Government of Western Australia
School Curriculum and Standards Authority

# MATHEMATICS SPECIALIST

## Unit 1 and Unit 2

## Formula Sheet

(*For use with Year 11 examinations and response tasks*)

2015/75004v10

This document is valid for teaching and examining from 1 July 2015.

## Measurement

| | |
|---|---|
| Circle: | $C = 2\pi r = \pi D$, where $C$ is the circumference,<br>$r$ is the radius and $D$ is the diameter<br>$A = \pi r^2$, where $A$ is the area |
| Triangle: | $A = \frac{1}{2}bh$, where $b$ is the base and $h$ is the perpendicular height |
| Parallelogram: | $A = bh$ |
| Trapezium: | $A = \frac{1}{2}(a + b)h$, where $a$ and $b$ are the lengths of the parallel sides |
| Prism: | $V = Ah$, where $V$ is the volume and $A$ is the area of the base |
| Pyramid: | $V = \frac{1}{3}Ah$ |
| Cylinder: | $S = 2\pi rh + 2\pi r^2$, where $S$ is the total surface area<br>$V = \pi r^2 h$ |
| Cone: | $S = \pi rs + \pi r^2$, where $s$ is the slant height<br>$V = \frac{1}{3}\pi r^2 h$ |
| Sphere: | $S = 4\pi r^2$<br>$V = \frac{4}{3}\pi r^3$ |

## Combinatorics

### Combinations

Number of arrangements: (of $n$ different objects in an ordered list)

$$n(n-1)(n-2)\times.....\times 3\times 2\times 1 = n!$$

Number of combinations: (of $r$ objects taken from a set of $n$ distinct objects)

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}; \qquad \binom{n}{r} = \binom{n}{n-r}; \qquad \binom{n}{0} = 1$$

Number of permutations: (of $r$ objects taken from a set of $n$ distinct objects)

$$^{n}P_{r} = n(n-1)(n-2)...(n-r+1) = \frac{n!}{(n-r)!}$$

Number of permutations with some identical objects: $\dfrac{n!}{r_1!r_2!r_3!...}$

Inclusion – exclusion principle: $|A\cup B| = |A| + |B| - |A\cap B|$

$$|A\cup B\cup C| = |A| + |B| + |C| - |A\cap B| - |A\cap C| - |B\cap C| + |A\cap B\cap C|$$

## Vectors in the Plane

### Representing vectors

Magnitude of a vector: $|\mathbf{a}| = |(a_1, a_2)| = \sqrt{a_1^2 + a_2^2}$

### Algebra of vectors

Unit vector: $\hat{\mathbf{a}} = \dfrac{\mathbf{a}}{|\mathbf{a}|}$

Scalar product: $\mathbf{a}\bullet\mathbf{b} = |\mathbf{a}||\mathbf{b}|\boldsymbol{cos\theta}$ or $\mathbf{a}\bullet\mathbf{b} = a_1b_1 + a_2b_2$

Vector projection (of **a** on **b**): $\mathbf{p} = (\mathbf{a}\bullet\hat{\mathbf{b}})\hat{\mathbf{b}} = |\mathbf{a}|cos\theta\ \hat{\mathbf{b}}$

## Trigonometry

### Basic trigonometric functions

$\sin(-\theta) = -\sin\theta$ $\quad\cos(-\theta) = \cos\theta$ $\quad\tan(-\theta) = -\tan\theta$

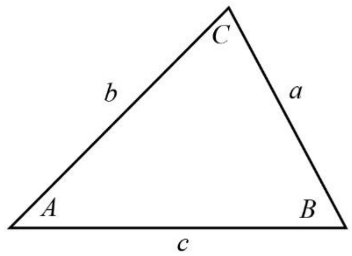
$\sin\left(\theta + \frac{\pi}{2}\right) = \cos\theta$ $\quad\cos\left(\theta - \frac{\pi}{2}\right) = \sin\theta$

### Cosine and sine rules

For any triangle $ABC$ with corresponding length of sides $a,b,c$

Cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

Sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$




Area of $\Delta$:

$$A = \frac{1}{2}ab\sin C$$

$$= \sqrt{s(s-a)(s-b)(s-c)} \text{ where } s = \frac{1}{2}(a+b+c)$$

### Circular measure and radian measure

In a circle of radius $r$ for an arc subtending angle $\theta$ (radians) at the centre

Length of arc: $\ell = r\theta$ $\quad$ Length of chord: $l = 2r\sin\frac{1}{2}\theta$

Area of sector: $A = \frac{1}{2}r^2\theta$ $\quad$ Area of segment: $A = \frac{1}{2}r^2(\theta - \sin\theta)$

### Compound angles

Angle sum and difference identites:

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

Double angle identities:

$$\sin 2A = 2\sin A\cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Reciprocal trigonometric functions

$$\sec\theta = \frac{1}{\cos\theta}, \quad \cos\theta \neq 0 \qquad \operatorname{cosec}\theta = \frac{1}{\sin\theta}, \quad \sin\theta \neq 0 \qquad \cot\theta = \frac{1}{\tan\theta}, \quad \tan\theta \neq 0$$

Trigonometric identities

Pythagorean identities: $\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad \cot^2\theta + 1 = \operatorname{cosec}^2\theta$

Product identities:

$$\cos A \cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$

$$\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$\cos A \sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$$

Auxiliary angle formulae:

$a\sin x \pm b\cos x = R\sin(x \pm \alpha)$ for $0 < \alpha < \frac{\pi}{2}$, where $R^2 = a^2 + b^2, \tan\alpha = \frac{b}{a}$

Triple angle identities:

$$\sin(3A) = 3\sin A - 4\sin^3 A$$

$$\cos(3A) = 4\cos^3 A - 3\cos A$$

$$\tan(3A) = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$$

## Matrices

### Matrix arithmetic

Identity matrix: If $\mathbf{A}$ is invertible, $\mathbf{AA}^{-1} = \mathbf{I}$ where $\mathbf{I}$ is the identity matrix

Inverse matrix: $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \dfrac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Determinant: If $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $\det \mathbf{A} = ad - bc$

### Transformation Matrices

Dilation: $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

Rotation: $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ where $\theta$ is an anti-clockwise rotation about the origin

Reflection: $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$ where the reflection is in the line $y = x\tan\theta$

## Real and Complex numbers

### Number Sets

Natural Numbers: $\mathbb{N} := \{1,2,3, \ldots\ldots\}$

Integer Numbers: $\mathbb{Z} := \{\ldots -2, -1,0,1,2, \ldots\}$

Rational Numbers: $\mathbb{Q} := \left\{q : q = \frac{a}{b},\ \text{where a and b are integers},\ b \neq 0\right\}$

Irrational Numbers: Numbers that cannot be expressed as the ratio of two integers

Real Numbers: The set of all rational and irrational numbers $(\mathbb{R})$

Complex Numbers: $\mathbb{C} := \{z : z = ai + b,\ \text{where } a, b \in \mathbb{R},\ i^2 = -1\}$

### Complex Numbers

For $z = a + ib$, where $a, b \in \mathbb{R}$, $i^2 = -1$

Modulus: $\text{mod } z = |z| = |a + ib| = \sqrt{a^2 + b^2}$

Product: $|z_1 z_2| = |z_1||z_2|$

Conjugate: $\bar{z} = a - ib$, $z\bar{z} = |z|^2$, $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$, $\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$

**Other useful results**

Binomial expansion: $(x+y)^n = x^n + \binom{n}{1} x^{n-1} y + \cdots + \binom{n}{r} x^{n-r} y^r + \cdots + y^n$

Binomial coefficients: $\binom{n}{r} = \dfrac{n!}{r!(n-r)!} = \dfrac{n \times (n-1) \times \cdots \times (n-r+1)}{r \times (r-1) \times \cdots \times 2 \times 1}$

Index laws:

For $a, b > 0$ and $m, n$ real,

$a^m b^m = (ab)^m$ $\quad a^m a^n = a^{m+n}$ $\quad (a^m)^n = a^{mn}$

$a^{-m} = \dfrac{1}{a^m}$ $\quad \dfrac{a^m}{a^n} = a^{m-n}$ $\quad a^0 = 1$

For $a > 0$, $m$ an integer and $n$ a positive integer, $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

Arithmetic sequences

For initial term $a$ and common difference $d$:

$T_n = a + (n-1)d, n \geq 1$

$T_{n+1} = T_n + d$, where $T_1 = a$

$S_n = \dfrac{n}{2}\left(2a + (n-1)d\right)$

Geometric sequences

For initial term $a$ and common difference $r$:

$T_{n+1} = rT_n$, where $T_1 = a$

$T_n = ar^{n-1}, n \geq 1$

$S_n = \dfrac{a(1-r^n)}{1-r}$

$S_\infty = \dfrac{a}{1-r}, \quad |r| < 1$

Lines and Linear relationships

For points $P(x_1, y_1)$ and $Q(x_2, y_2)$

Mid-point of $P$ and $Q$: $M = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

Gradient of the line through $P$ and $Q$: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Equation of the line through $P$ with slope $m$: $y - y_1 = m(x - x_1)$

Parallel lines: $m_1 = m_2$

Perpendicular lines: $m_1 m_2 = -1$

General equation of a line: $ax + by + c = 0$ or $y = mx + c$

## Quadratic relationships

For the general quadratic equation $ax^2 + bx + c = 0,\ a \neq 0$

Completing the square: $ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$

Discriminant: $\Delta = b^2 - 4ac$

Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

## Graphs and Relations

Equation of a circle: $(x-a)^2 + (y-b)^2 = r^2$

where, $(a,b)$ is the centre and $r$ is the radius

*Note: Any additional formulas identified by the examination writers as necessary will be included in the body of the particular question.*