Chapter 1 The force due to gravity

Section 1.1 Newton's law of universal gravitation

Worked example: Try yourself 1.1.1

GRAVITATIONAL ATTRACTION BETWEEN SMALL OBJECTS

Two bowling balls are sitting next to each other on a shelf so that the centres of the balls are 60 cm apart. Ball 1 has a mass of 7.0 kg and ball 2 has a mass of 5.5 kg. Calculate the force of gravitational attraction between them.	
Thinking	**Working**
Recall the formula for Newton's law of universal gravitation.	$F_g = G\frac{m_1 m_2}{r^2}$
Identify the information required, and convert values into appropriate units when necessary.	$G = 6.67 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2}$ $m_1 = 7.0\ \text{kg}$ $m_2 = 5.5\ \text{kg}$ $r = 0.60\ \text{m}$
Substitute the values into the equation.	$F_g = 6.67 \times 10^{-11} \times \frac{7.0 \times 5.5}{0.60^2}$
Solve the equation.	$F_g = 7.1 \times 10^{-9}\ \text{N}$

Worked example: Try yourself 1.1.2

GRAVITATIONAL ATTRACTION BETWEEN MASSIVE OBJECTS

Calculate the force of gravitational attraction between the Earth and the Moon, given the following data: $m_{\text{Earth}} = 6.0 \times 10^{24}\ \text{kg}$ $m_{\text{Moon}} = 7.3 \times 10^{22}\ \text{kg}$ $r_{\text{Moon–Earth}} = 3.8 \times 10^{8}\ \text{m}$	
Thinking	**Working**
Recall the formula for Newton's law of universal gravitation.	$F_g = G\frac{m_1 m_2}{r^2}$
Identify the information required.	$G = 6.67 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2}$ $m_1 = 6.0 \times 10^{24}\ \text{kg}$ $m_2 = 7.3 \times 10^{22}\ \text{kg}$ $r = 3.8 \times 10^{8}\ \text{m}$
Substitute the values into the equation.	$F_g = 6.67 \times 10^{-11} \times \frac{6.0 \times 10^{24} \times 7.3 \times 10^{22}}{(3.8 \times 10^{8})^2}$
Solve the equation.	$F_g = 2.0 \times 10^{20}\ \text{N}$

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Worked example: Try yourself 1.1.3

ACCELERATION CAUSED BY A GRAVITATIONAL FORCE

The force of gravitational attraction between the Sun and the Earth is approximately 3.6×10^{22} N. Calculate the acceleration of the Earth and the Sun caused by this force. Compare these accelerations by calculating the ratio $\frac{a_{Earth}}{a_{Sun}}$.

Use the following data:

$m_{Earth} = 6.0 \times 10^{24}$ kg

$m_{Sun} = 2.0 \times 10^{30}$ kg

Thinking	Working
Recall the formula for Newton's second law of motion.	$F = ma$
Transpose the equation to make *a* the subject.	$a = \frac{F}{m}$
Substitute values into this equation to find the accelerations of the Earth and the Sun.	$a_{Earth} = \frac{3.6 \times 10^{22}}{6.0 \times 10^{24}} = 6.0 \times 10^{-3}\,m\,s^{-2}$ $a_{Sun} = \frac{3.6 \times 10^{22}}{2.0 \times 10^{30}} = 1.8 \times 10^{-8}\,m\,s^{-2}$
Compare the two accelerations.	$\frac{a_{Earth}}{a_{Sun}} = \frac{6.0 \times 10^{-3}}{1.8 \times 10^{-8}} = 3.3 \times 10^{5}$ The acceleration of the Earth is 3.3×10^{5} times greater than the acceleration of the Sun.

Worked example: Try yourself 1.1.4

GRAVITATIONAL FORCE AND WEIGHT

Compare the weight of a 1.0 kg mass on the Earth's surface calculated using the formulae $F_{weight} = mg$ and $F_g = G\frac{m_1 m_2}{r^2}$.

Use the following dimensions of the Earth where necessary:

$g = 9.80\,m\,s^{-2}$

$m_{Earth} = 6.0 \times 10^{24}$ kg

$r_{Earth} = 6.4 \times 10^{6}$ m

Thinking	Working
Apply the weight equation.	$F_{weight} = mg$ $= 1.0 \times 9.80$ $= 9.8$ N (to two significant figures)
Apply Newton's law of universal gravitation.	$F_g = G\frac{m_1 m_2}{r^2}$ $F_g = 6.67 \times 10^{-11} \times \frac{6.0 \times 10^{24} \times 1.0}{(6.4 \times 10^{6})^2}$ $= 9.77$ N $= 9.8$ N (to two significant figures)
Compare the two values.	The equations give the same result to two significant figures.

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Worked example: Try yourself 1.1.5

CALCULATING APPARENT WEIGHT

A 79.0 kg student rides a lift down from the top floor of an office block to the ground. During the journey the lift accelerates downwards at 2.35 m s^{-2}, before travelling at a constant velocity of 4.08 m s^{-1} and then finally decelerating at 4.70 m s^{-2}.	
a Calculate the apparent weight of the student in the first part of the journey while accelerating downwards at 2.35 m s^{-2}.	
Thinking	**Working**
Ensure that the variables are in their standard units.	m = 79.0 kg a = 2.35 m s^{-2} down g = 9.80 m s^{-2} down
Apply the sign and direction convention for motion in one dimension. Up is treated as positive and down is negative.	m = 79.0 kg a = –2.35 m s^{-2} g = –9.80 m s^{-2}
Apply the equation for apparent weight (the normal force).	$F_{net} = F_N + F_{weight}$ $F_N = F_{net} - F_{weight}$ $= ma - mg$ $= (79.0 \times -2.35) - (79.0 \times -9.80)$ $= -185.65 + 774.2$ = 589 N
b Calculate the apparent weight of the student in the second part of the journey while travelling at a constant speed of 4.08 m s^{-1}.	
Thinking	**Working**
Ensure that the variables are in their standard units.	m = 79.0 kg a = 0 m s^{-2} down g = 9.80 m s^{-2} down
Apply the sign and direction convention for motion in one dimension. Up is positive and down is negative.	m = 79.0 kg a = 0 m s^{-2} g = –9.80 m s^{-2}
Apply the equation for apparent weight (the normal force).	$F_{net} = F_N + F_{weight}$ $F_N = F_{net} - F_{weight}$ $= ma - mg$ $= (79.0 \times 0) - (79.0 \times -9.80)$ $= +774.2$ = 774 N

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c Calculate the apparent weight of the student in the last part of the journey while travelling downwards and decelerating at $4.70\,m\,s^{-2}$.	
Thinking	**Working**
Ensure that the variables are in their standard units.	$m = 79.0\,kg$ $a = -4.70\,m\,s^{-2}$ down $g = 9.80\,m\,s^{-2}$ down
Apply the sign and direction convention for motion in one dimension. Up is positive and down is negative.	$m = 79.0\,kg$ $a = 4.70\,m\,s^{-2}$ $g = -9.80\,m\,s^{-2}$
Apply the equation for apparent weight (the normal force).	$F_{net} = F_N + F_{weight}$ $F_N = F_{net} - F_{weight}$ $= ma - mg$ $= (79.0 \times 4.70) - (79.0 \times -9.80)$ $= 371.3 + 774.2$ $= 1145.5\,N$ $= 1.1 \times 10^3\,N$

1.1 Review

1 The force of attraction between any two bodies in the universe is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

2 r is the distance between the centres of the two objects.

3 $F_g = G\frac{m_1 m_2}{r^2}$

$$= 6.67 \times 10^{-11} \times \frac{2.0 \times 10^{30} \times 6.4 \times 10^{23}}{\left(2.2 \times 10^{11}\right)^2} = 1.8 \times 10^{21}\,N$$

4 $F_{weight} = m_{Mars} \times a_{Mars}$

$1.8 \times 10^{21} = 6.4 \times 10^{23} \times a_{Mars}$

$1.8 \times 10^{21} \times a_{Mars} = 6.4 \times 10^{23}$

$a_{Mars} = 2.8 \times 10^{-3}\,m\,s^{-2}$

5 **a** Note: 1 million km $= 1 \times 10^6\,km = 1 \times 10^9\,m$

$F_g = G\frac{m_1 m_2}{r^2}$

$$= 6.67 \times 10^{-11} \times \frac{6.0 \times 10^{24} \times 6.4 \times 10^{23}}{\left(9.3 \times 10^{10}\right)^2}$$

$= 3.0 \times 10^{16}\,N$

b $F_g = G\frac{m_1 m_2}{r^2}$

$$= 6.67 \times 10^{-11} \times \frac{2.0 \times 10^{30} \times 6.0 \times 10^{24}}{\left(15.3 \times 10^{10}\right)^2}$$

$= 3.4 \times 10^{22}\,N$

c % comparison $= \frac{(3.0 \times 10^{16})}{(3.4 \times 10^{22})} \times 100 = 0.000\,088\%$. The Mars–Earth force was 0.000 088% of the Sun–Earth force.

6 The Moon has a smaller mass than the Earth and therefore experiences a larger acceleration from the same gravitational force.

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7 $a = g = G\frac{M}{r^2}$

$$g = 6.67 \times 10^{-11} \times \frac{3.3 \times 10^{23}}{(2\,500\,000)^2}$$
$$= 3.5\,\text{m}\,\text{s}^{-2}$$

8 $F_g = G\frac{m_1 m_2}{r^2}$

$$= 6.67 \times 10^{-11} \times \frac{6.4 \times 10^{23} \times 65}{\left(3.4 \times 10^6\right)^2}$$
$$= 240\,\text{N}$$

9 On Earth, weight is the gravitational force acting on an object near the Earth's surface whereas apparent weight is the contact force between the object and the Earth's surface. In many situations, these two forces are equal in magnitude but are in opposite directions. This is because apparent weight is a reaction force to the weight of an object resting on the ground. However, in an elevator accelerating upwards, the apparent weight of an object would be greater than its weight since an additional force would be required to cause the object to accelerate upwards.

10 **a** $F_{net} = F_N + F_{weight}$

$F_N = F_{net} - F_{weight}$
$= ma - mg$
$= (50 \times 1.2) - (50 \times -9.80)$
$= 60 + 490$
$F_N = 550\,\text{N}$

The person's apparent weight is 550 N.

b When the person is moving at a constant speed, their apparent weight is equal to their weight.

$F_N = F_{weight} = mg = 50 \times 9.80$
$= 490\,\text{N}$

11 $F_{net} = F_N + F_{weight}$

$F_N = F_{net} - F_{weight}$
$= ma - mg$
$= (45.0 \times -3.15) - (45.0 \times -9.80)$
$= -141.75 + 441$
$= 299\,\text{N}$

The child's apparent weight is 299 N.

12 D. Objects in orbit are in free-fall. While in orbit around the Earth, gravity is reduced, but it is still significant in magnitude.

13 D. At this altitude, gravity is reduced and so will be less than $9.80\,\text{N}\,\text{kg}^{-1}$; hence, acceleration is less than $9.80\,\text{m}\,\text{s}^{-2}$. Note: B is not correct, because although the speed of the satellite would be constant, its velocity is not.

14 A. Apparent weightlessness is felt during free-fall, when F_N is zero.

15 B. In order to be geostationary, the satellite must be in a high orbit.

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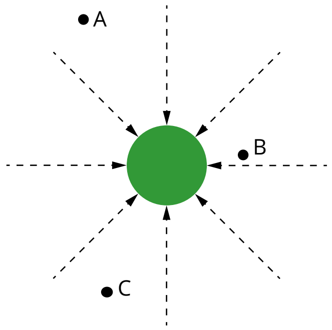
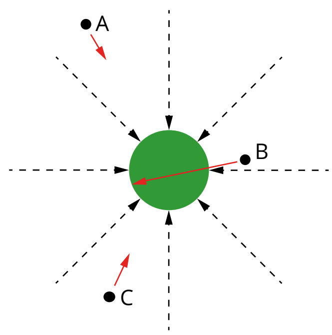
Section 1.2 Gravitational fields

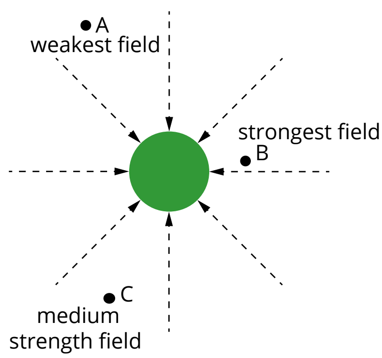
Worked example: Try yourself 1.2.1

INTERPRETING GRAVITATIONAL FIELD DIAGRAMS

The diagram below shows the gravitational field of a planet.



a Use arrows to indicate the magnitude and direction of the gravitational force acting at points A, B and C.	
Thinking	**Working**
The direction of the field arrows indicates the direction of the gravitational force, which is inwards towards the centre of the planet.	

b Describe the relative strength of the gravitational field at each point.	
Thinking	**Working**
The closer the field lines, the stronger the force.	

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Worked example: Try yourself 1.2.2

CALCULATING GRAVITATIONAL FIELD STRENGTH

A student uses a spring balance to measure the weight of a piece of wood as 2.5 N.
If the piece of wood is thought to have a mass of 260 g, calculate the gravitational field strength indicated by this experiment.

Thinking	Working
Recall the equation for gravitational field strength.	$g = \frac{F_{weight}}{m}$
Substitute in the appropriate values, converting the mass to kg.	$m = 260\,g$ $= 0.26\,kg$ $g = \frac{2.5}{0.26}$
Solve the equation.	$g = 9.6\,N\,kg^{-1}$

Worked example: Try yourself 1.2.3

CALCULATING GRAVITATIONAL FIELD STRENGTH AT DIFFERENT ALTITUDES

Commercial airlines typically fly at an altitude of 11 000 m. Calculate the gravitational field strength of the Earth at this height using the following data:
$r_{Earth} = 6.38 \times 10^6\,m$
$m_{Earth} = 5.97 \times 10^{24}\,kg$

Thinking	Working
Recall the formula for gravitational field strength.	$g = G\frac{M}{r^2}$
Add the altitude to the radius of the Earth.	$r = 6.38 \times 10^6 + 11\,000\,m$ $= 6.391 \times 10^6\,m$
Substitute the values into the formula.	$g = G\frac{M}{r^2}$ $= 6.67 \times 10^{-11} \times \frac{5.97 \times 10^{24}}{(6.391 \times 10^6)^2}$ $= 9.75\,N\,kg^{-1}$

Worked example: Try yourself 1.2.4

GRAVITATIONAL FIELD STRENGTH ON ANOTHER PLANET OR MOON

Calculate the strength of the gravitational field on the surface of Mars.
$m_{Mars} = 6.42 \times 10^{23}\,kg$
$r_{Mars} = 3390\,km$
Give your answer correct to two significant figures.

Thinking	Working
Recall the formula for gravitational field strength.	$g = G\frac{M}{r^2}$
Convert Mars' radius to m.	$r = 3390\,km$ $= 3.39 \times 10^6\,m$
Substitute values into the formula.	$g = G\frac{M}{r^2}$ $= 6.67 \times 10^{-11} \times \frac{6.42 \times 10^{23}}{(3.39 \times 10^6)^2}$ $= 3.7\,N\,kg^{-1}$

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1.2 Review

1 N kg^{-1}

2 $g = \frac{F_g}{m} = \frac{1.4}{0.15} = 9.3\ \text{N kg}^{-1}$

3 The distance has been increased to three times its original value, from 40 000 km to 120 000 km, so in terms of the inverse square law and the original distance, r:

$$F \propto \frac{1}{r^2}$$
$$\propto \frac{1}{(3r)^2}$$
$$\propto \frac{1}{9r^2}$$

The field strength is $\frac{1}{9}$ of the original value.

4 **a** $g = G\frac{M}{r^2}$

$$= 6.67 \times 10^{-11} \times \frac{5.97 \times 10^{24}}{((6380 + 2000) \times 10^3)^2}$$
$$= 5.67\ \text{N kg}^{-1}$$

b $g = G\frac{M}{r^2}$

$$= 6.67 \times 10^{-11} \times \frac{5.97 \times 10^{24}}{((6380 + 10\,000) \times 10^3)^2}$$
$$= 1.48\ \text{N kg}^{-1}$$

c $g = G\frac{M}{r^2}$

$$= 6.67 \times 10^{-11} \times \frac{5.97 \times 10^{24}}{((6380 + 20\,200) \times 10^3)^2}$$
$$= 0.56\ \text{N kg}^{-1}$$

d $g = G\frac{M}{r^2}$

$$= 6.67 \times 10^{-11} \times \frac{5.97 \times 10^{24}}{((6380 + 35\,786) \times 10^3)^2}$$
$$= 0.22\ \text{N kg}^{-1}$$

5 $g = G\frac{M}{r^2}$

$$= 6.67 \times 10^{-11} \times \frac{1 \times 10^{13}}{900^2}$$
$$= 0.0008\ \text{N kg}^{-1} \text{ or } 8 \times 10^{-4}\ \text{N kg}^{-1}$$

6 $g = G\frac{M}{r^2}$

Mercury: $g = 6.67 \times 10^{-11} \times \frac{3.30 \times 10^{23}}{(2.44 \times 10^6)^2} = 3.7\ \text{N kg}^{-1}$

Saturn: $g = 6.67 \times 10^{-11} \times \frac{5.69 \times 10^{26}}{(6.03 \times 10^7)^2} = 10.4\ \text{N kg}^{-1}$

Jupiter: $g = 6.67 \times 10^{-11} \times \frac{1.90 \times 10^{27}}{(7.15 \times 10^7)^2} = 24.8\ \text{N kg}^{-1}$

7 $g = G\frac{M}{r^2} = 6.67 \times 10^{-11} \times \frac{3.0 \times 10^{30}}{(10 \times 10^3)^2} = 2 \times 10^{12}\ \text{N kg}^{-1}$

8 $g_{poles} = G\frac{M}{r^2}$

$8.0 = 6.67 \times 10^{-11} \times \frac{M}{5\,000\,000^2}$

$M = 3 \times 10^{24}\ \text{kg}$

$g_{equator} = G\frac{M}{r^2} = 6.67 \times 10^{-11} \times \frac{3 \times 10^{24}}{6\,000\,000^2} = 5.6\ \text{N kg}^{-1}$

$8.0 \div 5.6 = 1.4$

The gravitational field strength at the poles is 1.4 times that at the equator. (Alternatively, the inverse square law could be used to find this relationship.)

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9 Let x be the distance from the centre of Earth to where Earth's gravity equals the Moon's gravity. Then:

$$g_{Earth} = \frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{x^2}$$

$$g_{Moon} = \frac{6.67 \times 10^{-11} \times 7.3 \times 10^{22}}{\left(3.8 \times 10^{8} - x\right)^2}$$

Equating these two expressions gives:

$$\frac{6.0 \times 10^{24}}{x^2} = \frac{7.3 \times 10^{22}}{\left(3.8 \times 10^{8} - x\right)^2}$$

$$\frac{82.2}{x^2} = \frac{1}{\left(3.8 \times 10^{8} - x\right)^2}$$

Taking square roots of both sides gives:

$$\frac{9.07}{x} = \frac{1}{\left(3.8 \times 10^{8} - x\right)}$$

Inverting both sides gives:

$$\frac{x}{9.07} = 3.8 \times 10^{8} - x$$

$x = 3.45 \times 10^{9} - 9.07x$

$10.07x = 3.45 \times 10^{9}$

$x = 3.4 \times 10^{8}$ m

10 g is proportional to $\frac{1}{r^2}$, so if g becomes $\frac{1}{100}$ of its value, r must become 10 times its value, so that $\frac{1}{r^2}$ becomes $\frac{1}{100}$.

10 times r means a distance of 10 Earth radii.

Section 1.3 Work in a gravitational field

Worked example: Try yourself 1.3.1

WORK DONE FOR A CHANGE IN GRAVITATIONAL POTENTIAL ENERGY

Calculate the work done (in MJ) to lift a weather satellite of 3.2 tonnes from the Earth's surface to the limit of the atmosphere, which ends at the Karman line (exactly 100 km up from the surface of the Earth). Assume $g = 9.80\ \text{N kg}^{-1}$.	
Thinking	**Working**
Convert the values into the appropriate units.	$m = 3.2$ tonnes $= 3200$ kg $h = 100$ km $= 100 \times 10^{3}$ m
Substitute the values into $E_g = mg\Delta h$. Remember to give your answer in MJ to two significant figures.	$E_g = mg\Delta h$ $= 3200 \times 9.80 \times 100 \times 10^{3}$ $= 3.136 \times 10^{9}$ J $= 3.1 \times 10^{3}$ MJ
The work done is equal to the change in gravitational potential energy.	$W = \Delta E = 3.1 \times 10^{3}$ MJ

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Worked example: Try yourself 1.3.2

SPEED OF A FALLING OBJECT

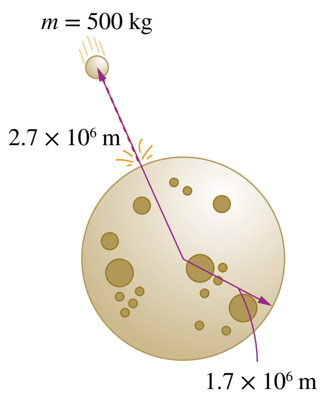
Calculate how fast a 450 g hammer would be going as it hit the ground if it was dropped from a height of 1.4 m on Earth, where $g = 9.80\ \text{N kg}^{-1}$.

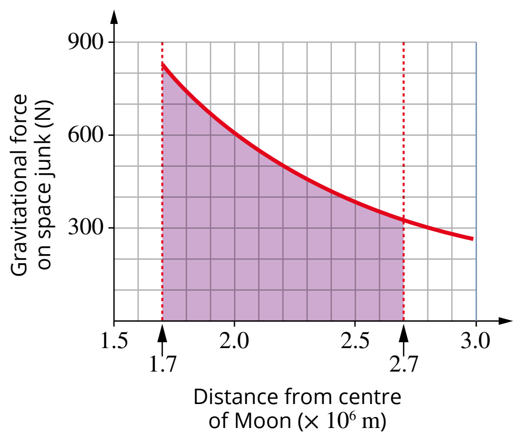
Thinking	Working
Calculate the gravitational potential energy of the hammer on Earth.	$E_g = mg\Delta h$ $= 0.45 \times 9.80 \times 1.4$ $= 6.2\ \text{J}$
Assume that when the hammer hits the surface of the Earth, all of its gravitational potential energy has been converted into kinetic energy.	$E_k = E_g = 6.2\ \text{J}$
Use the definition of kinetic energy to calculate the speed of the hammer as it hits the ground.	$E_k = \frac{1}{2}mv^2$ $6.2 = \frac{1}{2} \times 0.45 \times v^2$ $\frac{6.2 \times 2}{0.45} = v^2$ $v = 5.2\ \text{m s}^{-1}$

Worked example: Try yourself 1.3.3

CHANGE IN GRAVITATIONAL POTENTIAL ENERGY USING A FORCE–DISTANCE GRAPH

A 500 kg lump of space junk is plummeting towards the Moon. The Moon has a radius of 1.7×10^6 m. Using the force–distance graph, determine the decrease in gravitational potential energy of the junk as it falls to the Moon's surface.





Thinking	Working
Count the number of shaded squares. (Only count squares that are at least 50% shaded.)	Number of shaded squares = 52
Calculate the area (energy value) of each square.	$E_{square} = 0.1 \times 10^6 \times 100$ $= 1 \times 10^7\ \text{J}$
To calculate the energy change, multiply the number of shaded squares by the energy value of each square.	$\Delta E_g = 52 \times (1 \times 10^7)$ $= 5.2 \times 10^8\ \text{J}$

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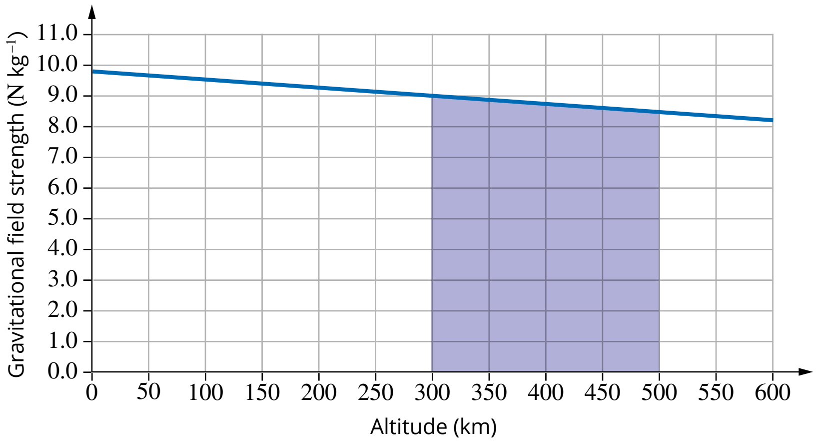
Worked example: Try yourself 1.3.4

CHANGE IN GRAVITATIONAL POTENTIAL ENERGY USING A GRAVITATIONAL FIELD STRENGTH–DISTANCE GRAPH

A 3000 kg Soyuz rocket moves from an orbital height of 300 km above the Earth's surface to dock with the International Space Station at a height of 500 km. Use the graph of the gravitational field strength of the Earth below to determine the approximate change in gravitational potential energy of the rocket.



Thinking	Working
Count the number of shaded squares. Only count squares that are at least 50% shaded.	Number of shaded squares = 36
Calculate the energy value of each square.	$E_{square} = 50 \times 10^3\,\text{m} \times 1\,\text{N kg}^{-1}$ $= 5 \times 10^4\,\text{J kg}^{-1}$
To calculate the energy change, multiply the number of shaded squares by the energy value of each square and the mass of the rocket.	$\Delta E_g = 36 \times 5 \times 10^4 \times 3000$ $= 5.4 \times 10^9\,\text{J}$

1.3 Review

1 C. A stable orbit suggests that the object is in a uniform gravitational field, hence its gravitational potential energy does not change. Its speed will also remain the same in a stable orbit.

2 g increases from point A to point D.

3 The meteor is under the influence of Earth's gravitational field, which will cause it to accelerate at an increasing rate as it approaches the Earth.

4 A, B and C are all correct. The total energy of the system does not change.

5 $W = E_g = 3\,000\,000 \times 9.80 \times 67\,000$

$= 2.0 \times 10^{12}\,\text{J}$

6 $E_g = mg\Delta h$

$= 0.4 \times 6.1 \times 7000$

$= 17\,100\,\text{J}$

$E_k = \frac{1}{2}mv^2$

$17\,100 = \frac{1}{2} \times 0.4 \times v^2$

$v = \sqrt{\frac{2 \times 17\,100}{0.4}}$

$= 292\,\text{m s}^{-1}$

7 **a** 100 km above the Earth's surface is a distance of $6.4 \times 10^6\,\text{m} + 100\,000\,\text{m} = 6.5 \times 10^6\,\text{m}$. According to the graph, F is between 9 N and 9.2 N at this height.

b According to the graph, 5 N occurs at approximately 9.0×10^6 m from the centre of the Earth. So, the height above the Earth's surface $= 9.0 \times 10^6 - 6.4 \times 10^6 = 2.6 \times 10^6$ m or 2600 km.

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8 **a** Convert $km\,s^{-1}$ to $m\,s^{-1}$ then apply the rule:

$$E_k = \frac{1}{2}mv^2$$
$$= \frac{1}{2} \times 1 \times 4000^2$$
$$= 8 \times 10^6\,J$$

b $\Delta E_k = \Delta E_g$

ΔE_g = area under the graph

Area = 19 squares $\times 2 \times 0.5 \times 10^6$

$= 1.9 \times 10^7\,J$

c New E_k = starting $E_k + \Delta E_k = 8 \times 10^6 + 1.9 \times 10^7 = 2.7 \times 10^7\,J$

d New speed $= \sqrt{\frac{2 \times 2.7 \times 10^7}{1}}$

$= 7348\,m\,s^{-1}$ or $7.3\,km\,s^{-1}$

9 600 km above the Earth's surface $= 6.4 \times 10^6 + 600\,000 = 7.0 \times 10^6\,m$ or 7000 km

Area under the graph between 7000 km and 8000 km is approximately 7 squares.

As the satellite comes to a stop, the increase in gravitational potential energy over the distance is the same as the E_k at its launch.

The graph is for a 1 kg object, but the satellite is 240 times that mass. So:

$\Delta E_k = \Delta E_g$ = area under the graph × mass of the satellite

$= 7$ squares $\times 2 \times 0.5 \times 10^6 \times 240$

$= 1.7 \times 10^9\,J$

10 600 km above the Earth's surface $= 6.4 \times 10^6 + 600\,000 = 7.0 \times 10^6\,m$ or 7000 km

2600 km above the Earth's surface $= 6.4 \times 10^6 + 2\,600\,000 = 9.0 \times 10^6\,m$ or 9000 km

The area under the graph between 7000 km and 9000 km is approximately 26 squares.

ΔE_g = area under the graph × mass of the satellite

$= 26 \times 1 \times 0.5 \times 10^6 \times 20\,000$

$= 2.6 \times 10^{11}\,J$

CHAPTER 1 REVIEW

1 $F_g = G\frac{m_1 m_2}{r^2}$

$$= 6.67 \times 10^{-11} \times \frac{6.0 \times 10^{24} \times 75}{(6.4 \times 10^6)^2}$$
$$= 730\,N$$

2 $F_g = G\frac{m_1 m_2}{r^2}$

$$2.79 \times 10^{20} = 6.67 \times 10^{-11} \times \frac{1.05 \times 10^{21} \times 5.69 \times 10^{26}}{r^2}$$
$$r^2 = \frac{6.67 \times 10^{-11} \times 1.05 \times 10^{21} \times 5.69 \times 10^{26}}{2.79 \times 10^{20}}$$

$r = 378\,000\,000\,m$

$= 3.78 \times 10^8\,m$

3 $F = ma_{Sun}$

$$a_{Sun} = \frac{F}{m}$$
$$= \frac{4.2 \times 10^{23}}{2.0 \times 10^{30}} = 2.1 \times 10^{-7}\,m\,s^{-2}$$

4 **a** The force exerted on Jupiter by the Sun is equal in magnitude to the force exerted on the Sun by Jupiter.

b The acceleration of Jupiter caused by the Sun is greater than the acceleration of the Sun caused by Jupiter.

5 $g = G\frac{M}{r^2}$

$$= 6.67 \times 10^{-11} \times \frac{6.4 \times 10^{23}}{(3\,400\,000)^2}$$
$$= 3.7\,m\,s^{-2}$$

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6 **a** $F_{net} = F_N + F_{weight}$

$F_N = F_{net} - F_{weight}$

$= ma - mg$

$= (50 \times -0.6) - (50 \times -9.80)$

$= -30 + 490$

$F_N = 460\,N$

The person's apparent weight is 460 N.

b When the person is moving at a constant speed, their apparent weight is equal to their weight: $F_{weight} = F_N = 490\,N$

7 **a** $F = G\frac{m_1 m_2}{r^2}$

$= \frac{6.67 \times 10^{-11} \times 1.9 \times 10^{27} \times 1000}{(7.15 \times 10^7)^2}$

$= 2.48 \times 10^4\,N$

b The magnitude of the gravitational force that the comet exerts on Jupiter is equal to the magnitude of the gravitational force that Jupiter exerts on the comet = $2.48 \times 10^4\,N$.

c $a = \frac{F_{net}}{m}$

$= \frac{2.48 \times 10^4}{1000}$

$= 24.8\,m\,s^{-2}$

d $a = \frac{F_{net}}{m}$

$= \frac{2.48 \times 10^4}{1.90 \times 10^{27}}$

$= 1.31 \times 10^{-23}\,m\,s^{-2}$

8 D. At a height of two Earth radii above the Earth's surface, a person is a distance of three Earth radii from the centre of the Earth.

Then $F = \frac{900}{3^2} = \frac{900}{9} = 100\,N$

9 **a** D. $F_{net} = F_N + F_{weight}$

$F_N = F_{net} - F_{weight}$

$= ma - mg$

$= (80 \times 30) - (80 \times -9.80)$

$= 2400 + 784$

$F_N = 3184\,N$

The person's apparent weight is 3200 N.

b B. From part (a), the apparent weight is greater than the weight of the astronaut.

c C. Weight is unchanged during lift-off as *g* is constant.

d A. During orbit, the astronaut is in free-fall.

e D. $F_{weight} = ma$

$= 80 \times 8.2$

$= 656\,N$ or $660\,N$

10 When representing a gravitational field with a field diagram, the direction of the arrowhead indicates the *direction* of the gravitational force and the space between the arrows indicates the *magnitude* of the field. In gravitational fields, the field lines always point towards the source of the field and never cross.

11 $g = \frac{F_{weight}}{m}$

$= \frac{600}{61.5}$

$= 9.76\,N\,kg^{-1}$

12 **a** $g = G\frac{M}{r^2}$

$= \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{(6378 \times 1000)^2}$

$= 9.79\,N\,kg^{-1}$

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b $g = G\frac{M}{r^2}$

$= \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{(6357 \times 1000)^2}$

$= 9.85\,\text{N kg}^{-1}$

$\% = \frac{9.85}{9.79} \times 100 = 100.61\%$

13 **a** $g = G\frac{M}{r^2}$

$= \frac{6.67 \times 10^{-11} \times 1.02 \times 10^{26}}{(2.48 \times 10^7)^2}$

$= 11.1\,\text{N kg}^{-1}$

b C. It will accelerate at a rate given by the gravitational field strength, *g*.

14 $G\frac{M}{(0.8R)^2} = G\frac{m}{(0.2R)^2}$

$\frac{M}{0.64} = \frac{m}{0.04}$

$\frac{M}{m} = \frac{0.64}{0.04} = 16$

15 **a** Increase in E_k = area under the graph between 3×10^6 m and 2.5×10^6 m

$= 6 \text{ squares} \times 10 \times 0.5 \times 10^6 = 3 \times 10^7$ J

b $E_{k(\text{initial})} = \frac{1}{2}mv^2 = \frac{1}{2} \times 20 \times 1000^2 = 1 \times 10^7$ J

$E_{k(\text{new})} = 1 \times 10^7 + 3 \times 10^7 = 4 \times 10^7$ J

c $v = \sqrt{\frac{2 \times E_k}{m}} = \sqrt{\frac{2 \times 4 \times 10^7}{20}} = 2000\,\text{m s}^{-1}$ or $2\,\text{km s}^{-1}$

d From the graph, $F = 70\,\text{N} = mg$

$g = \frac{70}{20} = 3.5\,\text{N kg}^{-1}$

16 300 km = 300 000 m or 3×10^5 m

From the graph, $g = 9\,\text{N kg}^{-1}$ at this altitude.

17 D. The units for the area under the graph are N m kg^{-1}, which are the same as J kg^{-1}.

18 C. As the satellite falls, its gravitational potential energy decreases. The units on the graph are J kg^{-1}, so therefore C is correct.

19 Increase in E_k = area under the graph × mass of the satellite

$= 35 \text{ squares} \times 1 \times 1 \times 10^5 \times 1000 = 3.5 \times 10^9$ J

20 No. Air resistance will play a major part as the satellite re-enters the Earth's atmosphere.

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