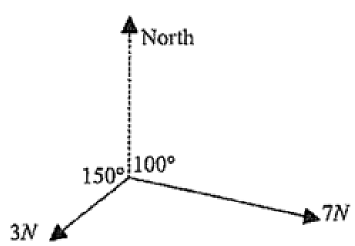
VECTORS REVISION

Question 1 **(9 marks)**

(a) A sailing boat leaves port and sails on a bearing of 300° for 4 km before turning to sail 1.9 km on a bearing of 080°. How far is the boat from the port and what is the bearing of final position of the boat from the port. (3 marks)

(b) Find the magnitude and direction of the resultant of the pair of forces in the diagram below. (3 marks)



(c) In still air, an aircraft can maintain a speed of 350 km/h. In what direction should the aircraft be pointing if it wished to travel in a direction 160°, and a 35 km/h wind is blowing from 075°? (3 marks)

Question 2 **(6 marks)**

(a) Vectors **a** and **b** have the same magnitude and vectors a and c are perpendicular, where a = $\begin{bmatrix} m \\ n \end{bmatrix}$, b = $\begin{bmatrix} -4 \\ 6 \end{bmatrix}$ and c = $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$. Determine the values of m and n. (3 marks)

(b) Determine the scalar projection of a velocity of 12 m/s on a bearing of 065° onto a velocity of 20 m/s on a bearing of 280°, giving your answer to two decimal places.

(3 marks)

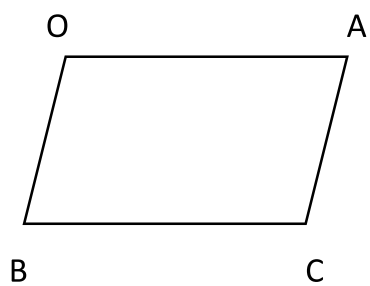
Question 3 **(8 marks)**

(a) A triangle has vertices as A$(-3, 1)$, B$(-1, 4)$ and C$(5, 0)$.

(i) Determine the vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{BC}$. (2 marks)

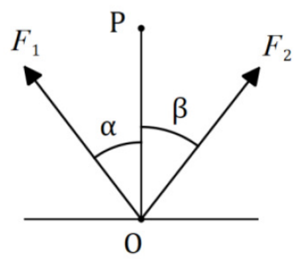
(ii) Use a vector method to prove that triangle ABC is right-angle. (2 marks)

(b) Use a vector method to prove that if the diagonals of a parallelogram are perpendicular to each other, then the parallelogram is a rhombus. (4 marks)



Question 4 **(8 marks)**

Two forces, F_1 = 550N and F_2 = 770N, act on a body at O, and make angles of $\alpha = 33°$, and $\beta = 18°$ respectively with the vertical OP, as shown in the diagram below.



(a) Determine the magnitude of resultant force and the angle it makes with the vertical. (5 marks)

(b) The magnitude of F_1 is to be adjusted so that the direction of the resultant is vertical. Determine the required magnitude of F_1. (3 marks)

Question 5 **(9 marks)**

(a) The work done, in joules, by a force of $\mathbf{F}$ Newtons in changing the displacement of an object by $\mathbf{s}$ metres, is given by the scalar product of $\mathbf{F}$ and $\mathbf{s}$.

(i) A force of 250 N acting due south moves an object 4.3 m in a south-westerly direction. Determine the work done. (2 marks)

(ii) Another force of 155 N does 269 joules of work in moving an object 190 cm. Determine the angle between the force and the direction of movement. (2 marks)

(b) A triangle is formed by three non-zero vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$, so that $\mathbf{c} = \mathbf{a} - \mathbf{b}$, and θ is the angle between $\mathbf{a}$ and $\mathbf{b}$.

(i) Sketch the triangle. (1 mark)

(ii) Explain why $\mathbf{c} \cdot \mathbf{c} = |\mathbf{c}|^2$. (1 mark)

(iii) Use $\mathbf{c} \cdot \mathbf{c} = (\mathbf{a} - \mathbf{b}) \cdot (\mathbf{a} - \mathbf{b})$ to deduce the cosine rule. (3 marks)

Question 6 **(11 marks)**

A small boat that can maintain a steady speed of 5 ms^{-1} is to cross a river from A to B, where $\overrightarrow{AB} = (35\mathbf{i} - 105\mathbf{j})$ m.

A current of $(-\mathbf{i} - 2\mathbf{j})$ ms^{-1} flows in the river.

The velocity vector that the pilot of the small boat must set to travel from A to B is $a\mathbf{i} + b\mathbf{j}$, where a and b are constants.

(a) Explain why $t(a - 1) = 35$ and $t(b - 2) = -105$, where t is a constant.(3 marks)

(b) Eliminate t from the equations in (a) and hence express b in terms of a, simplifying your expression. (3 marks)

(c) Explain why $a^2 + b^2 = 25$. (1 mark)

(d) Use your equations from (b) and (c) to determine the values of a and b. (3 marks)

(e) Determine the time that the small boat will take to travel from A to B. (1 mark)

Question 7 **(8 marks)**

Three vectors are given by $\mathbf{a} = 3\mathbf{i} - 4\mathbf{j}$, $\mathbf{b} = -3\mathbf{i} + 1.5\mathbf{j}$ and $\mathbf{c} = -2\mathbf{i} + y\mathbf{j}$, where y is a constant.

(a) Determine the vector projection of $\mathbf{b}$ on $\mathbf{a}$. (3 marks)

(b) Determine the value(s) of y if

(i) $\mathbf{a}$ and $\mathbf{c}$ are perpendicular. (2 marks)

(ii) the angle between the directions of $\mathbf{b}$ and $\mathbf{c}$ is 45°. (3 marks)