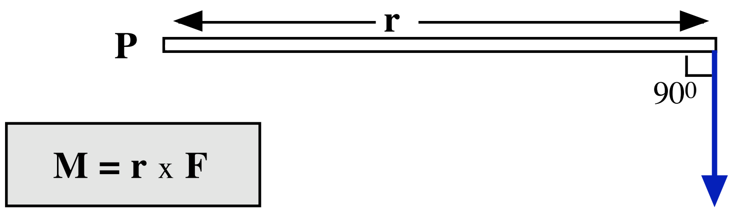
Electric motors.

7.1 The moment or torque of a force [M].

The moment of a force is it's turning effect about a point. Moment is often called torque [□] and in the case of two forces that are equal and opposite causing a torque but zero resultant force it is called a couple [**C**]. This is covered more fully in the chapter on equilibrium in structures.

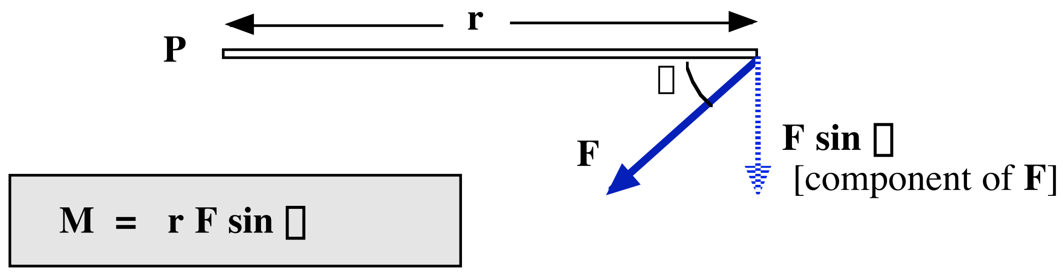
Fig.7[i] Torque for a perpendicular force.



The moment of the force **F** about the point **P** is defined by the formula above when **F** and **r** are perpendicular as shown. When the force is **not** perpendicular to the displacement **r** from the point of rotation P the component of **F** that is at right angles to **r** is used.

Fig.7[ii] The moment of a force in general.

P is the axis of rotation, into the page.

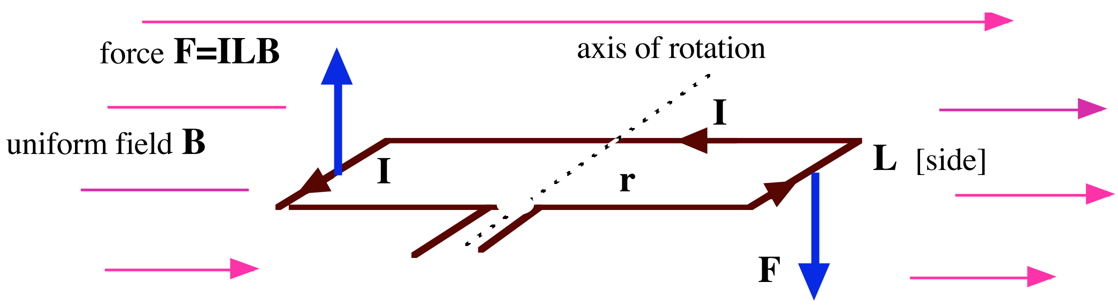


The units of torque are newton metre **Nm .** Torque is a vector quantity. The direction of the torque or moment is given by the rules for a cross product, which is into the page in the example above. This is not required for year twelve studies, even in mathematics.

7.2 The torque on a coil in a magnetic field:

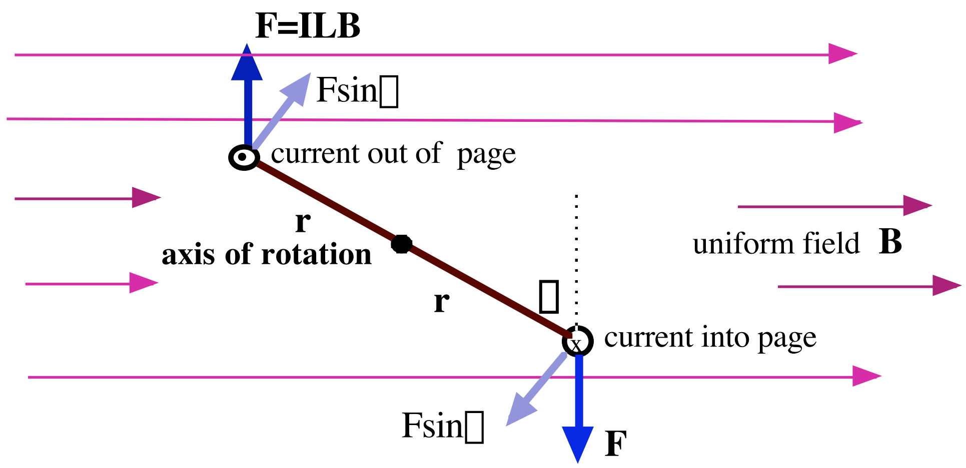
If a plane rectangular coil is pivoted centrally as shown and is carrying a current in a magnetic field perpendicular to the axis then it experiences a torque or couple. The term couple is usually preferred since there is no resultant force.

Fig.7[iii] Forces on a rectangular coil.



The torque or moment of the forces about the axis or the couple on the coil is given by **C = 2 r F**
As the coil turns the forces stay vertical and the moment or torque reduces.

Fig.7[iv] End elevation of the coil.



Dimensions of coil are **L** x 2**r** and the area **A = 2Lr**

□ = 2 **r Fsin**□ = **2r ILB sin** □ where □ is the angle the angle between **F** and **r** .

T [or **C**] = **B A I sin**□ and for a coil with '**N**' turns

the couple or torque is given by □ = **C = B A N I sin**□

Of course the coil will only turn until it is vertical if the forces remain as shown.

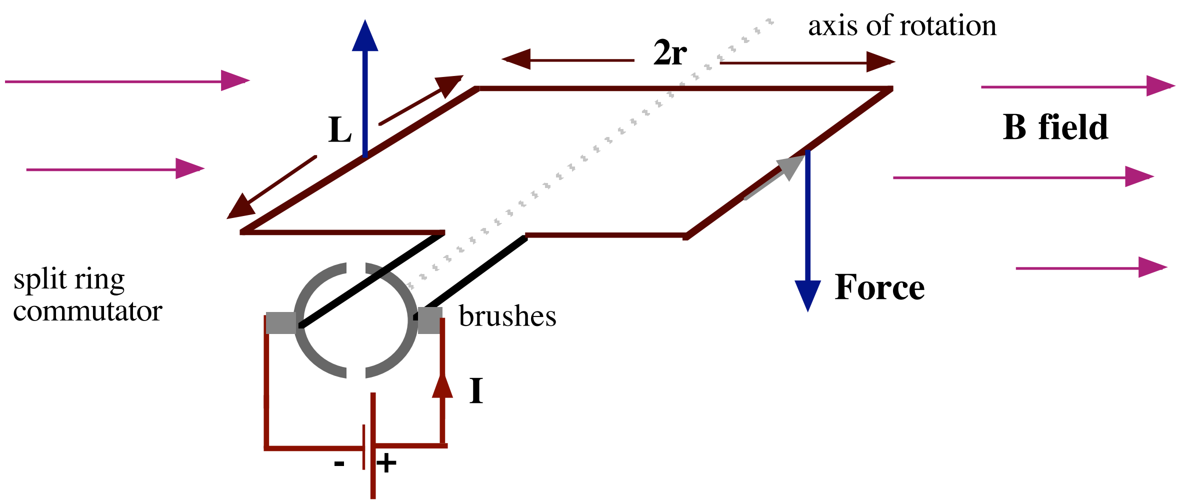
7.3 The electric motor.

The simple d.c. motor is designed to produce a continuous torque and has the following features :

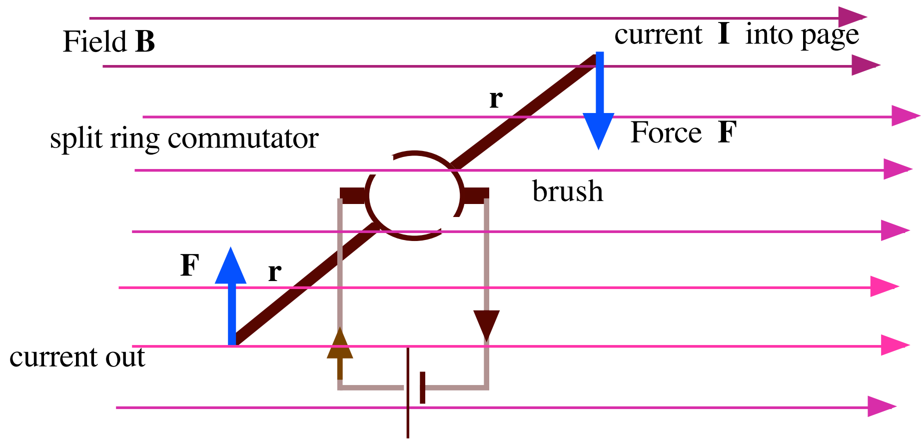
[i] Commutator and brushes.

The forces must reverse to keep the torque clockwise once it passes the vertical. This is achieved using a split ring commutator and the sliding conducting brushes. The current is fed into the coil near the axis by sliding contacts, usually made of graphite, called brushes which rub on a copper ring, that rotates with the coil. If the coil is to turn continuously then the current must be reversed every 180^{o} in the vertical position and this is achieved by splitting the copper ring in to two halves and connecting each half to the ends of the wires forming the coil as shown.
This is naturally called a split ring commutator.

Fig.7[v] [a] Commutator and brushes.

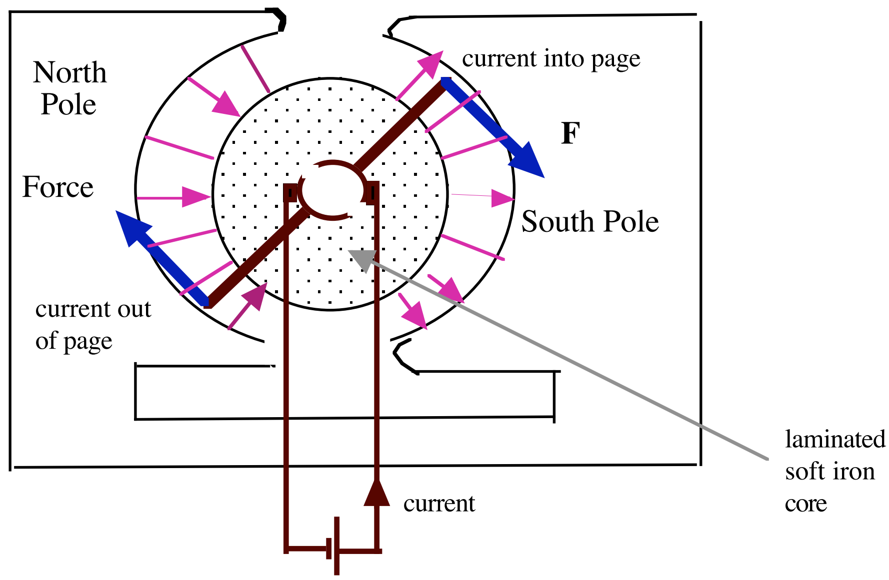


[b] side elevation



The torque varies from maximum when the plane of the coil is parallel to the field [horizontal in the figure] to zero when it is perpendicular or normal to the field. This is improved by using a **radial field**, which keeps the forces on the sides of the coil perpendicular to the radius as below:-

Fig.7[vi] A d.c. electric motor.

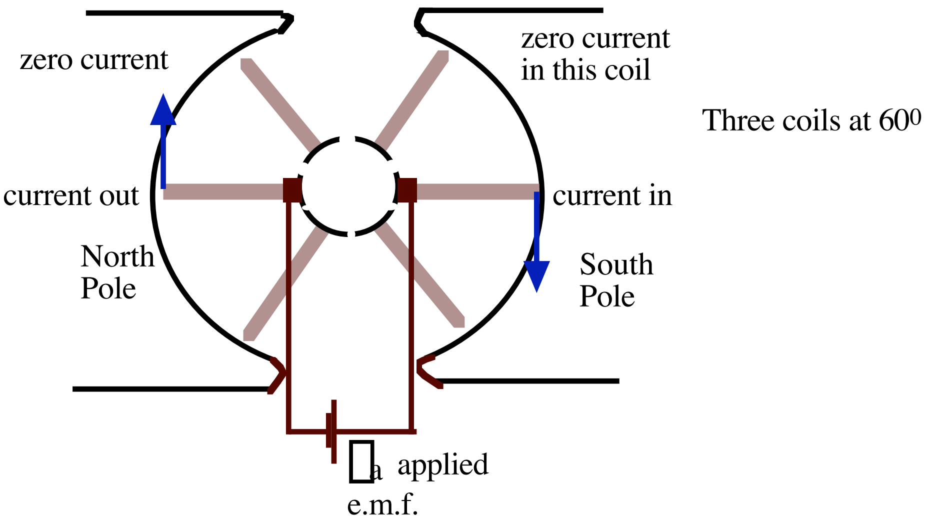


The forces are perpendicular to the coil while it is in the field and this provides maximum torque at all times. The iron core has two functions. Firstly it greatly increases the flux density and secondly it keeps the field radial. The commutator reverses the current every half revolution and there is a flat spot as the coil nears the vertical where the torque is near zero.

[ii] Multiple Coils.

By using several coils and a multi-split ring commutator it is possible to produce a reasonably constant torque. This passes the current through a coil that is in the strongest part of the field at all times.

Fig.7[vii] A motor armature with three coils.



7.4 The factors affecting torque in a motor.

The torque produced depends on several factors as can be seen from the formula:

Couple or Torque [or moment] **C = B A N I sin θ**

if $\theta = 90^0$ then the couple is **C = B A N I**

[i] Obviously the torque depends directly on the value of the **current** and this can be increased by applying a greater p.d.

[ii] The **number of turns** in the armature coil also increases the torque or couple; but remember that increasing the value of 'N' will increase the resistance of the coil and reduce the current.

[iii] The **area**, rather than the **radius** affects the torque.
n.b. larger area means longer wire and higher resistance, hence less current!

[iv] The **flux density B** affects the torque directly. Flux density can be increased by winding the coil on a cylindrical soft iron core that rotates with the coil. This rotating part is the armature.

[v] The iron core is **laminated** to stop eddy currents causing a force which would oppose the motion and hence the net torque. *See also transformers and later notes on eddy currents and laminations*

[vi] The **speed** has a dramatic effect on the current and therefore the torque. This is covered in the section on **back e.m.f.** in the next section.

[vii] Series and shunt wound motors have different torque/current and torque/speed characteristics. These are discussed in detail later but will definately not be examined.

The following pages should be studied after the chapter on induction.

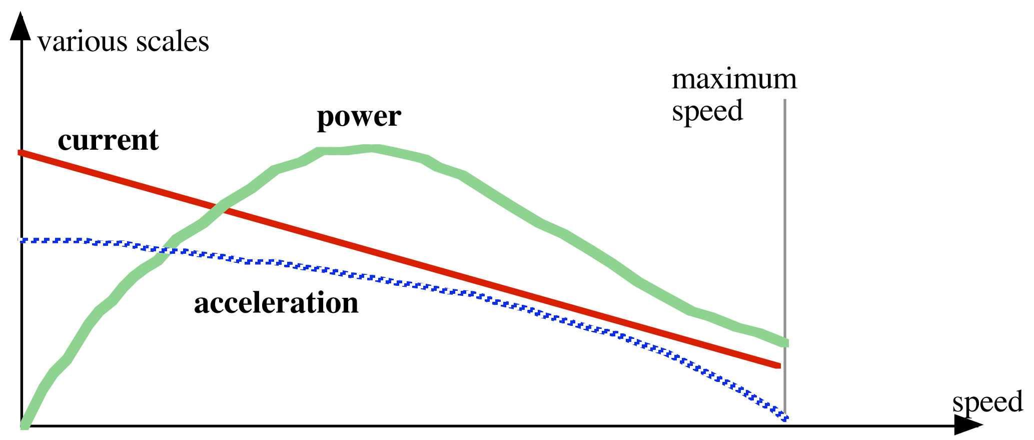
7.4 The back e.m.f. in a motor.

As the armature rotates in the field a back e.m.f. is induced in it. The back e.m.f. induced in the coil opposes the applied e.m.f. This means that as the motor goes faster and the back e.m.f. increases the net p.d. across the coil is reduced and therefore the current is also reduced. Ohm's Law for the circuit must include the back e.m.f.

$$\varepsilon_{applied} - \varepsilon_{back} = \varepsilon_a - \varepsilon_b = RI \qquad \textit{[Ohm's Law]}$$

As the motor speeds up it produces a bigger back e.m.f. and the current drops. Therefore the torque is less. The maximum torque is when starting where the speed is zero. This is when the current is maximum and in large motors such as on a car the current is large enough to cause dangerous over heating if the motor does not speed up quickly. This also means that there is a theoretical maximum speed that any particular motor is capable of which is when the back e.m.f. equals the applied e.m.f. At this speed the current and torque will be zero. Since there are always air resistance and other friction forces to overcome then the current never drops to zero in practice.

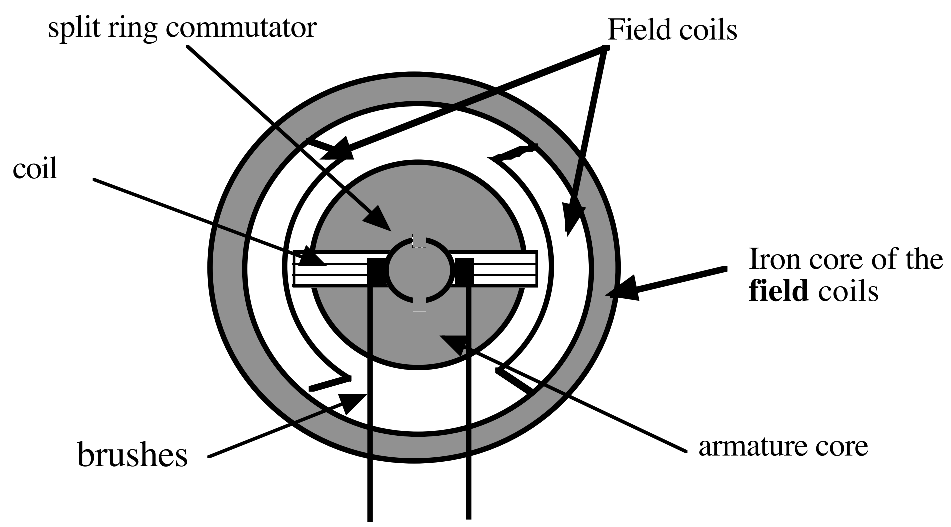
Fig.7[viii] **Characteristics of a d.c. motor.**



7.5 The use of field coils.

The strength of the field can be increased by using electromagnets to produce it . They are wound around the iron either in series or in parallel with the armature. Parallel wound motors are also called shunt wound. These two types have different running characteristics: A combination of these two is called a compound motor.

Fig.7[ix] **A Practical Motor.**



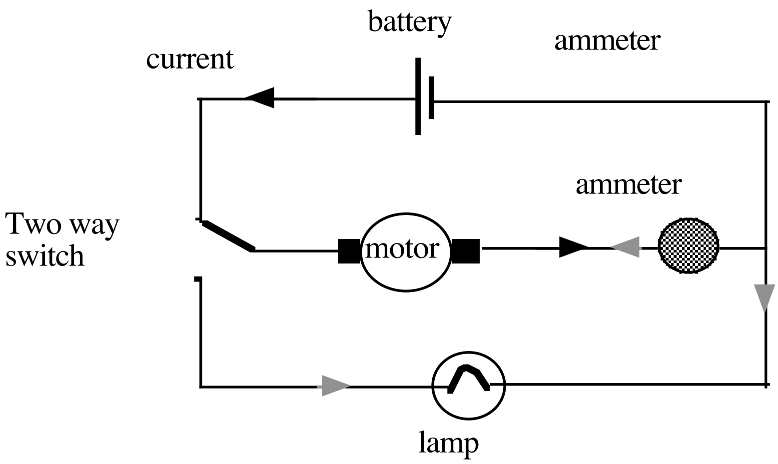
In practice a series wound motors produce a larger starting torque.
Shunt wound motors produce a steadier torque at various speeds.

7.6 Electrodynamic braking.

If the motor is switched off by disconnecting from the supply and then shorted through a heavy duty low resistance the back e.m.f. causes a large current. The current will be in the opposite direction to the original current from the supply and will therefore cause a force that opposes the motion. This is of course just Lenz's Law in action. The forces are large enough to slow the motor, which may be driving a train or car, considerably at at high speeds but at slow speeds the back e.m.f. is small and the forces drop off exponentially with speed meaning that the vehicle will never stop by this method.

It is possible to use the e.m.f. in the armature to recharge batteries or even feed energy back into the mains grid but it is very complex and rarely done. The back e.m.f. will of course be less than the e.m.f. of the battery or supply. It is d.c. and not simply transformed but with modern electronics it is possible and, where energy savings justify it, regenerative braking is used. Westrail use big resistors on the new Fastrak trains and do not utilise the energy. The presence of a back e.m.f. and braking is easily demonstrated as follows.

Fig.7[x]. Demonstration of a back e.m.f.



The motor needs to be driving say a flywheel to represent the mass and momentum of the train. When driving the flywheel the current flows through the motor and the ammeter from left to right as shown by the heavy arrows. It is also obvious that the current from the battery goes down as the motor speeds up. This is due to the increasing back e.m.f. If the flywheel is slowed by hand the current increases as the back e.m.f. falls.

When the two way switch is moved down the battery is disconnected from the motor and the 'motor' is now connected to the lamp.The lamp lights up and the ammeter shows a current that flows the opposite way through the motor. This current is due to the back e.m.f. that was opposing the battery while the motor being driven by the battery. The motor is now acting as a d.c. generator. The flywheel slows down quite quickly, in fact much quicker than if the motor is just switched off.

The maximum speed of a motor can be increased by using a battery with a greater e.m.f. but less obviously and surprisingly the speed is faster for less turns in the armature and a weaker field. In practice the top speed is not important. It is the torque produced at lower speeds that is needed to do work. This is greater for a strong field and also a small resistance in the armature which gives a bigger current and torque.

7.7 Universal motors and a.c.motors.

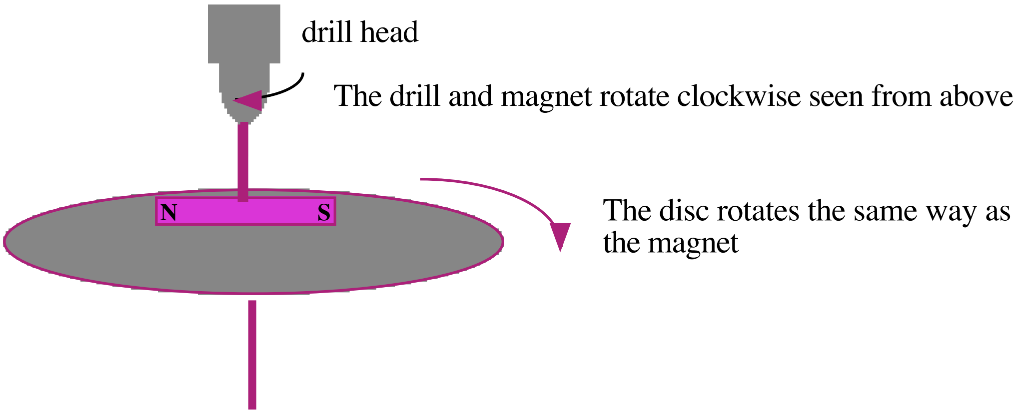
A series wound d.c. motor will run on a.c. The current reverses at the same time in the armature and the field coils and therefore if the commutator is still used the motor runs as if on d.c. These motors are called universal motors.

An a.c. motor is very robust and has less parts to go wrong. They work by using the varying current in the field coils inducing currents in the armature. The induced currents cause the armature to follow the magnetic field of the field coils which is apparently rotating around it. This apparent rotation is very pronounced if three phase power is used. The rotating coils are of thick copper rods and the induced currents in them are large. They are effectively short circuited and there is no connection to any of the non rotating parts of the motor. This is a distinct advantage as no brushes are needed.

The induced currents are greatest when the armature is not moving and reduces as the speed approaches the apparent speed of rotation of the magnetic field. Thus maximum torque is at starting and the torque is low at high speeds. The maximum speed is altered by using different arrangements of coils but since the motor does not operate near maximum speed due to the low torque this is not very important.

The principle of an a.c. motor can be shown using a metal [say aluminium] disc and a magnet attached to a small hand operated drill.

Fig. 7[xi] Demonstration of ac motor principle.



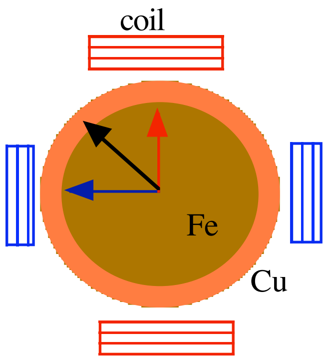
The magnetic field of the magnet passes through the aluminium disc. As the magnet moves the field moves and the moving flux passing through the disc induces an emf in the disc. Lenz's law tells us that the direction of the subsequent induced currents will exert a force on the magnet opposing it's motion.

Newton's third law tells us that there will therefore be a force on the disc in the opposite direction to the force on the magnet. This is such as to cause the disc to rotate the **same way** as the magnet.

In an ac motor the rotating magnetic field is provided by field coils that have out of phase currents in them. These can be from a three phase power supply. The disc is a solid conducting cylinder with an iron central core [to increase the flux density] and a copper sleeve to give low resistance and hence larger currents.When the current in the field coils is switched on the armature is stationary and there is maximum rate of change of flux through the cylinder. Maximum torque is exerted on the cylinder [armature]. As the armature begins to rotate the relative speed of rotation of the field to the armature is decreased and the induced emf, the current and the torque all decrease. There is little torque at the top speed, but the motor is designed to work at lower speed and give more torque.The drive of the motor is connected directly to the rotating armature.

Anyone who has stalled a hand drill when drilling say a hole in some hard wood will know how the torque suddenly becomes very high and the drill can wrench itself from one's hand. The ac motor has no brushes or slip rings and the only moving part is a solid metal cylinder. It is very robust and such motors usually last for decades.

The rotating field can be seen from the simpler case of two perpendiculr sets of coils with the currents 90^{o} out of phase. The blue coils provide a horizontal field and the red coils a vertical field.



The coloured arrows show the directions of the two fields at a time **t =** $^{T}/_{8}$ [see graph in Fig7[xii]]

The black arrow shows the **resultant field**.

At **t = 0** there is no red field and the resultant is to the left.

As the fields change the resultant is seen to rotate and thus it induces a current in the armature which causes the armature to rotate the same way as the field.

The graphs below show the currents in the coils and the figure below shows the filds over one time period.

Fig.7[xii] Graphs of current v. time in the two pairs of coils

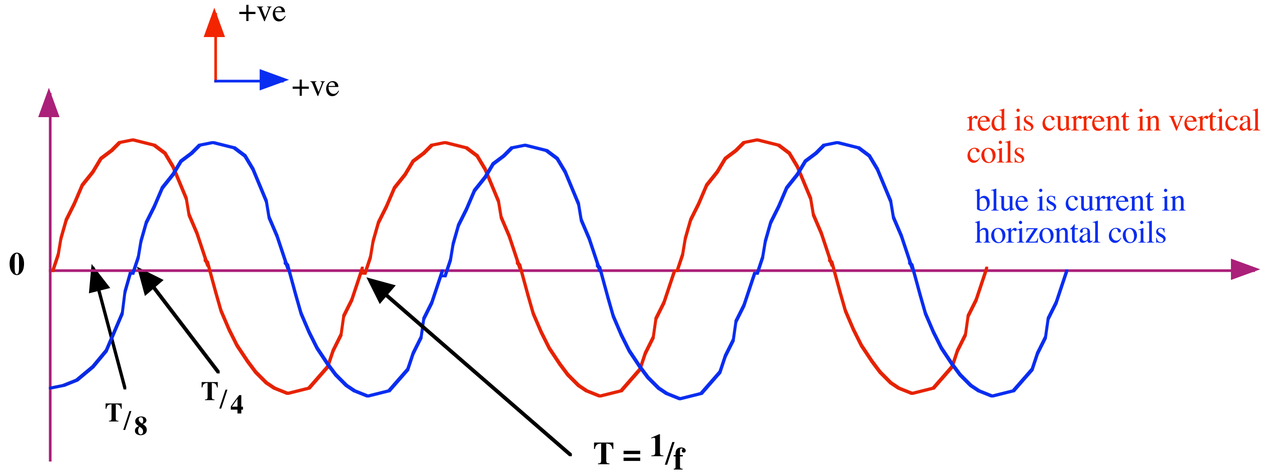
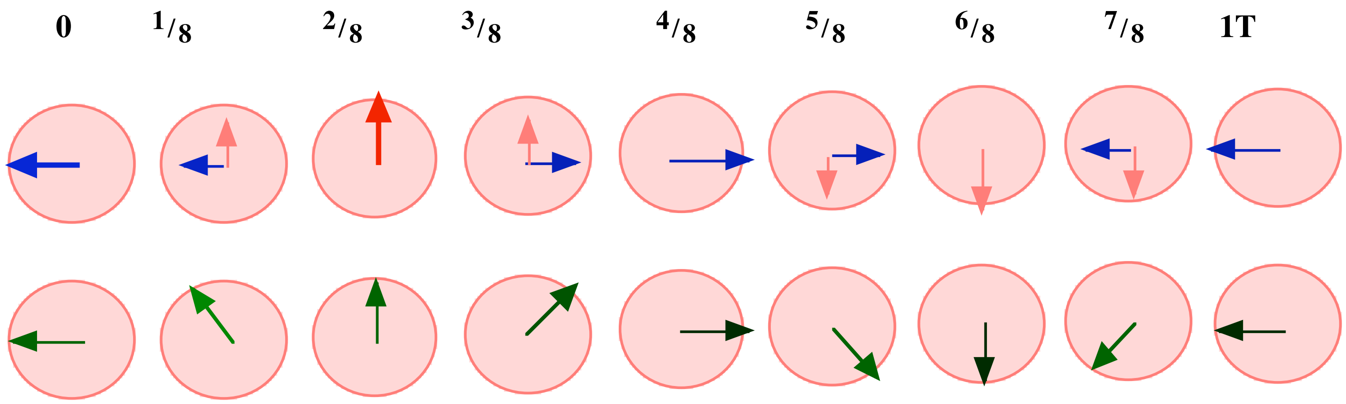


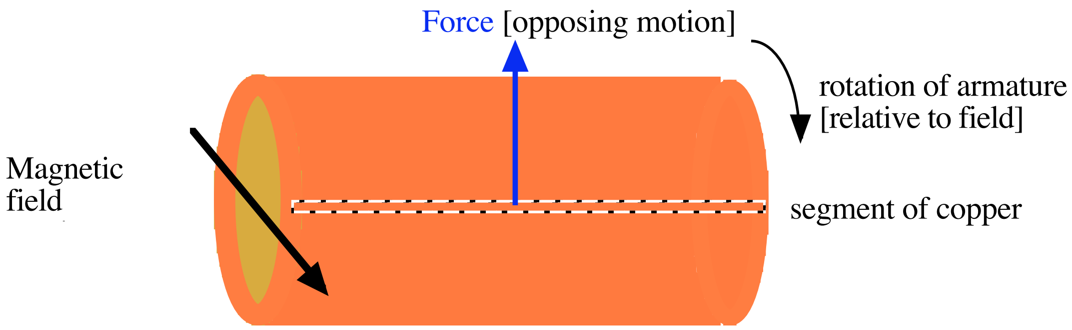
Figure 7[xiii] below shows the direction and strength of the fields due to the red and blue coils at intervals of $^{T}/_{8}$ for one complete cycle. The black arrows show the resultant of the red and blue vectors which is seen to rotate in an anticlockwise direction.

Fig.7[xiv] Magnetic fields combining to produce a rotating field



The direction of the force on the armature can be understood a little more easily by considering the situation from the frame of reference of the rotating field. In this frame the field is stationary and the armature rotates. The conductor which can be thought of as a straight section of the copper sleeve, is moving perpendicular to the field and a current is induced which according to Lenz's law will flow in such a direction as to oppose the motion of the armature. In the normal frame of reference this is the way that the field rotates.

Fig.7[xv] Frames of reference.



7.8 Practical series and shunt wound motors.

If the field coils are in series then they must not have a high resistance or they will limit the armature current. Therefore the field coils are made of thick wire and they do not have a large number of turns. Series motors produce a large starting torque because the armature current also flows through the field coils. When starting the back e.m.f. is zero and the current is very large through both coils. Large motors need a protective resistor in series to limit the starting current and stop the damage due to heating. Torque is proportional to current and field and hence the torque is very large when starting.

Shunt wound motors have higher resistance field coils with a large number of turns. The current in the field coils is small but the field is strong and of course constant. Shunt wound motors will tend to keep a near constant speed under varying load. They also have a large starting torque but in practice the series motor starting torque is larger.

A series motor does not keep a steady speed under varying loads as well as a shunt wound motor. When the motor is required to work against a larger force and a larger torque is needed the current must increase and therefore the back e.m.f. must fall . The back e.m.f. in a series wound motor does not fall as quickly as it does in a shunt wound motor as the speed falls. The smaller drop in back e.m.f. for a series motor is explained as follows:

[i] when the speed falls the back e.m.f. falls .

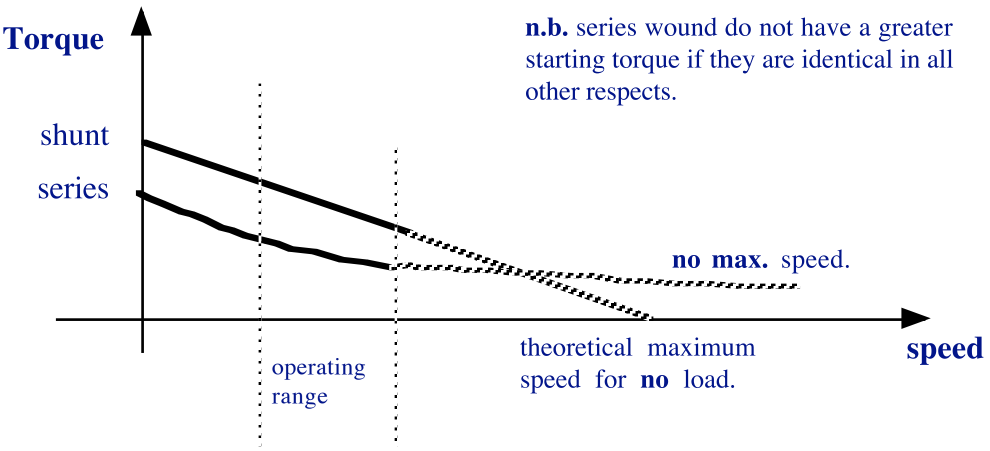
[ii] the current in both armature and field coils increases.

[iii] the flux density increases.

[iv] the larger flux density tends to make the back e.m.f. drop less for a given drop in speed than in a shunt wound motor where the flux density is constant.

However, the torque depends on the flux density and the current , both of which have increased. Thus it is still not obvious that the speed will fall more than for a shunt wound motor. It can be shown that the change of speed needed to increase the torque by a certain amount is greater for a series wound motor than for a shunt wound motor by using calculus and the full expression relating torque to speed for each type of motor. The variation of torque with speed is shown in the graphs below.

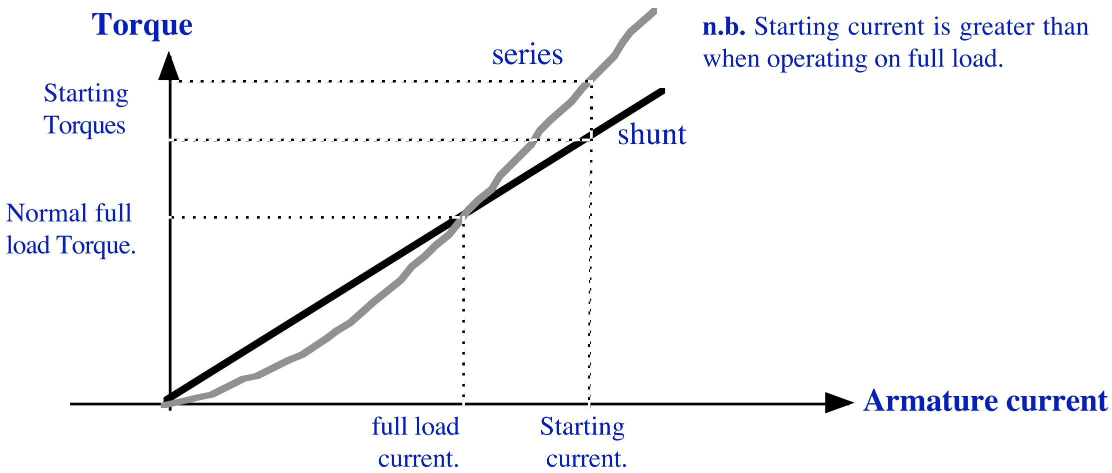
Fig.7[xii] Graphs of torque against speed.



The expressions for these functions are derived in 6.10. These show that the starting torque is greatest in a shunt wound motor, if all aspects of the motors are the same. The graphs also show that the gradient of the shunt wound motor is greater than that of the series motor and this is why the speed drops more.

Another pair of graphs for a practical series and shunt wound motor is of torque against armature current. These are shown below for motors that have the same current for the same full load. It can be seen that a series motor has a greater starting torque.

Fig.7[xiii] Graphs of torque against current.



7.10 Starting torques and response to changes in load.

The following is a treatment of series and shunt wound motors based on several simplifying assumptions

[i] Assumptions.

[i] The motor has a radial field and multiple coils.

[ii] Field coils resistance = **R** and armature resistance = **r**

[iii] Constant applied e.m.f. = $\mathcal{E}_a$

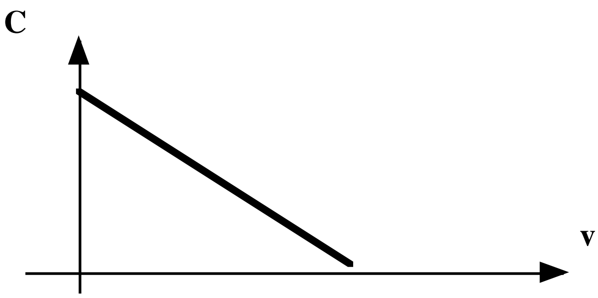
[iv] Magnetic field in field coils $\mathbf{B \propto I_f}$ $\mathbf{B = KI_f}$

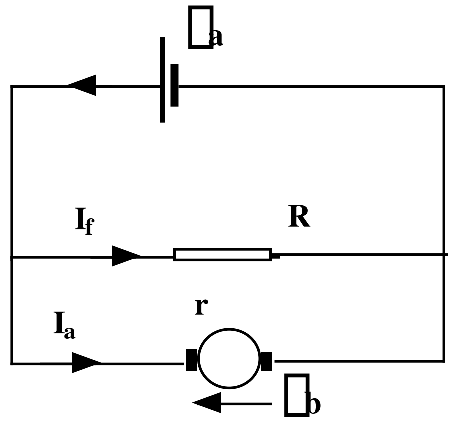
Couple on coil $\mathbf{C = BANI_a}$

Back e.m.f. $\mathbf{\mathcal{E}_b = BAN\omega = BAN\ ^{v}/_{d} = KI_fAN\ ^{v}/_{d}}$

where **d** is the radius of the wheels and **v** is the speed.

[ii] Shunt [parallel] Wound Motors.





Field coil current $I_f = □_a/R$ Armature current $I_a = [□_a - □_b]/r$

Torque $C = BANI_a = KANI_aI_f = KANI_f[□_a - □_b]/r$

$$C = KANI_f[□_a - KI_fAN\ v/d]/r = KAN \frac{□_a^2 [R - KAN v/d]}{R^2\ r}$$

equation [1]

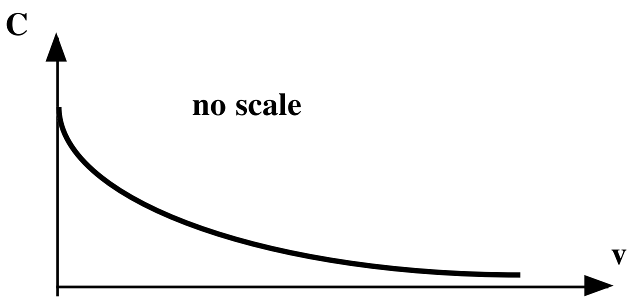
[ii] Series wound motors.

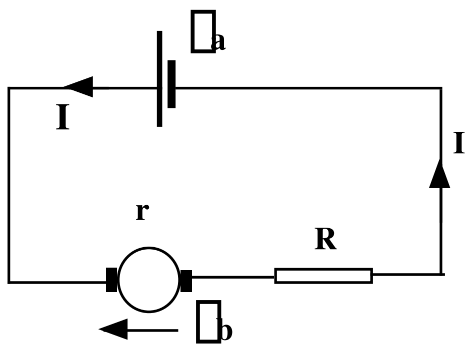
$C = BANI$ $B = KI$ $□_b = BAN□ = KIAN v/d$

$□_a - □_b = [R + r]I$ $□_a = [R + r + KAN v/d]\ I$

$$C = KAN\ I^2 = \frac{KAN\ □_a^2}{[R + r + KAN v/d]^2}$$

equation [2]





Shunt Series

Starting $C = KAN \frac{\square_a^2}{R\,r}$ $C = \frac{KAN\,\square_a^2}{[R + r]^2}$

Differentiating [1] and [2] we get the rate of change of torque with speed.

$$\frac{dC}{dv} = -\frac{[KAN\square_a]^2}{R^2 r d} \qquad \frac{dC}{dv} = -\frac{2\,[KAN\square_a]^2}{[R + r + KAN\,v/d]^3\, d}$$

However, we are concerned with the effect of increased load on the speed. That is which motor gives the higher value for the rate of change of speed with load dv/dC

Shunt	**Series**
$\frac{dv}{dC} = -\frac{R^2 r d}{[KAN\square_a]^2}$ **[constant]**	$\frac{dv}{dC} = -\frac{d\,[R + r + KAN\,v/d]^3}{2\,[KAN\square_a]^2}$ **[biggest changes at high speeds]**

Which of these is greater depends on the values of **R** and **r**!!!!