



# **MATHEMATICS SPECIALIST**

## **Unit 1 and Unit 2**

### **Formula Sheet**

*(For use with Year 11 examinations and response tasks)*

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This document is valid for teaching and examining from 1 July 2015.

## Measurement

Circle:  $C = 2\pi r = \pi D$ , where  $C$  is the circumference,  
 $r$  is the radius and  $D$  is the diameter  
 $A = \pi r^2$ , where  $A$  is the area

Triangle:  $A = \frac{1}{2}bh$ , where  $b$  is the base and  $h$  is the perpendicular height

Parallelogram:  $A = bh$

Trapezium:  $A = \frac{1}{2}(a + b)h$ , where  $a$  and  $b$  are the lengths of the parallel sides

Prism:  $V = Ah$ , where  $V$  is the volume and  $A$  is the area of the base

Pyramid:  $V = \frac{1}{3}Ah$

Cylinder:  $S = 2\pi rh + 2\pi r^2$ , where  $S$  is the total surface area  
 $V = \pi r^2h$

Cone:  $S = \pi rs + \pi r^2$ , where  $s$  is the slant height  
 $V = \frac{1}{3}\pi r^2h$

Sphere:  $S = 4\pi r^2$   
 $V = \frac{4}{3}\pi r^3$

## Combinatorics

### Combinations

Number of arrangements: (of  $n$  different objects in an ordered list)

$$n(n-1)(n-2)\times\dots\times 3\times 2\times 1 = n!$$

Number of combinations: (of  $r$  objects taken from a set of  $n$  distinct objects)

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}; \quad \binom{n}{r} = \binom{n}{n-r}; \quad \binom{n}{0} = 1$$

Number of permutations: (of  $r$  objects taken from a set of  $n$  distinct objects)

$${}^n P_r = n(n-1)(n-2)\dots(n-r+1) = \frac{n!}{(n-r)!}$$

Number of permutations with some identical objects:  $\frac{n!}{r_1!r_2!r_3!\dots}$

Inclusion – exclusion principle:  $|A \cup B| = |A| + |B| - |A \cap B|$

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

## Vectors in the Plane

### Representing vectors

Magnitude of a vector:  $|\mathbf{a}| = \left| (a_1, a_2) \right| = \sqrt{a_1^2 + a_2^2}$

### Algebra of vectors

Unit vector:  $\hat{\mathbf{a}} = \frac{\mathbf{a}}{|\mathbf{a}|}$

Scalar product:  $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$  or  $\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2$

Vector projection (of  $\mathbf{a}$  on  $\mathbf{b}$ ):  $\mathbf{p} = (\mathbf{a} \cdot \hat{\mathbf{b}})\hat{\mathbf{b}} = |\mathbf{a}|\cos\theta \hat{\mathbf{b}}$

## Trigonometry

### Basic trigonometric functions

$$\sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\tan(-\theta) = -\tan \theta$$

$$\sin\left(\theta + \frac{\pi}{2}\right) = \cos \theta$$

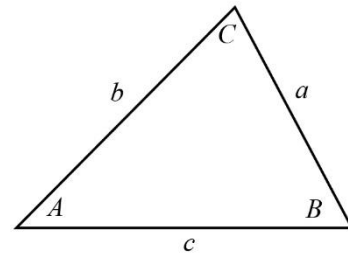
$$\cos\left(\theta - \frac{\pi}{2}\right) = \sin \theta$$

### Cosine and sine rules

For any triangle  $ABC$  with corresponding length of sides  $a, b, c$

Cosine rule:  $c^2 = a^2 + b^2 - 2ab \cos C$

Sine rule:  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$



Area of  $\Delta$ :  $A = \frac{1}{2}ab \sin C$   
 $= \sqrt{s(s-a)(s-b)(s-c)}$  where  $s = \frac{1}{2}(a+b+c)$

### Circular measure and radian measure

In a circle of radius  $r$  for an arc subtending angle  $\theta$  (radians) at the centre

Length of arc:  $\ell = r\theta$

Length of chord:  $l = 2r \sin \frac{1}{2}\theta$

Area of sector:  $A = \frac{1}{2}r^2\theta$

Area of segment:  $A = \frac{1}{2}r^2(\theta - \sin \theta)$

### Compound angles

Angle sum and difference identities:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

Double angle identities:

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Reciprocal trigonometric functions

$$\sec \theta = \frac{1}{\cos \theta}, \cos \theta \neq 0 \quad \operatorname{cosec} \theta = \frac{1}{\sin \theta}, \sin \theta \neq 0 \quad \cot \theta = \frac{1}{\tan \theta}, \tan \theta \neq 0$$

Trigonometric identities

Pythagorean identities:  $\sin^2 \theta + \cos^2 \theta = 1$      $1 + \tan^2 \theta = \sec^2 \theta$      $\cot^2 \theta + 1 = \operatorname{cosec}^2 \theta$

Product identities:  $\cos A \cos B = \frac{1}{2} [\cos(A - B) + \cos(A + B)]$

$$\sin A \sin B = \frac{1}{2} [\cos(A - B) - \cos(A + B)]$$

$$\sin A \cos B = \frac{1}{2} [\sin(A + B) + \sin(A - B)]$$

$$\cos A \sin B = \frac{1}{2} [\sin(A + B) - \sin(A - B)]$$

Auxiliary angle formulae:

$$a \sin x \pm b \cos x = R \sin(x \pm \alpha) \text{ for } 0 < \alpha < \frac{\pi}{2}, \text{ where } R^2 = a^2 + b^2, \tan \alpha = \frac{b}{a}$$

Triple angle identities:  $\sin(3A) = 3 \sin A - 4 \sin^3 A$

$$\cos(3A) = 4 \cos^3 A - 3 \cos A$$

$$\tan(3A) = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$$

## Matrices

### Matrix arithmetic

Identity matrix: If  $\mathbf{A}$  is invertible,  $\mathbf{A}\mathbf{A}^{-1} = \mathbf{I}$  where  $\mathbf{I}$  is the identity matrix

Inverse matrix:  $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Determinant: If  $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  then  $\det \mathbf{A} = ad - bc$

### Transformation Matrices

Dilation:  $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

Rotation:  $\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$  where  $\theta$  is an anti-clockwise rotation about the origin

Reflection:  $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$  where the reflection is in the line  $y = x \tan \theta$

## Real and Complex numbers

### Number Sets

Natural Numbers:  $\mathbb{N} := \{1, 2, 3, \dots\}$

Integer Numbers:  $\mathbb{Z} := \{\dots - 2, -1, 0, 1, 2, \dots\}$

Rational Numbers:  $\mathbb{Q} := \left\{q: q = \frac{a}{b}, \text{ where } a \text{ and } b \text{ are integers, } b \neq 0\right\}$

Irrational Numbers: Numbers that cannot be expressed as the ratio of two integers

Real Numbers: The set of all rational and irrational numbers ( $\mathbb{R}$ )

Complex Numbers:  $\mathbb{C} := \{z : z = ai + b, \text{ where } a, b \in \mathbb{R}, i^2 = -1\}$

### Complex Numbers

For  $z = a + ib$ , where  $a, b \in \mathbb{R}, i^2 = -1$

Modulus:  $\text{mod } z = |z| = |a + ib| = \sqrt{a^2 + b^2}$

Product:  $|z_1 z_2| = |z_1| |z_2|$

Conjugate:  $\bar{z} = a - ib, z\bar{z} = |z|^2, \overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2, \overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$

**Other useful results**

Binomial expansion:  $(x + y)^n = x^n + \binom{n}{1}x^{n-1}y + \dots + \binom{n}{r}x^{n-r}y^r + \dots + y^n$

Binomial coefficients:  $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n \times (n-1) \times \dots \times (n-r+1)}{r \times (r-1) \times \dots \times 2 \times 1}$

Index laws:

For  $a, b > 0$  and  $m, n$  real,

$$\begin{array}{lll} a^m b^m = (ab)^m & a^m a^n = a^{m+n} & (a^m)^n = a^{mn} \\ a^{-m} = \frac{1}{a^m} & \frac{a^m}{a^n} = a^{m-n} & a^0 = 1 \end{array}$$

For  $a > 0$ ,  $m$  an integer and  $n$  a positive integer,  $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

**Arithmetic sequences**

For initial term  $a$  and common difference  $d$ :

$$\begin{array}{l} T_n = a + (n-1)d, n \geq 1 \\ T_{n+1} = T_n + d, \text{ where } T_1 = a \\ S_n = \frac{n}{2}(2a + (n-1)d) \end{array}$$

**Geometric sequences**

For initial term  $a$  and common difference  $r$ :

$$\begin{array}{l} T_{n+1} = rT_n, \text{ where } T_1 = a \\ T_n = ar^{n-1}, n \geq 1 \\ S_n = \frac{a(1-r^n)}{1-r} \\ S_\infty = \frac{a}{1-r}, \quad |r| < 1 \end{array}$$

**Lines and Linear relationships**

For points  $P(x_1, y_1)$  and  $Q(x_2, y_2)$

Mid-point of  $P$  and  $Q$ :

$$M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Gradient of the line through  $P$  and  $Q$ :

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Equation of the line through  $P$  with slope  $m$ :

$$y - y_1 = m(x - x_1)$$

Parallel lines:

$$m_1 = m_2$$

Perpendicular lines:

$$m_1 m_2 = -1$$

General equation of a line:

$$ax + by + c = 0 \text{ or } y = mx + c$$



### Quadratic relationships

For the general quadratic equation  $ax^2 + bx + c = 0$ ,  $a \neq 0$

Completing the square:  $ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$

Discriminant:  $\Delta = b^2 - 4ac$

Quadratic formula:  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

### Graphs and Relations

Equation of a circle:  $(x - a)^2 + (y - b)^2 = r^2$   
where,  $(a, b)$  is the centre and  $r$  is the radius

*Note: Any additional formulas identified by the examination writers as necessary will be included in the body of the particular question.*