# Electric Power

## 5.1 Electrostatics.

**Charge.** symbol **q** unit coulomb [C] scalar

This is a property of subatomic particles. There are two types of charge that are called positive and negative because they cancel each other or have opposite effects. Protons are positively charged and electrons are negatively charged. They both have charges of magnitude **$1.60 \times 10^{-19}$ C.**

Like charges repel and opposite charges attract and we attribute this force to the existence of charge. The electric force keeps the electrons orbiting the nucleus in the atom. In an atom there are equal numbers of protons and electrons which means that the atom is **neutral** or has no net charge.

Charge is measured in **coulomb [C]** and the definition of one coulomb is from the **fundamental unit** of electric current the **ampere**. The ampere is defined in terms of electromagnetic effects.

Charge is **quantised**. At our level of treatment it only comes in integer multiples of the electron or proton charge of $\mathbf{e} = +1.60 \times 10^{-19}$ C.

$$\mathbf{q = n\,e} \qquad \text{[where } \mathbf{n} \text{ is an integer]}$$

Charge is **conserved** in all interactions [including relativistic transformations].

### Charging.

Atoms or molecules can be **ionised** by several means. In solution many metal salts, acids and bases separate into positive and negative ions freely. In flames the collisions between fast moving molecules ionises many of them. **Metals** consist of a crystal lattice of positive ions and a cloud of **'free'** electrons. The metal ions and the electrons hold each other together. The free electrons can move through the metal if influenced by an electric force field. This is why **metals are good conductors** of electricity [and heat].

Many non metallic materials can be **'charged'** by friction, rubbing with a different substance. The electrons are loosely held by the molecules of one material. In this case we do not refer to it as ionising. One substance will lose electrons, becoming positive and the other will gain electrons, become negatively charged. Notice there is never a creation of charge, only a separation of positive and negative charges. This is another of the conservation laws:-

### The Law of Conservation of Charge;

This states that charge can neither be created nor destroyed, the total charge in the cosmos is constant and zero. The origin of the word 'electron' is from the Greek for amber 'electros', which when rubbed becomes charged negatively. Charging by friction was known to the Greeks three thousand years ago, although they did not know of electrons, they did suggest the existence of atoms.

## 5.2 Coulombs Law.

The force between two point charges, or charged spherical conductors, depends on the magnitude of each charge and on the inverse of the square of their distance apart.

$$F = q_1 q_2 / 4\pi\varepsilon\, d^2 \qquad \text{in a medium}$$

Where $\varepsilon$ [epsilon] is a constant called the absolute **permittivity** of the medium.

In a vacuum $\varepsilon$ is the **permittivity of free space** denoted $\varepsilon_0$

$\varepsilon > \varepsilon_0$ i.e. the force is greatest in vacuo.

The permittivity of free space $\varepsilon_0 = 8.84 \times 10^{-12}\ \mathrm{Fm^{-1}}$ $[\ \mathrm{C^2N^{-1}m^{-2}}\ ]$

**n.b.** This relationship only applies to point charges or charged spherical metal conductors, not to any other shapes unless they are very far apart compared to their size, in which case it is approximately true.

## 5.3 Electric Fields.

The region around any charges in which a forces will act on any other charge is called an electric field. The field is denoted [visualised] by drawing 'lines of force' which show the direction of the force on a small positive charge at any point. The 'small' charge has to be infinitesimally small otherwise it influences the shape of the field.

**Electric Field Strength** [or Electric Intensity] symbol **E**

The strength of the field at any point is defined as the force per unit charge at that point.

$\mathbf{E} = {}^{\mathbf{F}}/_{\mathbf{q}}$ units $NC^{-1}$ **vector** in direction of **F**

In the region around a point charge **q** or a charged [conducting] sphere with the same charge **q** the field strength is given by:-

$\mathbf{E} = {}^{q}/_{4\pi\varepsilon r^2}$ where **r** is the distance from the centre of the sphere or from the point charge.

**Fig. 5[i]** **The field of a positively charged sphere.**

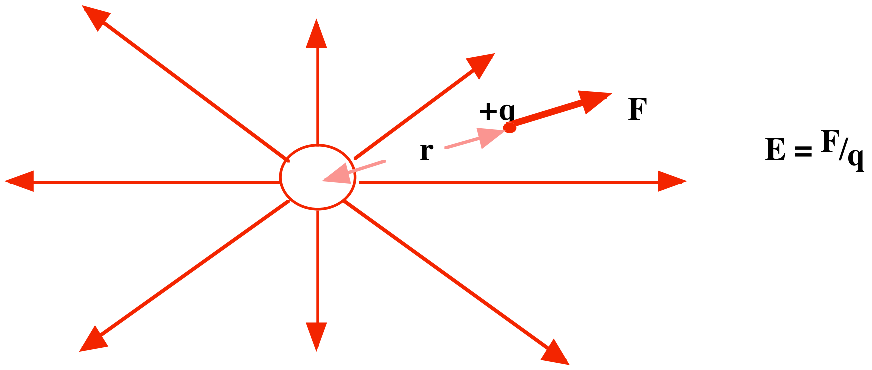


**Fig.5[ii] The field of a negatively charged sphere**

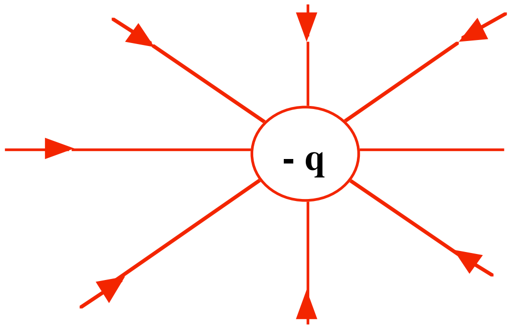


**Fig.5[iii] The field of equal and opposite charges**

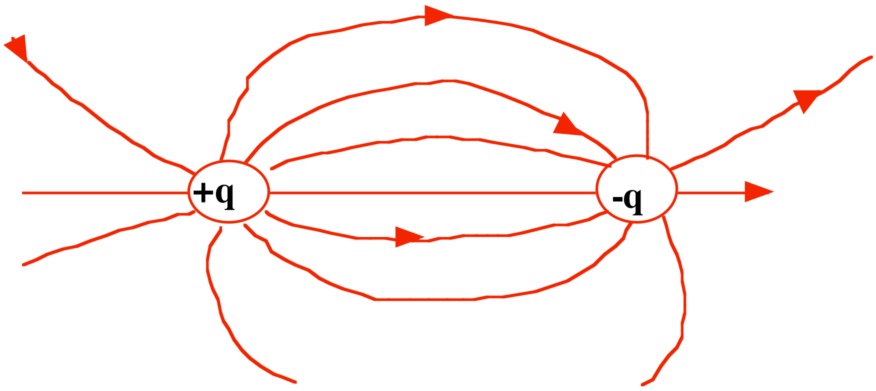
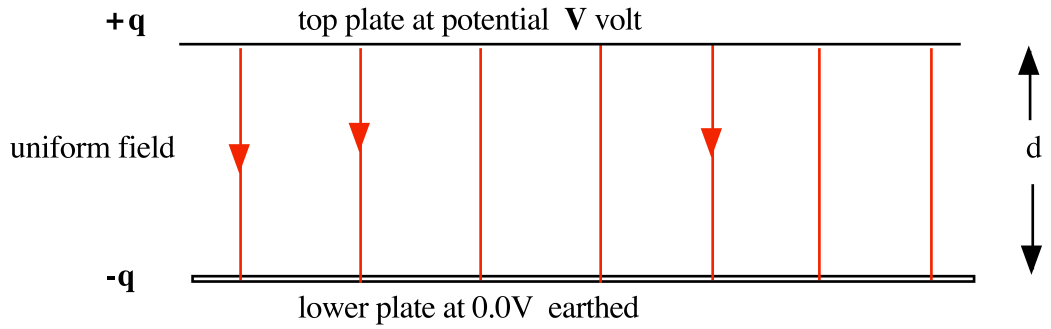


**Fig.5[iv]** **The field between parallel plates [oppositely charged or one is earthed]**



The field is uniform if the plates are relatively close together.

### 5.4 Potential.

symbol **V** unit volt [V] scalar

When a charge is moved around in the field work is done on the charge either by an outside agent or by the field, depending on the direction the charge is moved. The absolute potential at a point in the field is defined as the work done per unit charge in moving a small charge from infinity, or outside the field, to that point.

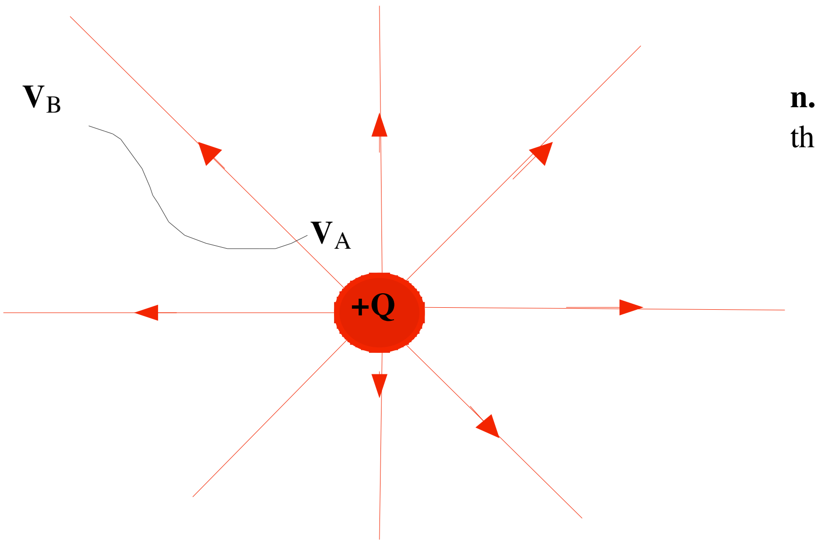
$$V = {}^{W}/_{q}$$

### 5.5 Potential Difference.

symbol **V** unit volt [V] scalar.

The potential difference [p.d.] between two points A and B is the work done **per unit charge** in moving a small charge from one point [B] to the other [A] in the field.

**Fig.5[v]** **Potential difference.**



$$V_{AB} = {}^{W}/_{q}$$

**n.b.** the work done is independent of the path taken from A to B.

**e.g.1** If 6.5J of work is done [by an outside agency] in moving 1.3C of charge from B to A then the potential difference between A and B is :-

$$V_{AB} = V_A - V_B = {}^{W}/_{q} = {}^{6.5}/_{1.3} = +5.0V$$

**n.b. A is at a higher potential than B.**

Hence the potential difference between B and A is negative :-

$V_{BA} = -5.0V$ and the field would do work on the charge, accelerating it if it was free to move.

The definition applies equally to a circuit and the p.d. across components. Potential difference or e.m.f. is measured by a voltmeter, which should have a **very high resistance** and be connected **in parallel or across** the device being measured. The sum of the p.d.'s in a series circuit equals the e.m.f. of the source of energy.

## 5.6 Parallel Plates. [extension notes]

The uniform field in e.g.4 is of strength **E** [$NC^{-1}$] thus the work done in moving charge **q** [coulomb] is from :-

$$W = F s = E q d$$

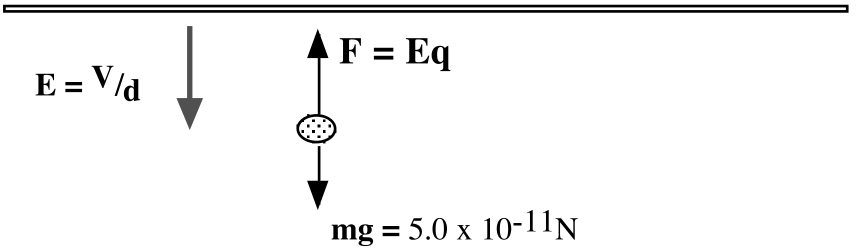
$${}^{W}/_{q} = V = E d \qquad \text{thus} \qquad E = {}^{V}/_{d}$$

Therefore the field strength is equal to the number of volts per metre across the gap between the plates. This is called the **potential gradient** and **alternative units** for electric intensity are volts per metre. [$\mathbf{Vm^{-1}}$] This applies to non uniform fields as well but applications are beyond the requirements of this course.

## 5.7 Further examples [extension]

The p.d. between the parallel plates shown above is $5.00 \times 10^{3}$ V and their separation is 0.500cm. How fast would an oil drop released at the bottom plate, carrying a charge of 1250 excess electrons, be moving if it was of mass 5.00ng and air resistance was negligible?

**Fig.5[vi] Parallel plates**



[take $\mathbf{g} = 10.0\ Nkg^{-1}$ and $\mathbf{e} = 1.60 \times 10^{-19}$ C]

method 1.

Using Forces: $u = 0$ $\mathbf{v^2 = u^2 + 2as}$ up is +ve

$$E = {}^{V}/_{d} = 1.0 \times 10^{6}\ Vm^{-1}$$

$$F = Eq = 1.0 \times 10^{6} \times 1.60 \times 10^{-19} \times 1250 = 2 \times 10^{-10}\ N$$

$$\Sigma F = F - mg = ma \qquad 2 \times 10^{-10} - 5 \times 10^{-11} = 1.5 \times 10^{-10} = 5.0 \times 10^{-12}\, a$$

$$a = 30.0 ms^{-2} \qquad v^2 = 0 + 2 \times 30.0 \times 5 \times 10^{-3} = 0.300$$

$$\mathbf{v = 5.48 \times 10^{-1}\ ms^{-1}}$$

method 2. Using Conservation of Energy:

$$W = V q = 5000 \times 1250 \times 1.60 \times 10^{-19}\ J = 1.00 \times 10^{-12}\ J$$

$$Vq = {}^{1}/_{2} mv^{2} + mgh$$

therefore $v^2 = 2 \times [1.0 \times 10^{-12} - 5 \times 10^{-11} \times 5 \times 10^{-3}] / 5 \times 10^{-12}$

$\mathbf{v = 5.48 \times 10^{-1}\ ms^{-1}}$

When dealing with ions and electrons moving in a vacuum the electric forces are much greater than the gravitational forces, since the masses are so small, and the latter can be neglected.

**e.g.3.** What is the speed of an electron in a T.V. tube that has been accelerated through a p.d. of 5000V ?
$[\ m_e = 9.11 \times 10^{-31}$kg, $g = 9.80$Nkg$^{-1}]$

$\mathbf{W = Vq = {}^1/_2\ mv^2} = 5000 \times 1.6 \times 10^{-19} = {}^1/_2 \times 9.11 \times 10^{-31}\ \mathbf{v^2}$

therefore $\mathbf{v = 4.19 \times 10^7 ms^{-1}}$

**n.b.** This speed is approaching relativistic values and could in fact be less than this figure.

## 5.8 Electron Volt.

This is an **alternative energy unit**, symbol **eV**, and it is used for the energy of ions and fundamental particles such as protons, electrons and alpha particles. When any charged particle is accelerated freely through an electric field it acquires kinetic energy. Since most such particles have charges of '**e**' the electronic charge, and others have a simple multiple of this **ne** where **n** is an integer, then the energy of the particle is proportional to the p.d. that it was passed through.

$$\mathbf{E_k = W = Vq = V\ ne}$$

A singly charged particle [n = 1] will have an energy **V** electron volt. Thus one electron volt is the energy acquired by a particle of charge **e** that has been accelerated freely through a p.d. of **one** volt.

n.b. $\mathbf{1.00eV = 1.0 \times 1.6 \times 10^{-19}\ J}$

If the particle has a charge **2e** and it is accelerated through a p.d. **V** then it's kinetic energy is given by

$$\mathbf{E_k = W = q\ v = 2e\ V}\ \text{[joule]} = \mathbf{2V}\ \text{[electron volt]}$$

**e.g.4.** A proton has kinetic energy of 5500eV. What is it's speed?

[mass of proton = $1.67 \times 10^{-27}$kg]

$\mathbf{E_k = {}^1/_2 mv^2}$ [n.b. energy must be in joule]

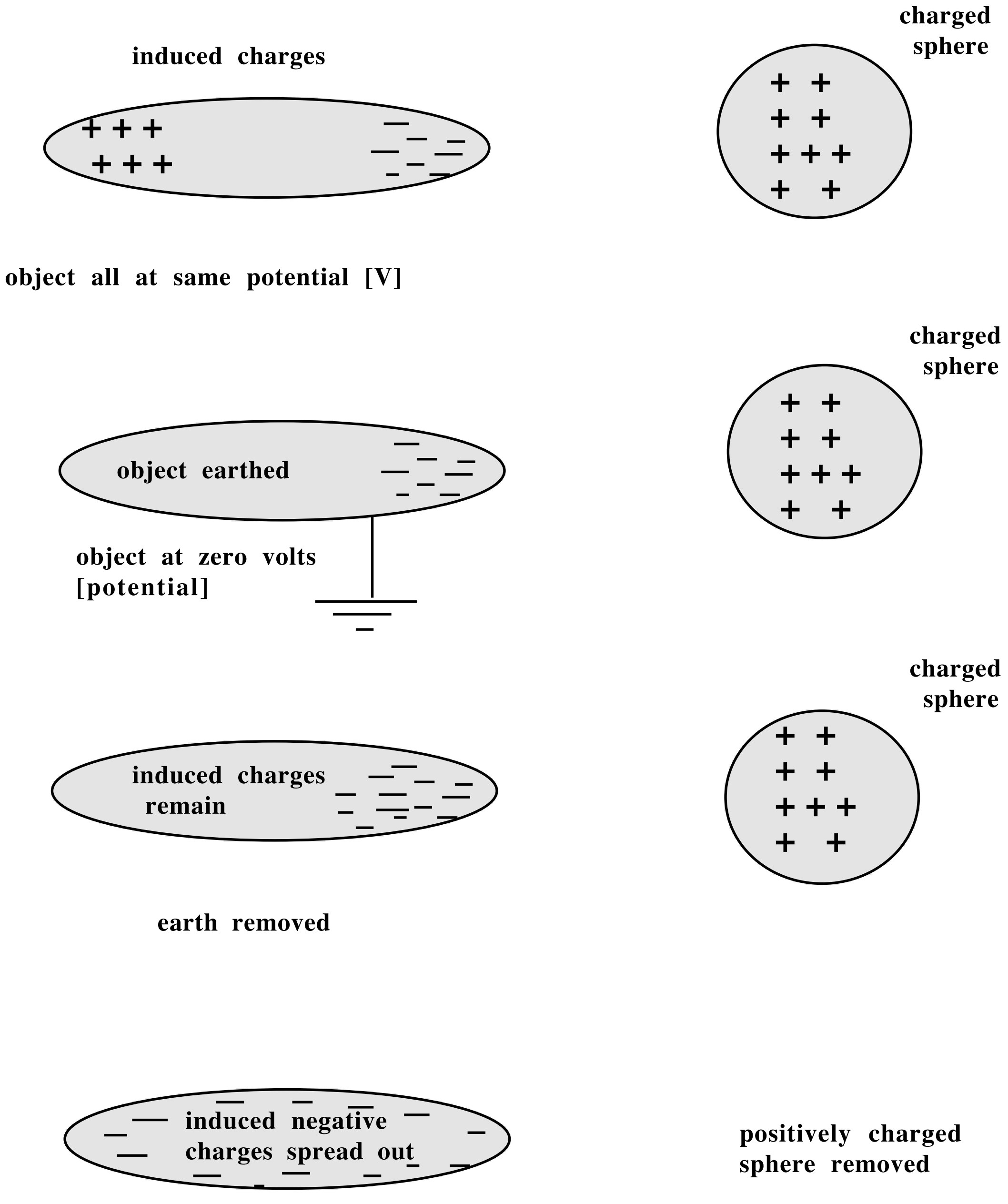
$5500\text{eV} = 5500 \times 1.60 \times 10^{-19}\ \text{J} = {}^1/_2 \times 1.67 \times 10^{-27}\ \mathbf{v^2}$

$\mathbf{v = 1.0 \times 10^6\ ms^{-1}}$

## 5.9 Induction.

When a charged object is near a conductor the electrons in the conductor move until there is no electric field in the metal of the conductor or inside a hollow conductor. If there were a field in the metal there would be a current and since there is no current then the field must be zero. Also the lines of force must leave the surface of a conductor normally [90°] or there would be a force along the surface and again a current would flow. There is no current in the static situation.

**fig.5[vii] Charging by Induction.**



**The object is negatively charged and at a negative potential**

**5.10 Electric Current**. Symbol [**I**] . unit ampere. [**A**]. scalar

Current is the **rate** of flow of [positive] charge. Current is one of the seven fundamental quantities and the ampere is defined in terms of the electromagnetic force. This is covered later.

$$\mathbf{I} = {}^{q}/_{t} \qquad \{ \text{ i.e. } A = C\, s^{-1} \}$$

Current is measured by an ammeter, which should have a **very low resistance** and be connected **in series** with the device through which the current is being measured.

**5.11 Electromotive force**. **[e.m.f.]** symbol [**ε**] . unit volt [**V**]. scalar

A **source** of electric potential energy has a p.d. in volts across it's electrodes which is maximum when no current is drawn. This value is called the e.m.f. of the source. It is the energy per unit charge developed by the source. Engineers may call this the open circuit voltage.

[a] e.m.f. is **not** a force.

[b] The e.m.f. of a source of p.d. is often just called a voltage, and the symbol is also often **V** instead of **ε** .

**e.g.** Chemical cell, generator or dynamo, frictional charging, pizo electric crystals, solar cells, thermocouple.

**5.12 Resistance.** Symbol [**R**] . unit ohm [**Ω** ] . scalar

Resistance is a measure of the difficulty with which charge can flow through a conductor. It is defined as the ratio of potential difference to current for a resistor.

$$\mathbf{R} = {}^{V}/_{I} \qquad \{ \Omega = VA^{-1} \}$$

Resistance is rarely constant and can be affected by temperature and pressure.

**5.13 Power.** Symbol **[P]** unit watt **[W]** scalar.

Power is the rate of transformation of energy, or the rate of doing work.

Power = ${}^{\text{work done}}/_{\text{time taken}}$ = ${}^{\text{energy used}}/_{\text{time taken}}$

$$\mathbf{P = {}^{W}/_{t}} \qquad \textbf{or} \qquad \mathbf{P = {}^{E}/_{t}}$$

Power = Potential Difference x Current **$P = VI$**

Energy = Power x time **$E = P\,t = VIt$**

**Formulae**

Resistors in Series **$R = R_1 + R_2 + \text{etc.}$**

Resistors in Parallel $$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

## 5.14 Circuit rules

**1.** The current through resistors connected in series is the same.

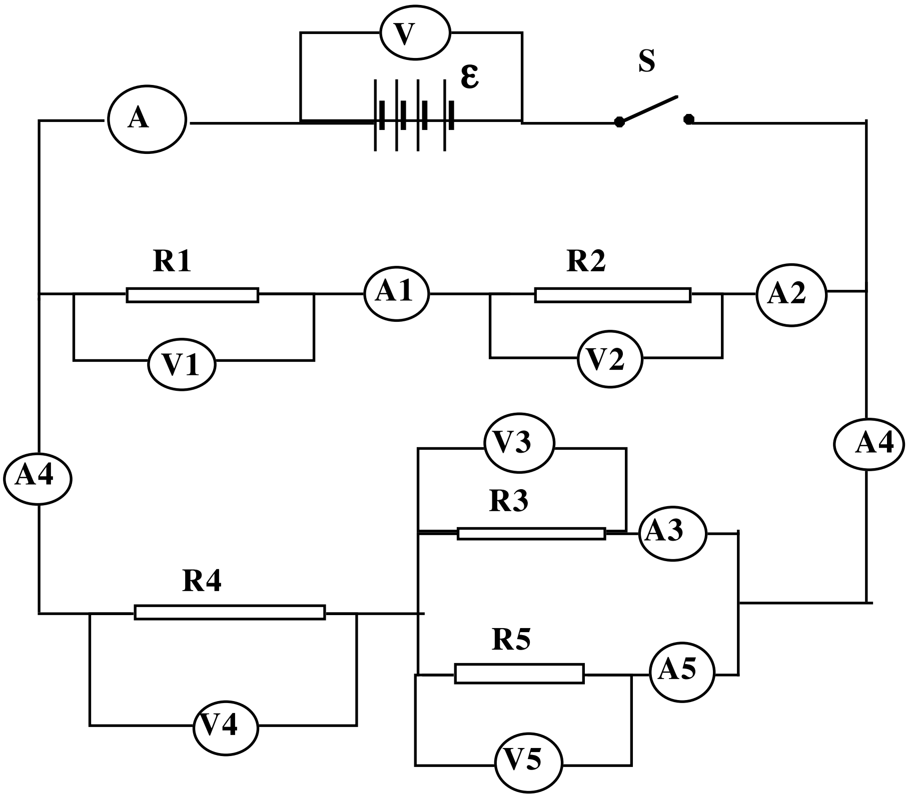
**2.** The potential differences across parallel resistors are equal.

**3.** The sum of the currents entering a junction equals the sum of the currents leaving.

**4a.** The sum of the p.d. across resistors in series is equal to the p.d. across them all.

**4b.** The sum of the p.d.s around a complete circuit equals the e.m.f. of the supply.

**Fig.5[viii] Example circuit.**



The e.m.f. of the battery is ε and the voltmeter **V** reads the e.m.f. if the switch S is open. The definitions and rules described earlier can be applied to the circuit above. If we call the currents through the resistors $\mathbf{I_1}$ , $\mathbf{I_2}$ , etc. then the following are true.

$\mathbf{V = V_1 + V_2}$ [ series connection] $\mathbf{V_1 + V_2 = V_3 + V_4}$ [or = $\mathbf{V_4 + V_5}$]

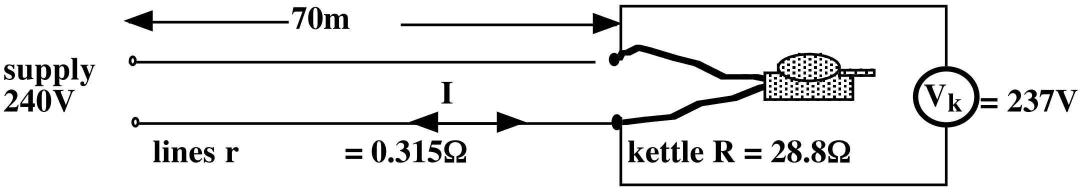
since $\mathbf{V_5 = V_3}$ [parallel connection]

$\mathbf{I_1 = I_2}$ [in series] **and** $\mathbf{I_4 = I_5 + I_3}$ [junction]

$\mathbf{I = I_1 + I_4}$ [junction]

## Example calculation.

[a] A house is 70m from a step down transformer which gives a 240V output. The line resistance is $2.25 \times 10^{-3}\Omega m^{-1}$ and a kettle with a resistance of 28.8Ω is switched on in a house at the end of the line. Calculate the power of the kettle.



The resistance of the lines is $\mathbf{r = 2 \times 70 \times 2.250 \times 10^{-3} = 0.315\Omega}$

The total resistance is $\mathbf{R + r = 28.8 + 0.315 = 29.115\Omega}$

The current is given by $\mathbf{I = {}^{V}/_{R} = {}^{240}/_{29.1} = 8.243A}$

The power of the kettle is $\mathbf{P = RI^2 = 28.8 \times 8.243^2 = 1957W}$

**n.b.** A common error in these problems is to use the formula for power incorrectly. That is use **P = VI** and then use the **incorrect p.d.** such as the total p.d. of 240V. The correct value for the p.d is that across the kettle, which is given by

$$\mathbf{V_k = RI = 28.8 \times 8.243 = 237.4V}$$

**n.b.** Keep more than three **sig. fig.** until your answer. Sig. fig. in this question and many others were completely ignored, but do the right thing anyway. The 70m is one sig. fig. but may be considered to be two by the examiner. It seems best to give all answers to **three** significant figures.

[b] A second kettle which is identical to the first is switched on while the first kettle is still in operation.

[i] What is the power output of the first kettle now ?

The kettles are in parallel. $\mathbf{R = {}^{28.8}/_{2} = 14.4\ \Omega}$

The total resistance is $\mathbf{14.4 + 0.315 = 14.715\Omega}$

The total current is $\mathbf{I = {}^{240}/_{14.715} = 16.31\ A}$

**Total** Power $\mathbf{P = RI^2 = 14.4 \times 16.13^2 = 3831W}$

Power of one kettle is **1915W**

[ii] What is the potential difference [voltage] across the kettles ?

p.d. $\mathbf{V = R\,I = 14.4 \times 16.31 = 235\ V}$

**[c]** Compare the p.d. across the kettle in part [a] and that across them in part [b]. Comment on any difference if there is any.

The p.d. is less for two kettles. **235V** [2] **<** **237V** [1]

This is because there is a greater current in the 70m power lines and this means **a greater potential drop along the lines.**