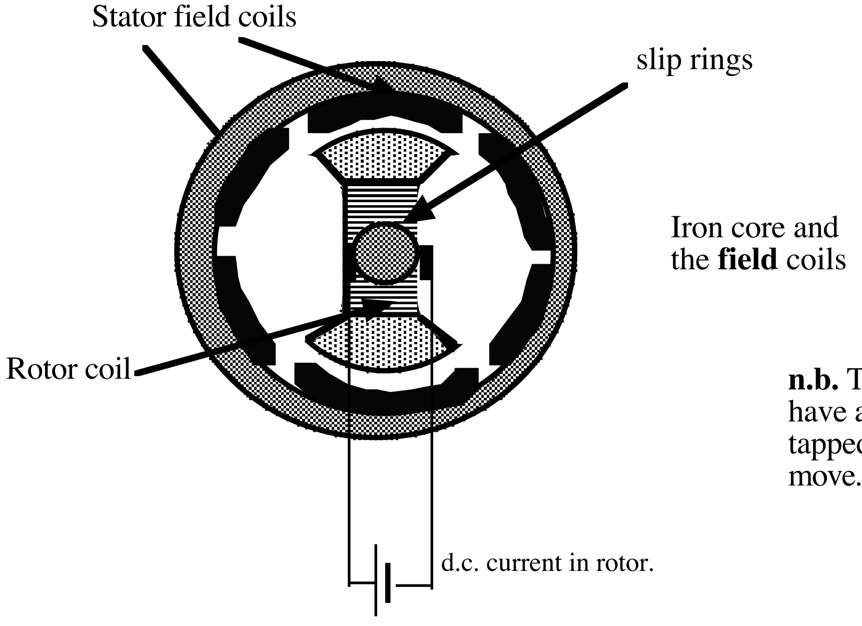
Output for multiple coils and a radial field



9.6 Alternators.

The problem with the a.c. generator discussed so far is that if large currents are drawn the rings will be heated, arcing can occur and wear out the rings. An alternator has a very different arrangement, the output is from stationary coils wound on the iron frame, called the stator.

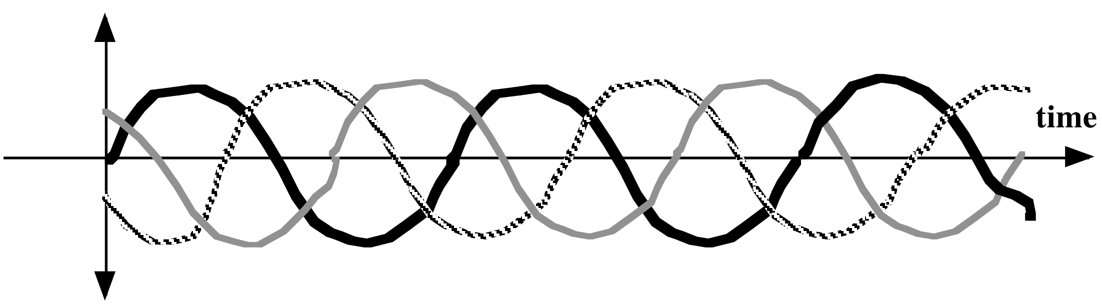
Fig.9[vi] Diagram of alternator.



n.b. The stator coils each have an output that is easily tapped since they do not move.

The varying flux is obtained by rotating an electromagnet between these coils. This operates on d.c. and the small current is fed in through two slip rings as before. However, the small current and absence of split rings means much less chance of sparking and much less wear and tear. Each of the pairs of stator coils will have a sinusoidal e.m.f. induced in it and the three pairs of coils will be 120^0 out of phase with each other. The effective value of the voltage using three phase power is much higher than the 240V r.m.s. on single phase. It is about 415 V r.m.s.

Fig.9[vii] Three Phase Output.

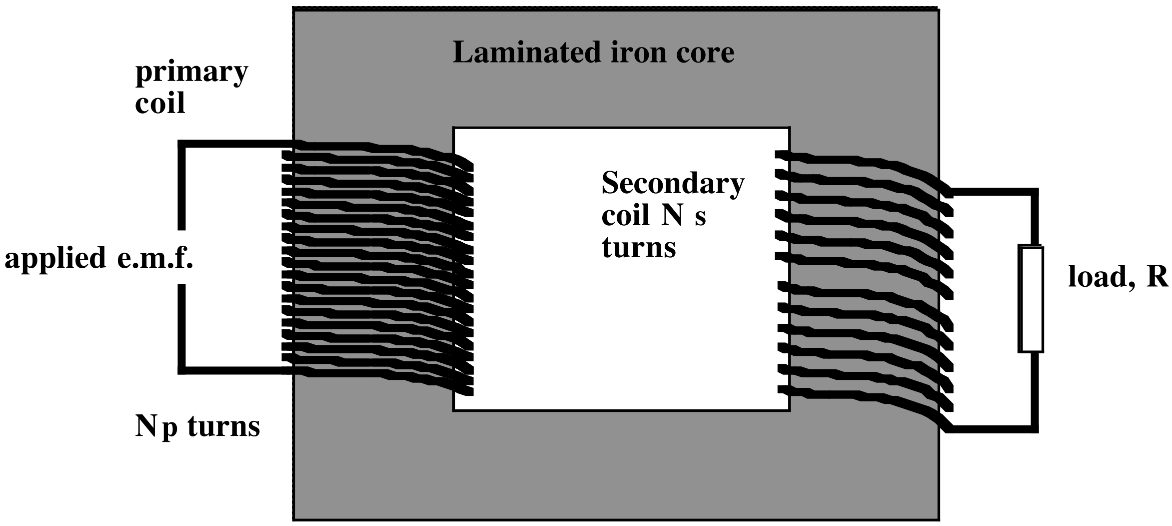


9.7 Transformers.

[i] Construction.

A transformer is a device to change the voltage of a supply. They do not supply energy and in ideal cases do not use energy. The reasons why the e.m.f. needs to be altered are covered a little later but some should be obvious.

Fig.9[viii] Simple diagram of a transformer.



The primary coil has $\mathbf{N_p}$ turns and the secondary $\mathbf{N_s}$ turns . Both coils have a very low resistance to reduce energy wastage caused by heating.

If the applied a.c. voltage in the primary is $□_p$ then the voltage induced in the secondary is $□_s$ then :-

$$\frac{□_s}{□_p} = \frac{N_s}{N_p}$$

e.g. In the example shown if the input $□_p$ was **106V** r.m.s.; $N_p = 1000$ and $N_s = 400$; then the output would be given by :-

$$□_s = 240 \times {}^{400}/_{1000} = \mathbf{96\ V}$$

[ii] Explanation of the action of a Transformer.

The applied e.m.f. in the primary coil causes a changing current Ip in the primary.This produces a changing flux which in turn produces a back e.m.f. in the primary. This back e.m.f. is proportional to the rate of change of flux in the primary and is produced by self induction. Assuming the resistance of the coils is negligible then the current must produce flux at such a rate that the back e.m.f. is equal and opposite to the primary applied e.m.f. at all times. The applied e.m.f. in the primary is $□_p$

Thus $\mathbf{□_p + □_b = 0}$

and $\mathbf{□_p = -□_b = N_p} \times {}^{d□}/_{dt}$ [rate of change of flux]

The same flux all passes through the secondary coil due to the high permeability of the iron core.

Therefore $\square_s = - N_s \times {}^{d\square}/_{dt}$ therefore $^{[ii]}/_{[i]}$

$$\frac{\square_s}{\square_p} = \frac{N_s}{N_p}$$

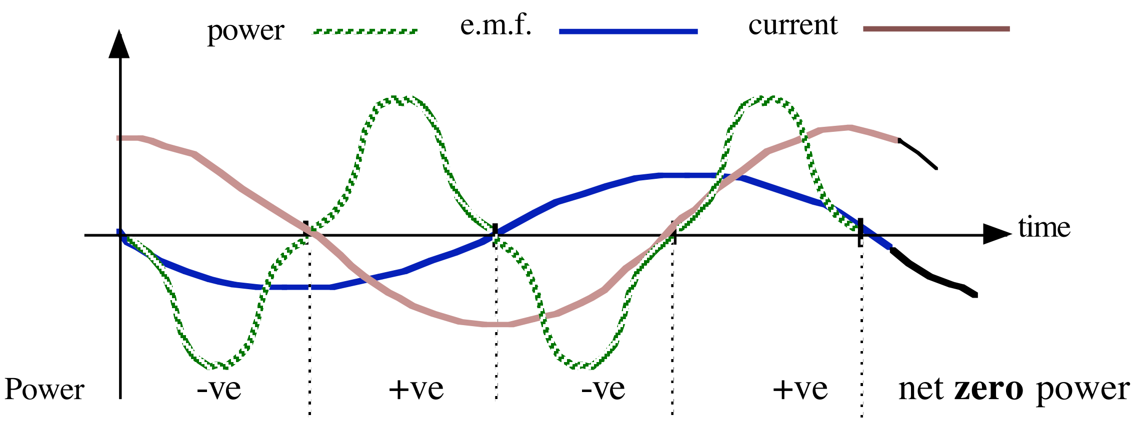
The minus sign is usually ignored at this level, but it tells us that the e.m.f. in the secondary opposes the changing flux in the iron core while the applies e.m.f. is causing the change in the flux.

[iii] Zero load characteristics.

The minus sign shows a phase difference between the secondary and the primary e.m.f.'s of 180^0 [they are out of phase]. There is no current in the secondary but there is in the primary.

Assuming the resistance of the primary is negligible there is a phase difference between the primary e.m.f. and the primary current of 90^0 when zero power is used in the secondary. This means that the transformer functions without wasting energy when not in use even though there is a current in the primary. This is shown by the power curve which is the product of the current and the e.m.f.

Fig.9[ix] Graphs of applied e.m.f. and primary current against time



Power **P = V I** The average power over one cycle is zero.

There is a detailed discussion of this in the section on induction. If you are lucky you may see these phase differences demonstrated on a double beam C.R.O. or on a computer using current and voltage probes.

[iv] Action of the transformer under load.

When energy is drawn from the secondary the phase difference becomes less than 90^0. The current in the secondary produces flux that opposes the changes in the flux due to the primary. However the primary must produce a back e.m.f. equal to the applied e.m.f. and therefore the primary current increases in order to do this. The actual power used in either primary or secondary is not given by

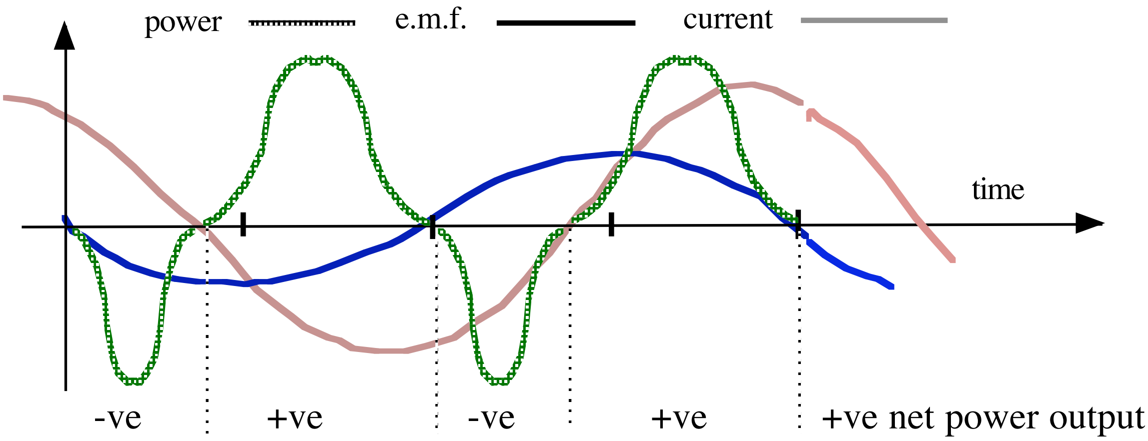
$$P = V I \qquad \text{or even } P = V_{rms} I_{rms}$$

The latter expression is called a rating and measured in **VA**. It is used by engineers and commercial operators. The actual mean power is given by the expression

$P = V_{rms} I_{rms} \cos \square$ where $\cos \square$ is called the power factor.

The symbol □ is the phase difference between the primary current and the primary voltage. This must be less than 90o to give a net positive output from the primary which is of course the source of the energy dissipated in the secondary. Power factors of quite small values are obtained with large inductive devices such as motors. Since consumption is measured and costed by meters that actually measure I^2 [current squared] then allowances must be made. Big industry takes steps to reduce the value of the phase difference by using large capacitors.

Fig.9[x] The phase difference under load.



[v] Impedance matching.

There is another very important aspect of efficiency and that is Impedance matching. If the resistance of the load, assuming it is a resistive load, is very different to the resistance [impedance] of the secondary then the power output is low. The maximum output power is obtained when the impedance of the load is equal to the internal impedance of the secondary.

When this is not the case and energy losses would be high a 1 : 1 transformer can be used to match the impedances. This is easily demonstrated using a small transformer with various coils with different numbers of turns and measuring the efficiency under several different loads. However, the values of current and voltage using a.c. meters will not give the actual power in watts but the rating in VA. The phase differences are not as easily measured but the output will show a maximum and the p.d. across the load drops as the current increases.

[vi] The function of the iron core.

The iron core serves the function of maintaining the flux linkage between the coils at nearly 100% and thus the ratio of secondary to primary e.m.f. is as shown. This could be described as increasing the mutual inductance between the coils. The function of the core is also to increase the self inductance of the primary keeping the primary current down and therefore reducing energy losses due to heating. The resistance of both primary and secondary is usually low and it is essential that their self inductances are high or huge currents would flow and the e.m.f. in the secondary would drop considerable as power in the secondary increases.

If the primary is inadvertently switched on without the iron core in place it will melt very quickly. When assembling transformers always make sure that the mains is off before plugging the primary in. One function of the iron core is to increase the flux density in the primary coil per unit of current so that it does not draw a large current from the supply. A second function is to maintain the linkage between the coils under load. The core does indeed increase the magnitude of the flux but it is more important that it all passes through the secondary. Induction coils can give an output at high potential and low current without an iron core.

However , it is often said that it is the function of the core to increase the flux density and it is advisable to keep your answers simple. You only need to know that transformers change the 'voltage'. See your text books for examples and applications. See the notes on mutual induction on this CD.

9.8 Transmission of electric energy.

[i] The efficiency of transformers.

The efficiency of good transformers is close to 100% and therefore the power output nearly equals the power input. Assuming this is the case then the currents in primary and secondary are in the opposite ratio to the e.m.f.'s

Power in = Power out or $V_p I_p = V_s I_s$

A good transformer is very efficient. This means that very little energy is lost or wasted in the process of transforming the voltage. The following all affect efficiency.

[i] The windings must be tight
[ii] The resistance of the coils should be very low to reduce losses due to heating in the coils.
[ii] Eddy currents induced in the iron core due to the changing flux cause heating and loss of energy.
[iv] Hysteresis losses in the core due to the continual remagnetisation as the current altenates.

Transformers are nearly 100% efficient if there are very low resistive losses and the the eddy currents induced in the iron core are reduced to a minimum by laminations. Magnetic losses due to hysteresis are the only significant cause of energy loss and the use of a soft iron alloy called mumetal reduces this. However the thermal energy from all sources produced in big transformers is still enough to require cooling systems to keep the temperature down.

[ii] The power grid.

Ignoring the phase differences and output changes discussed in the previous section then it can be shown that energy is best transmitted at high potential. Energy is transmitted over large distances and the energy losses in the overhead power lines due to heating can be considerable. The losses are reduced to very low percentages by using transformers to send power at high voltage.

Since $\mathbf{P = V\,I}$ then, for a given power, if the voltage is high then the current is low. The energy used in the lines and causing heating is dependent on the resistance and the current in the lines.

Power wasted in lines $P_{loss} = V_{drop}\, I = R\, I^2$

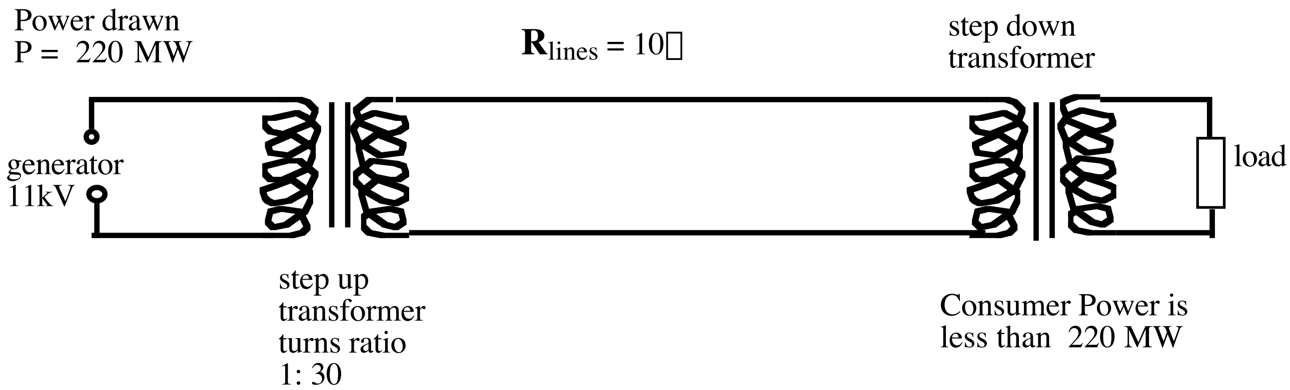
where $V_{d\,rop}$ is the p.d. across the lines or the drop in p.d. [voltage] at the consumer's end of the lines and **R** is the resistance of the lines. The p.d. drop along the lines depends on the power used by the consumer.

This also affects the efficiency of the system.

Of course in practice there is still a current in the lines when there is no load. How else can there be an e.m.f. in the primary of the step down transformer at the consumer's end ? Therefore there are losses even when no energy is being used. The r.m.s. values of the currents and voltages do not give the actual power input or output. However, a simplistic approach can be used to give a reasonable approximation of the losses :-

e.g. The power station shown below generates 11 kV r.m.s. and this is then transformed up to 330 kV. At the consumer the voltage is stepped down in stages to 240V. If all the consumers draw 220MW from the power station then the values for losses and voltage drops are as follows.

Fig.9[xi] A reticulation circuit.



The consumers draw 220MW **from the power station** not the step down transformer. The current in the lines assuming transformers are 100% efficient is found from

$\mathbf{P = V\,I\; = \;220 \times 10^6\; = \;330000 \times I}$ $\qquad\qquad$ $\mathbf{I = 667A}$

Power losses in lines $\mathbf{= R\,I^2 = 10 \times 667^2 = 4.44MW}$

Efficiency = output/input = $\mathbf{{}^{[220\,-\,4.44]}/_{220} \times 100\%}$

Efficiency = 98% or about 2% lost or wasted

The p.d. at consumers end of lines is **330000 - 10 x 667 = 323330 V**

If the p.d. [voltage] at the power station end of the lines is $\mathbf{V_{ps}}$ and this drops to $\mathbf{V_c}$ at the consumer, then the percentage loss of power [energy] is as follows.

Power out $\mathbf{= V_{ps}\,I}$ $\qquad\qquad$ Power used $\mathbf{= V_c\;I}$

Power lost $\mathbf{= [V_{ps} - V_c]\,I}$

Efficiency $\mathbf{= {}^{V_c}/V_{ps}}$ x $\mathbf{100\% = {}^{323330}/_{330000}}$ x $\mathbf{100\% = 98\%}$

Due to the drop along the lines the power delivered to the consumers will be a little less than 220 MW but these differences are very small. The voltage drop along the lines can be allowed for by altering the step down ratio in the street transformers and so consumers all get the same operating voltage.

Transformers allow energy to be transmitted over large distances without great loss of energy in the overhead or under ground lines. They also enable all consumers to operate at nearly the same voltage.

In the Fastrak Train system the potential drop along the lines is small when there is not much power drawn, so that when it is stepped down to 600V the trains get nearly the same voltage, however far they are from the sub station. However, when many trains are running the potential drop along the lines is larger and the trains furthest from the sub station work on as little as 550V.

e.g. The trains on the Northern Suburbs line draw power from overhead lines kept at 25kV at Claisebrooke. The p.d. at Warwick drops to 24kV when the line is not busy. The power used by two trains operating near Warwick at this time is 2.4MW and just at this moment no other trains were running.

[i] What is the current drawn by the trains?
[ii] What is the resistance of the overhead lines?
[iii] What is the power drawn from Claisebrooke?
[iv] What is the percentage power loss in the lines?

Solution.

[i] $P = V\,I$ $\quad 2.4 \times 10^6 = 24 \times 10^3\,I$ $\quad$ **$I = 100A$**

[ii] p.d. across lines $V_d = 25kV - 24kV = 1000 = R\,I = R \times 100$ $\quad$ **$R = 10.0\Omega$**

[iii] $P = V\,I = 25 \times 10^3 \times 100 = 5.0 \times 10^6 W =$ **2.5 MW**

[iv] $P = R\,I^2 = 10.0 \times 100^2 = 0.10MW$

Percentage loss = $\{^{0.1}/_{2.5}\} \times 100\%$ **= 4.0%** **or** $[^{[2.5 - 2.4]}/_{2.5}] \times 100\%$ **= 4.0%**

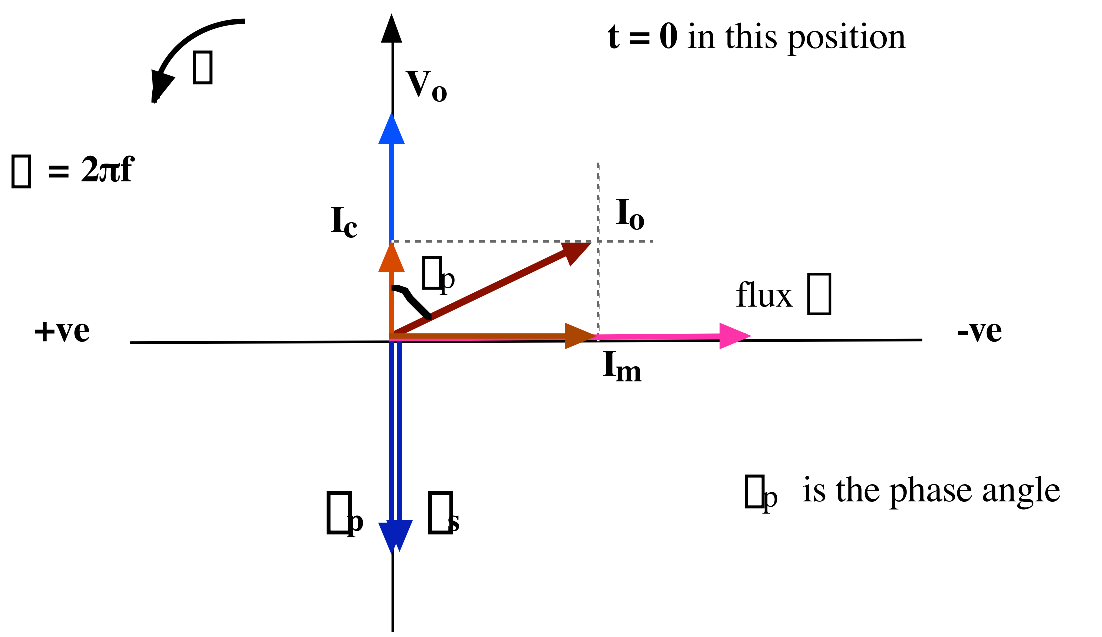
9.9 Phase differences and phasors in transformers,.

[i] Secondary with no load.

The flux in the iron core is produced by the primary current **Ip** which normally lags behind the applied e.m.f. of the primary **Vp** by 90°. However there are resistive losses in the primary as well as losses due to eddy currents and hysteresis and part of the primary input voltage is used across the coils. The resistance is usually very low and the component $\mathbf{I_c}$ which is in phase with $\mathbf{V_p}$ is quite small. There is of course a back e.m.f. in the primary □$_p$ which is equal and opposite to the input except for the very small difference due to the resistive power losses [which would be $\mathbf{V_p I_c}$]. The flux in the iron core is in phase with the component of the primary current $\mathbf{I_m}$. All of these quantities can be represented by a vector rotating in an anticlockwise direction with frequency **f** as shown below. These are called **phasors**. The e.m.f. in the secondary depends on the turns ratio which we shall take to be unity for simplicity.

Thus $\mathbf{N_p = N_s}$ and **□$_p$ = □$_s$ = $\mathbf{V_p}$**

Fig.9[xii] Values for a transformer with no load



All instantaneous values can taken as the component of the phasor along one of the axes, say the x axis, but taking to the left as positive, we have :-

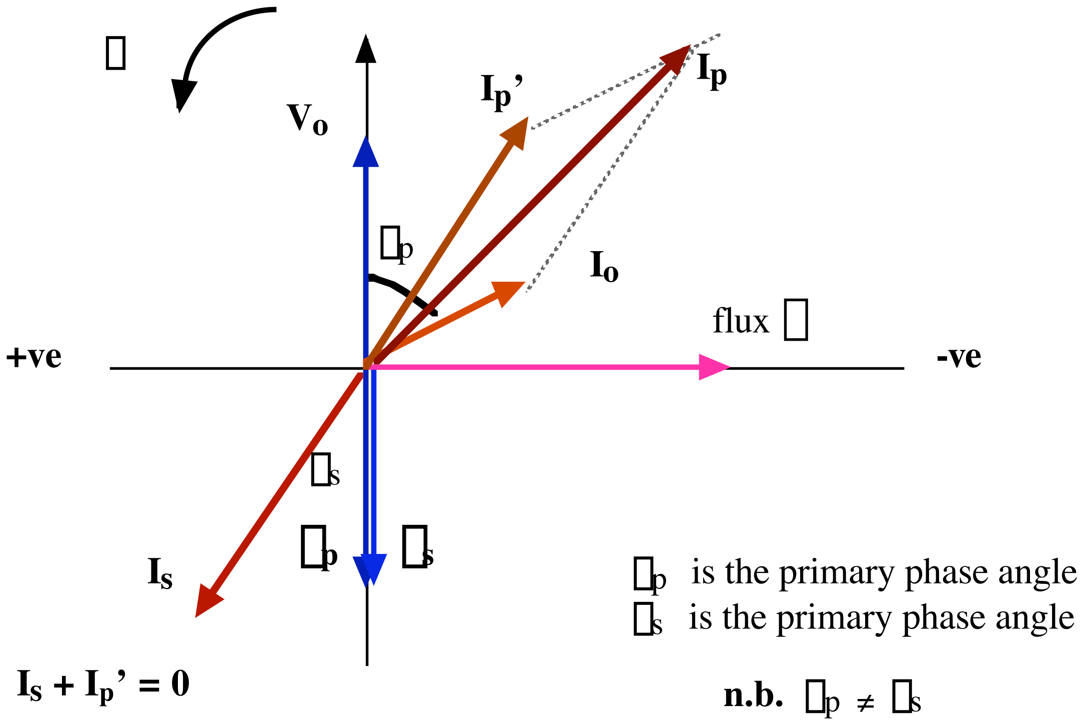
Instantaneous primary input $\mathbf{V_p = V_o}$ **sin [□t]**

Instantaneous primary current $\mathbf{I_p = I_o}$ **sin [□t -□]** [where I_o is the peak [no load] current]

[ii] Secondary with a load.

If there is a current in the secondary $\mathbf{I_s}$ this will oppose the changes in the flux and therefore the current in the primary must rise and completely counteract the effect of the secondary current. However, the secondary current is not in phase with the secondary voltage or e.m.f. □$_s$. There will be a lag □$_s$, which depends on the nature of the load connected in the secondary. The primary current must rise to produce an equal and opposite component $\mathbf{I_p}$'. If we ignore any losses due to resistance in the coils then the phase diagram becomes as shown below.

Fig.9[xiii] Transformer under load.



Hence we can now see that the secondary current must cause a rise in the primary current and a phase shift. Remember the flux in the core must change at such a rate that the back e.m.f. in the primary is equal and opposite [or nearly] to the input. This flux in the iron core is still caused by the component of $\mathbf{I_o}$ which is $\mathbf{I_m}$ in fig.8[xii] and the flux is the same under load as it is with no load. As the phasors rotate the power drawn in the primary will be negative for some of the time, but if □$_p$ is less than 90° the power is mostly positive.