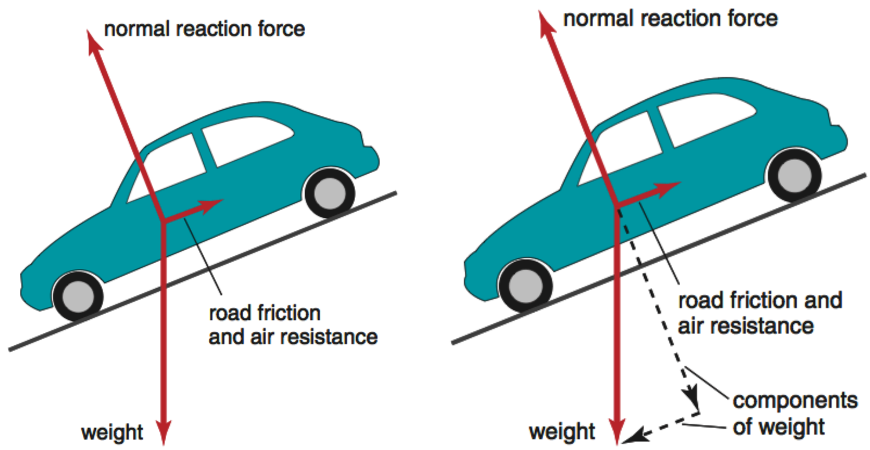
GRAVITATIONAL FORCES ALONG A SLOPE

- A force acting in a two-dimensional plane can be divided into two vectors
    - These two vectors are called the **vector components** of the force
        - Thus it makes sense that when the components are added together, the original force is the resultant
- The gravitational force (i.e. weight) of an object on a slope can be divided into:
    - Components parallel to the slope
    - Components perpendicular to the slope
- E.g. Consider the forces acting on a car rolling down an inclined plane are shown below



    - In order to simplify the diagram, all the forces are modelled as if they were acting through the centre of mass of the car
    - The net force on a car can be found by finding the vector sum of the forces acting on it. It is also helpful in analysing the forces and subsequent motion of the car to 'break up', or resolve, the forces into components
        - The weight can be resolved into two components:
            - One parallel to the surface
            - One perpendicular to the surface
        - By resolving the weight into these components, the analysis of the forces and subsequent motion of the car is made simpler
            - Consider the forces perpendicular to the inclined plane: The magnitude of the normal reaction force is equal to the component of weight that is perpendicular to the surface. I.e. $\boldsymbol{N = mg\cos\theta}$. Thus the net force has no component perpendicular to the surface
            - Consider the forces parallel to the inclined plane. The horizontal component of the weight is greater than the sum of road friction and air resistance. The net force is therefore parallel to the surface. The car will accelerate down the slope. I.e. $\boldsymbol{\Sigma F = mg\sin\theta - F_{friction}}$

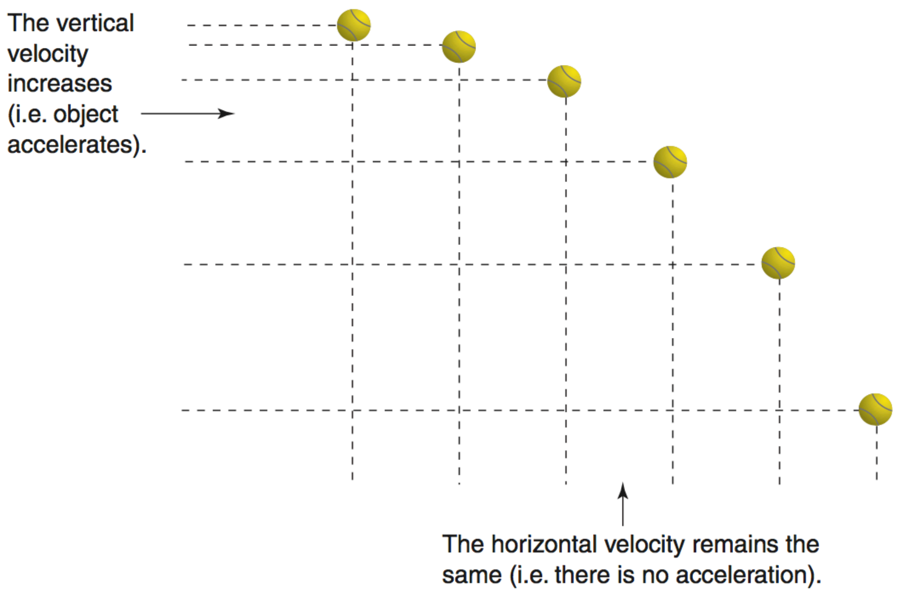
PROJECTILE MOTION

Force of Gravity

- Any object that is launched into the air is a projectile
- Except for those projectiles whose motion is initially straight up or down, or those that have their own power source (e.g. a guided missile), projectiles generally follow a parabolic path
- Imagine a ball that has been released some distance above the ground. Once the ball is set in motion, the only forces acting on it are gravity (straight down) and air resistance (straight up)
- Often the force exerted on the ball by air resistance is very small in comparison to the force of gravity, and so can be ignored. This makes it possible to model projectile motion by assuming that the acceleration of the ball is due only to gravity and is a constant 9.8 ms^{-2} downwards

- If a ball is thrown horizontally, the only force acting on the ball once it has been released is again just the weight force due to gravity (ignoring air resistance)
  - As the force of gravity is the same regardless of the motion of the ball, the ball will still accelerate downwards at the same rate as if it were dropped
  - For this reason, the equations of motion for uniform acceleration are used for calculations involving the vertical dimension
    - $v = u + at$
    - $v^2 = u^2 + 2as$
    - $s = ut + \frac{1}{2}at^2$
- There will not be any horizontal acceleration as there is no net force acting in the horizontal dimension. This means that while the ball's vertical velocity will change, its horizontal velocity remains the same throughout its motion
  - For this reason, only the formula $s = vt$ is used for calculations involving the horizontal dimension

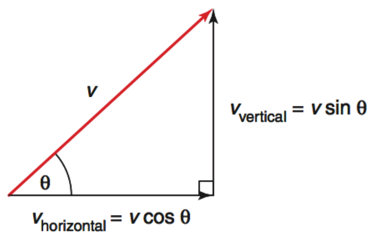
- It is the constant horizontal velocity and changing vertical velocity that give projectiles their characteristic parabolic motion
  - Note in the diagram below that the vertical distance travelled by the ball in each time period increases, but that the horizontal distance is constant



Position of a ball at constant time intervals

- In modelling projectile motion, the vertical and horizontal components of the motion are treated separately
  - The total time taken for the projectile motion is determined by the vertical part of the motion as the projectile cannot continue to move horizontally once it has hit the ground, the target or whatever else it might collide with
  - This total time can then be used to calculate the horizontal distance, or **range**, over which the projectile travels

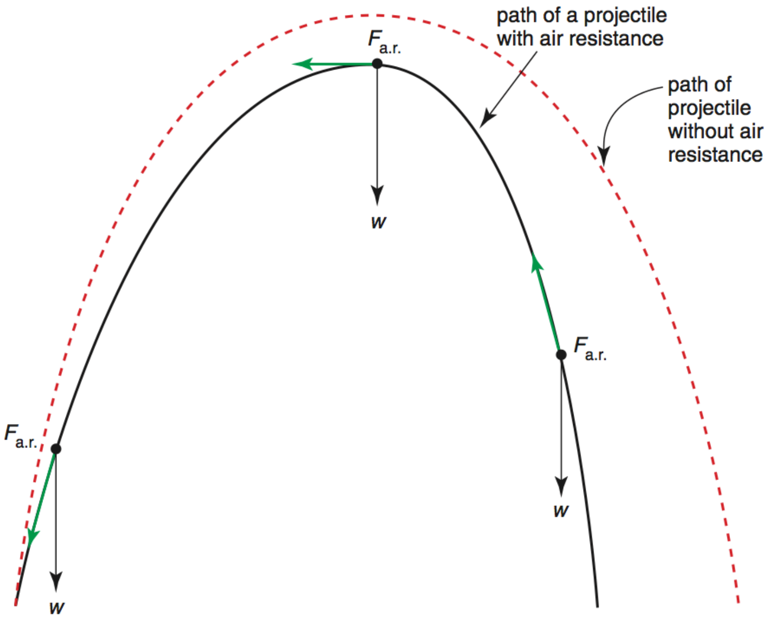
- Generally, projectiles are shot, thrown or driven at some angle to the horizontal
  - In these cases the initial velocity may be resolved into its horizontal and vertical components to help simplify the analysis of the motion
- If the velocity and the angle to the horizontal are known, the size of the components can be calculated using trigonometry
- If an object has an initial projection velocity v at an angle of θ to the horizontal
  - $\boldsymbol{u_H = v\,\sin\theta}$
  - $\boldsymbol{u_V = v\,\sin\theta}$



The velocity can be resolved into a vertical and a horizontal component.

Air Resistance

- Air resistance is a retarding force and it acts in a direction that is opposite to the motion of the projectile
    - I.e. It is tangential to the flight curve directly opposing the instantaneous velocity



While the magnitude of air resistance changes throughout the motion, it always opposes the direction of the motion.

- The effect of air resistance on the projectile's path will:
    - Reduce its calculated range
    - Reduced its calculated maximum height
    - Increase its angle of descent, producing an asymmetrical path

CIRCULAR MOTION

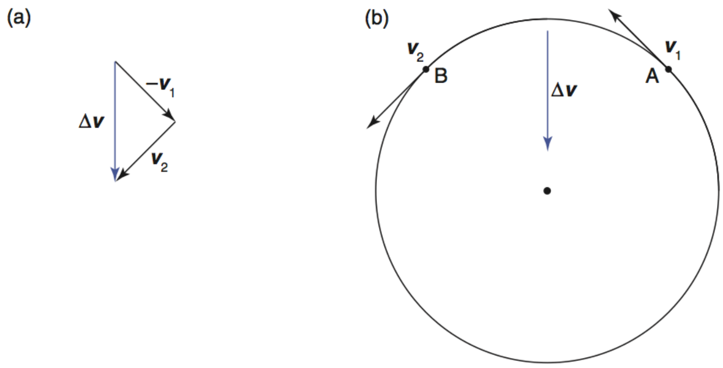
Circular Motion in a Horizontal Plane

- Circular motion is studied because of its common occurrence throughout the Universe
    - E.g. On the smallest scale: Electrons travel around atomic nuclei in circular paths
    - E.g. On a bigger scale: Satellites orbit planets in circular paths

- An object moving with a **uniform speed** in a circular path of radius r and with a period T has an average speed v that is given by the formula:

$$\boldsymbol{v} = \frac{s}{t} = \frac{circumference}{time} = \frac{\boldsymbol{2\pi r}}{\boldsymbol{T}} = \boldsymbol{2\pi r f}$$

- The velocity of an object moving with a constant speed in a circular path is continually changing and is at a tangent to the circular path



(a) Vector addition (b) The change in velocity is towards the centre of the circle.

Centripetal Acceleration and Force

- Because the velocity of the object moving in a circular path is continually changing, it can be said that it is constantly accelerating (even though its speed is not changing)
    - This acceleration is known as **centripetal acceleration**
        - The magnitude of centripetal acceleration F_C is given by the formula:

$$\boldsymbol{a_C} = \frac{\boldsymbol{v^2}}{\boldsymbol{r}} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{\boldsymbol{4\pi^2 r}}{\boldsymbol{T^2}}$$

- Centripetal means 'centre-seeking', which indicates that the direction of centripetal acceleration is always towards the centre
    - However, even though the object is accelerating towards the centre of the circle, it never gets any closer to the centre

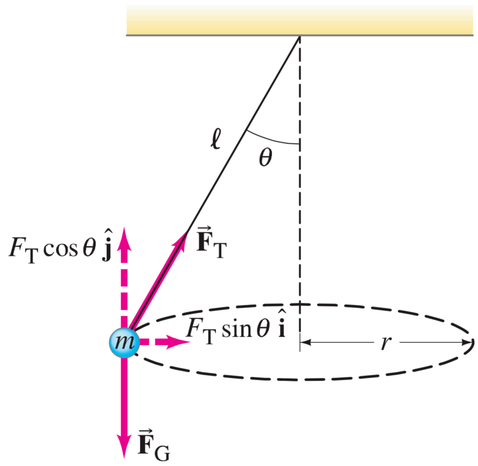
- It follows from Newton's laws, that there must be an unbalanced force continuously acting on the object to cause this acceleration
  - The unbalanced force that gives an object its acceleration towards the centre of the circle is called the **centripetal force**
- In every case where there is circular motion, **a real force** is necessary to provide the centripetal force
  - In a hammer throw or for any other object rotated while attached to an arm or wire, the **tension** in the arm or wire provides the centripetal force
  - For a car on a roundabout, the **friction** between the tyres and the road provides the centripetal force
  - For planets and satellites, the **gravitational attraction** to the central body provides the centripetal force
  - For charged particles in an electromagnetic field, **electrostatic forces** provides the centripetal force
- The magnitude of the centripetal force (F_C) is given by the formula:

$$F_C = m\boldsymbol{a_C} = \frac{\boldsymbol{mv^2}}{\boldsymbol{r}} = \frac{m\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{\boldsymbol{m4\pi^2 r}}{\boldsymbol{T^2}}$$

- Centripetal force, like centripetal acceleration, is a vector directed towards the centre of the circle
- It is important to consider what happens to people and objects inside larger objects that are travelling in circles. E.g. Consider the scenario of passengers travelling inside a bus
  - The sideways frictional forces of the road on the bus tyres act towards the centre of the circle, which provides centripetral force on the bus and keeps the bus moving around the circle
  - If the passengers are also to move in a circle (therefore keeping the same position in the bus) they need, too, to have a net force towards the centre of the circle. Without such a force, they would continue to move in a straight line and probably hit the side of the bus! Usually the friction between the seat and a passenger's legs is sufficient to prevent this happening
  - However, if the bus is moving quickly, friction alone may not be adequate. In such cases, passengers may grab hold of the seat in front, thus adding a force of tension through their arms

Conical Pendulum

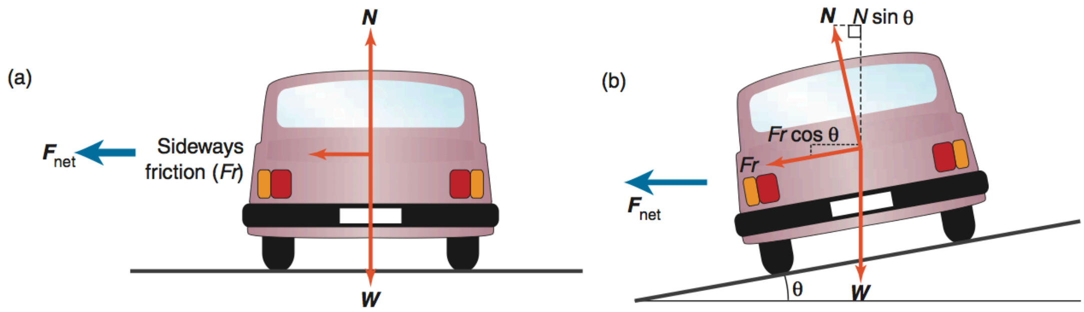
- In a conical pendulum, an object moves in a circular path attached to string or cable that is not horizontal (i.e. the centre of the circular path is not the end of the string, but in the same horizontal plane as the object itself)



- The horizontal component of the tension is the (net) centripetal force
    - $\Sigma F = F_C = F_T \sin\theta$
- The vertical component of the tension balances the force due to gravity on the object
    - $F_g + F_T \cos\theta = 0$

Banking

- Track athletes, cyclists and other vehicles rely on sideways frictional forces to enable them to move around corners
- Consider the forces acting on a vehicle of mass m travelling around a curve with a radius r at a constant speed v. The forces acting on the car are weight W, friction F_f, and the normal reaction N



(a) For the vehicle to take the corner safely, the net force must be towards the centre of the circle. (b) Banking the road allows a component of the normal reaction to contribute to the centripetal force.

  - On a level road the only force with a component towards the centre of the circle is the 'sideways' friction. This sideways friction makes up the whole of the magnitude of the net force (and therefore the centripetal force) on the vehicle, as shown by diagram (a) above
    - $\Sigma F = F_f = F_c = \frac{mv^2}{r}$
    - If you drive the vehicle around the curve with a speed so that F_C is greater F_f, the motion is no longer circular and the vehicle will skid off the road
- To increase the size of the sideways centripetal force, hence move around the corner faster than if they had to rely on friction alone:
  - The athlete/cyclist will **lean** into the corner. The lean means that they are pushing on the surface at an angle, so the reaction force is no longer normal to the ground, but now has a component towards the centre of their circular motion
  - The track/road is **banked**. The banking has an identical effect to a lean, as a component of the normal reaction acts towards the centre of their circular motion, thus increasing the net force in this direction
- Consider the forces on the car of mass m again
  - If the road is banked at an angle θ towards the centre of the circle, a component of the normal reaction N can also contribute to the centripetal force as shown in diagram (b) above
    - $\Sigma F = F_c = F_f \cos\theta + N \sin\theta$
  - The larger centripetal force means that for a given curve, banking the road makes a higher speed possible

Circular Motion in a Vertical Plane

- Objects that are moving in vertical circular paths also experience a centripetal acceleration and centripetal force that acts towards the centre of their path
- Circular motion in a vertical plane, however, is not always uniform because the speed of the body can vary
    - The speed of such an object is not constant because at the top of the loop, the object has gravitational potential energy (as well as kinetic energy), while at the bottom of the loop, its potential energy is converted into extra kinetic energy
    - Note the formulae for mechanical energy:
        - $\boldsymbol{E_k = \frac{1}{2}mv^2}$
        - $\boldsymbol{E_p = mgh}$
- The fundamental concepts that should be understood for questions dealing with circular motion in a vertical plane are:
    - F_C represents the net force force (i.e. it is the resultant of all the real forces acting on the object
        - $F_C = F_g + F_R$
    - F_C always faces the centre
        - F_C faces upwards at the bottom of the curve
        - F_C faces downwards at the top of the curve
    - F_g always faces downwards
    - The reaction force F_R an object experiences from a surface (e.g. seat, road, etc), or the tension T in the string attached to a weight is the apparent weight of the object
- On the basis of the above points, the following conclusions can be made:
    - At the top of circular motion: $F_R = |F_g| - |F_c|$, where F_R is upwards
        - Therefore an object at the top of circular motion experiences **apparent weightlessness** (i.e. $F_R = 0$) when $F_g = F_C$
    - At the bottom of circular motion: $F_R = |F_g| + |F_c|$, where F_R is upwards

GRAVITATIONAL FORCES AND FIELDS AND SATELITTE MOTION

Gravitational Forces and Fields

- Features of **gravitational forces**: For two bodies of mass m and M with a centre-centre separation distance r
    - There is a force of attraction acting between the two
    - The force acts equally and oppositely on each mass (consistent with Newton's third law: $F_1 = -F_2$)
    - The force is directly proportional to the two individual masses

      $F_G \propto mM$
    - The force is inversely proportional to the centre-centre separation distance

      $F_G \propto \frac{1}{r^2}$
    - Newton put these two relationships together and introduced a constant of proportionality G, the gravitational constant to give **Newton's Law of Universal Gravitation**:

      $$\boldsymbol{F_g = \frac{GMm}{r^2}}$$
        - The universal gravitational constant G is equal to $\mathbf{6.67 \times 10^{-11}\ Nm^2kg^{-2}}$
- Features of **gravitational fields:**
    - A gravitational field is a region in which any object with mass will experience a force
    - The magnitude of a gravitational field is known as the gravitational field strength, g
        - This is defined as the gravitational force that acts on each kilogram of a (hypothetical) body in the field (i.e. the gravitational force per unit mass)

          $$\boldsymbol{g} = \frac{F_g}{m} = \frac{\frac{GMm}{r^2}}{m} = \boldsymbol{\frac{GM}{r^2}}$$
    - Since $g = \frac{F_g}{m} \Longrightarrow \boldsymbol{F_g = mg}$

Satellite Motion

- A **satellite** is an object that is in a stable orbit around another larger object
- Satellites may be:
    - Natural satellites:
        - The planets are natural satellites of the sun
        - The moons are natural satellites of the planets
    - Artificial satellites:
        - E.g. International Space Stations, Hubble Telescope, etc

- The only force acting on a satellite is the gravitational attraction between it and the central body
    - The gravitational force that acts on a satellite is always directed towards the centre of the central mass (i.e. gravitational force is the real force that provides a centripetal force)
        - If the gravitational force that acts on the satellite is always perpendicular to the velocity of the satellite, the orbit is circular (like centripetal motion)
        - The gravitational force does not cause the satellite to speed up or slow down; it only acts to change its direction of motion (like centripetal motion)
- Because satellites follows centripetal motion due to a gravitational force, the orbital properties of a satellite can be calculates using the same equations as those used in the section for centripetal motion in the horizontal dimension and/or gravitational forces and fields
    - The speed of a satellite is given by:

$$\boldsymbol{v = \frac{2\pi r}{T}}$$

    - The acceleration of a satellite is given by:

$$\boldsymbol{a_C = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2} = g = \frac{GM}{r^2}}$$

    - The gravitational or centripetal force is given by:

$$\boldsymbol{F_C = ma_C = \frac{mv^2}{r} = \frac{m4\pi^2 r}{T^2} = F_g = \frac{GMm}{r^2} = mg}$$

Kepler's Law

- **Kepler's Third Law** summarises an important relationship

$$F_C = F_g$$

$$\frac{m4\pi^2 r}{T^2} = \frac{GMm}{r^2}$$

$$\boldsymbol{\frac{GM}{4\pi^2} = \frac{r^3}{T^2}}$$

- This relationship is particularly important, as the value of $\frac{r^3}{T^2}$ is constant for every object revolving around a particular mass
    - E.g. $\frac{r^3}{T^2}$ is constant for all the planets in our solar system, as they rotate about the Sun
    - E.g. $\frac{r^3}{T^2}$ is a different constant for all the moons revolving around Jupiter

- **Geosynchronous Orbit**
  - The term 'geosynchronous' refers to the satellite's orbital period being exactly one day (i.e. 24 hours), which enables it to be synchronised with the rotation of the Earth
  - Along with this orbital period requirement, to be '**geostationary**' as well, the satellite must be placed in an orbit that puts it in the vicinity over the equator
  - These two requirements make the satellite appear in an unchanging area of visibility when viewed from the Earth's surface, enabling continuous operation from one point on the ground. The special case of a geostationary orbit is the most common type of orbit for communications satellite
- Satellites are said to be in continual **free-fall**
  - When a projectile is launched at its orbital speed, the projectile will fall towards the Earth with a trajectory that matches the curvature of the Earth
  - The projectile will fall around the Earth, always accelerating towards it under the influence of gravity, yet never colliding into it since the Earth is constantly curving at the same rate. Such a projectile is an orbiting satellite
- Objects in satellite motion experience **apparent weightlessness**
  - This is because the acceleration of an object towards is Earth is dependent on the mass of the orbited body and independent of the mass of the orbiting
  - E.g. The acceleration of an orbiting spacecraft towards Earth is equation to the acceleration of the astronaut towards Earth. The astronaut is therefore in **continues freefall** and experiences apparent weightlessness

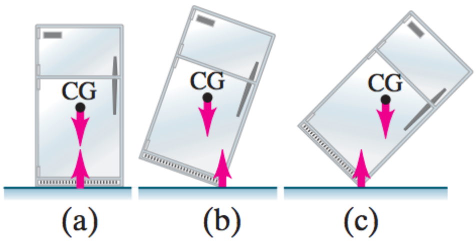
TORQUE AND EQUILIBRIUM

Torque

- A force that acts to cause a rotation is said to provide a turning effect or **torque** (τ) or **moment**
  - A torque/moment can result in a rotation that can be given as either clockwise or anticlockwise
- When analysing a rotating system, the axis of rotation (i.e. the **pivot**) is an important reference point
- The torque acting on a body is given by:
  - $\boldsymbol{\tau = Fr(\perp)\;\; F(\perp)r}$ or $\boldsymbol{\tau = Fr\sin\theta}$
    - τ is the torque in Newton metres (Nm)
    - F is the force applied to the lever arm in Newtons (N)
    - $r(\perp)$ is the perpendicular distance from the line of action of the force to the pivot point or axis of rotation in metres (m)
    - $F(\perp)$ is the component of the force applies that is perpendicular to the distance from the line of action of the force to the pivot point or axis of rotation in Newtons (N)
    - θ is the angle between F and r

Balance and Stability

- The **balance** or **stability** of an object will depend on the relative positions of the CoG and the base or point of support
  - Consider a standing refrigerator (figure a)



    - If it is tipped slightly, it will return to its original position because the torque pulls the object back on its original base of support (figure b)
    - If it is tipped too far, it will fall over (figure c)
      - The critical point is reached when the CoG shifts from one side of the pivot point, to the other
  - In general, an object whose CoG is above its base of support will stable if a vertical line projected downward from the CoG falls within the base of support

Equilibrium

- Equilibrium (of a body/system) may occur in relation to various aspects
  - **Translational Equilibrium:** This is when the sum of the forces is zero
    - I.e. $\Sigma F = 0$
    - If the sum of the forces is zero, the sum of the individual components of the forces must also be zero
      - I.e. $\Sigma F_x = 0$, $\Sigma F_y = 0$ and $\Sigma F_z = 0$
  - **Rotational Equilibrium:** This is when the sum of the torques acting about a point is zero
    - I.e. $\Sigma \tau = 0$
    - I.e. $\Sigma \tau_{CW} = \Sigma \tau_{ACW}$
  - **Static Equilibrium:** This is when the body/system is in both translational as well as rotational equilibrium
    - I.e. $\Sigma F = 0$ **AND** $\Sigma \tau = 0$