Solutions to

Unit 3A Specialist Mathematics

by A.J. Sadler

Prepared by:
Glen Prideaux

2009

Preface

The answers in the Sadler text book sometimes are not enough. For those times when your really need to see a fully worked solution, look here.

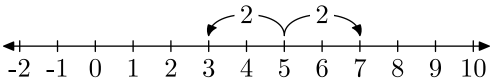
It is essential that you use this sparingly!

You should not look here until you have given your best effort to a problem. Understand the problem here, then go away and do it on your own.

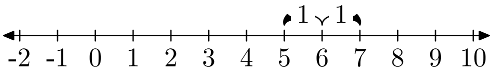
Contents

Chapter 1

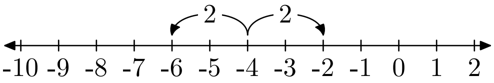
Exercise 1A

1. 

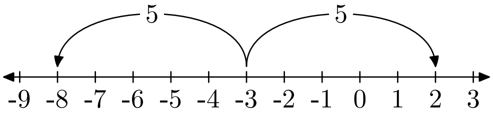
$x = 3$ or $x = 7$

2. 

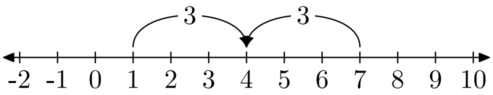
$x = 5$ or $x = 7$

3. 

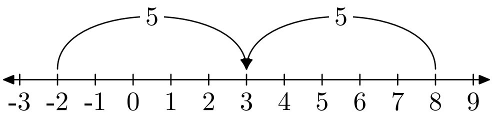
$x = -6$ or $x = -2$

4. 

$x = -8$ or $x = 2$

5. 

$x = 4$

6. 

$x = 3$

7. $x + 3 = 7$ or $x + 3 = -7$
   $x = 4$ $x = -10$

8. $x - 3 = 5$ or $x - 3 = -5$
   $x = 8$ $x = -2$

9. No solution (absolute value can never be negative).

10. $x - 2 = 11$ or $x - 2 = -11$
    $x = 13$ $x = -9$

11. $2x + 3 = 7$ or $2x + 3 = -7$
    $2x = 4$ $2x = -10$
    $x = 2$ $x = -5$

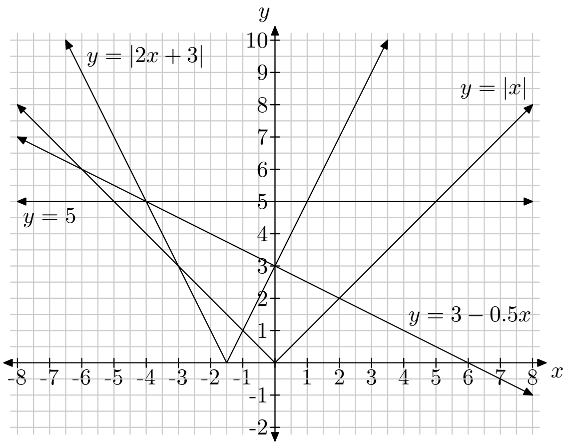
12. $5x - 8 = 7$ or $5x - 8 = -7$
    $5x = 15$ $5x = 1$
    $x = 3$ $x = \frac{1}{5}$

13. Find the appropriate intersection and read the x-coordinate.

    (a) Intersections at (3,4) and (7,4) so $x = 3$ or $x = 7$.

    (b) Intersections at (-2,4) and (6,4) so $x = -2$ or $x = 6$.

    (c) Intersections at (4,2) and (8,6) so $x = 4$ or $x = 8$.

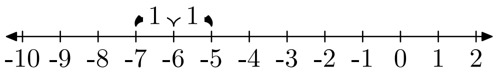
14. 

    (a) Intersections at (-4,5) and (1,5) so $x = -4$ or $x = 1$.

    (b) Intersections at (-6,6) and (2,2) so $x = -6$ or $x = 2$.

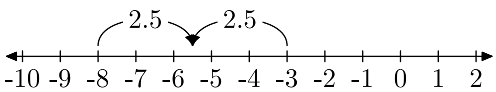
    (c) Intersections at (-4,5) and (0,3) so $x = -4$ or $x = 0$.

    (d) Intersections at (-3,3) and (-1,1) so $x = -3$ or $x = -1$.

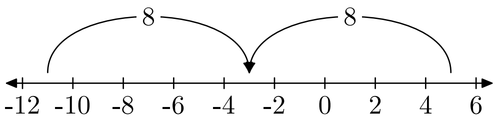
15. 

$x = -7$ or $x = -5$

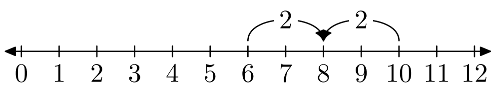
16. No solution (absolute value can never be negative).

17. 

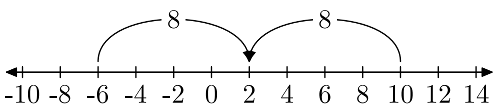
$x = -5.5$

18. 

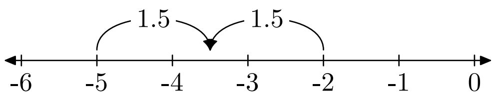
$x = -3$

19. 

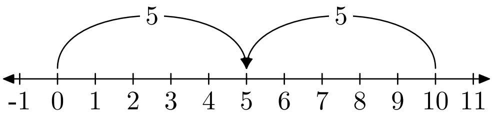
$x = 8$

20. 

$x = 2$

21. 

$x = -3.5$

22. 

$x = 5$

23. $$x + 5 = 2x - 14$$
$$x = 19$$

$$|19 + 5| = |2 \times 19 - 14|$$
$$|24| = |24| \quad \checkmark$$

or

$$x + 5 = -(2x - 14)$$
$$x + 5 = -2x + 14$$
$$3x = 9$$
$$x = 3$$

$$|3 + 5| = |2 \times 3 - 14|$$
$$|8| = |-8| \quad \checkmark$$

24. $$3x - 1 = x + 9$$
$$2x = 10$$
$$x = 5$$

$$|3 \times 5 - 1| = |5 + 9|$$
$$|14| = |14| \quad \checkmark$$

or

$$-(3x - 1) = x + 9$$
$$-3x + 1 = x + 9$$
$$-4x = 8$$
$$x = -2$$

$$|3 \times -2 - 1| = |-2 + 9|$$
$$|-7| = |7| \quad \checkmark$$

25. $$4x - 3 = 3x - 11$$
$$x = -8$$

$$|4 \times -8 - 3| = |3 \times -8 - 11|$$
$$|-35| = |-35| \quad \checkmark$$

or

$$4x - 3 = -(3x - 11)$$
$$4x - 3 = -3x + 11$$
$$7x = 14$$
$$x = 2$$

$$|4 \times 2 - 3| = |3 \times 2 - 11|$$
$$|5| = |-5| \quad \checkmark$$

26. $$5x - 11 = 5 - 3x$$
$$8x = 16$$
$$x = 2$$

$$|5 \times 2 - 11| = |5 - 3 \times 2|$$
$$|-1| = |-1| \quad \checkmark$$

or

$$-(5x - 11) = 5 - 3x$$
$$-5x + 11 = 5 - 3x$$
$$6 = 2x$$
$$x = 3$$

$$|5 \times 3 - 11| = |5 - 3 \times 3|$$
$$|4| = |-4| \quad \checkmark$$

27. $$x - 2 = 2x - 6$$
$$-x = -4$$
$$x = 4$$

$$|4 - 2| = 2 \times 4 - 6$$
$$|2| = 2 \quad \checkmark$$

or

$$-(x - 2) = 2x - 6$$
$$-x + 2 = 2x - 6$$
$$-3x = -8$$
$$x = \frac{8}{3}$$

$$|\frac{8}{3} - 2| = 2 \times \frac{8}{3} - 6$$
$$|\frac{2}{3}| \neq -\frac{2}{3}$$

The second 'solution' is not valid. The only solution is $x = 4$.

28. $$x - 3 = 2x$$
$$x = -3$$

$$|-3 - 3| = 2 \times -3$$
$$|-6| \neq -6$$

or

$$-(x - 3) = 2x$$
$$-x + 3 = 2x$$
$$-3x = -3$$
$$x = 1$$

$$|1 - 3| = 2 \times 1$$
$$|-2| = 2 \quad \checkmark$$

The first 'solution' is not valid. The only solution is $x = 1$.

29. $$x - 2 = 0.5x + 1$$
$$0.5x = 3$$
$$x = 6$$

$$|6| - 2 = 0.5 \times 6 + 1$$
$$4 = 4 \quad \checkmark$$

or

$$\begin{aligned}-x-2&=0.5x+1\\-1.5x&=3\\x&=-2\end{aligned}$$

$$\begin{aligned}|-2|-2&=0.5\times-2+1\\0&=0\quad\checkmark.\end{aligned}$$

30. $$\begin{aligned}x+2&=-3x+6\\4x&=4\\x&=1\end{aligned}$$

$$\begin{aligned}|1+2|&=-3\times1+6\\|3|&=3\quad\checkmark\end{aligned}$$

or

$$\begin{aligned}-(x+2)&=-3x+6\\-x-2&=-3x+6\\2x&=8\\x&=4\end{aligned}$$

$$\begin{aligned}|4+2|&=-3\times4+6\\|6|&\neq3-6\end{aligned}$$

The second solution is invalid. The only solution is $x=1$.

31. $x\geq1$:

$$\begin{aligned}x+5+x-1&=7\\2x+4&=7\\2x&=3\\x&=1.5\quad\checkmark\end{aligned}$$

$-5\leq x\leq1$:

$$\begin{aligned}x+5-(x-1)&=7\\x+5-x+1&=7\\6&\neq7\quad\implies\text{no sol'n}\end{aligned}$$

$x\leq-5$:

$$\begin{aligned}-(x+5)-(x-1)&=7\\-x-5-x+1&=7\\-2x-4&=7\\-2x&=11\\x&=-5.5\quad\checkmark\end{aligned}$$

32. $x\geq4$:

$$\begin{aligned}x+3+x-4&=2\\2x-1&=2\\2x&=3\\x&=1.5\\&\implies\text{no sol'n (out of domain)}\end{aligned}$$

$-3\leq x\leq4$:

$$\begin{aligned}x+3-(x-4)&=2\\x+3-x+4&=2\\7&\neq2\quad\implies\text{no sol'n}\end{aligned}$$

$x\leq-3$:

$$\begin{aligned}-(x+3)-(x-4)&=2\\-x-3-x+4&=2\\-2x+1&=2\\-2x&=1\\x&=-0.5\\&\implies\text{no sol'n (out of domain)}\end{aligned}$$

The equation has no solution.

33. $x\geq3$:

$$\begin{aligned}x+5+x-3&=8\\2x+2&=8\\2x&=6\\x&=3\quad\checkmark\end{aligned}$$

$-5\leq x\leq3$:

$$\begin{aligned}x+5-(x-3)&=8\\x+5-x+3&=8\\8&=8\end{aligned}$$

$\implies$ all of $-5\leq x\leq3$ is a solution.

$x\leq-5$:

$$\begin{aligned}-(x+5)-(x-3)&=8\\-x-5-x+3&=8\\-2x-2&=8\\-2x&=10\\x&=-5\end{aligned}$$

Solution is $-5\leq x\leq3$.

34. $x\geq8$:

$$\begin{aligned}x-8&=-(2-x)-6\\x-8&=-2+x-6\\-8&=-8\end{aligned}$$

$\implies$ all of $x\geq8$ is a solution.

$2\leq x\leq8$:

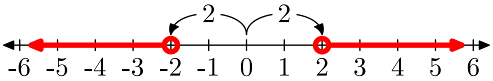
$$\begin{aligned}-(x-8)&=-(2-x)-6\\-x+8&=-2+x-6\\2x&=16\\x&=8\end{aligned}$$

$x\leq2$:

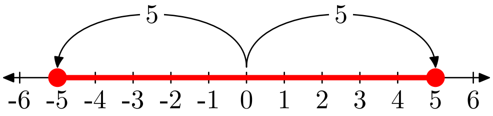
$$\begin{aligned}-(x-8)&=2-x-6\\-x+8&=-x-4\\8&\neq-4\quad\implies\text{no sol'n}\end{aligned}$$

Solution is $x\geq8$.

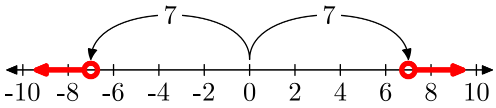
Exercise 1B

1. 

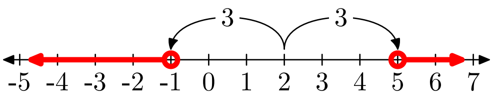
$-2 < x < 2$

2. 

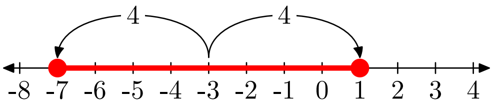
$-5 \leq x \leq 5$

3. 

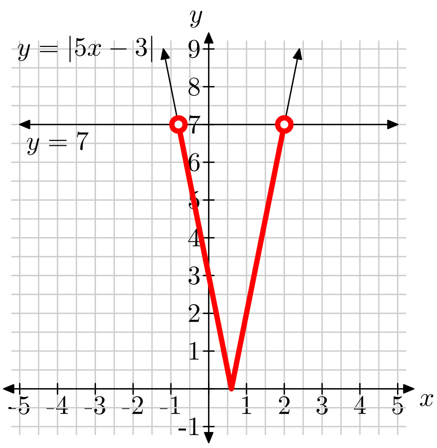
$x < -7$ or $x > 7$

4. 

$x < -1$ or $x > 5$

5. 

$-7 \leq x \leq 1$

6. 

Algebraically:

For $5x - 3 \geq 0$:

$5x - 3 < 7$

$5x < 10$

$x < 5$

For $5x - 3 \leq 0$:

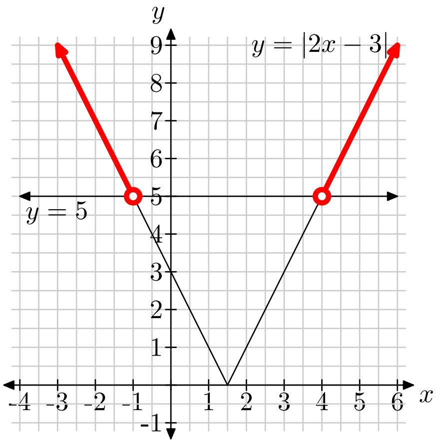
$-(5x - 3) < 7$

$5x - 3 > -7$

$5x > -4$

$x > -\frac{4}{5}$

$-\frac{4}{5} < x < 2$

7. 

Algebraically:

For $2x - 3 \geq 0$:

$2x - 3 > 5$

$2x > 8$

$x > 4$

For $2x - 3 \leq 0$:

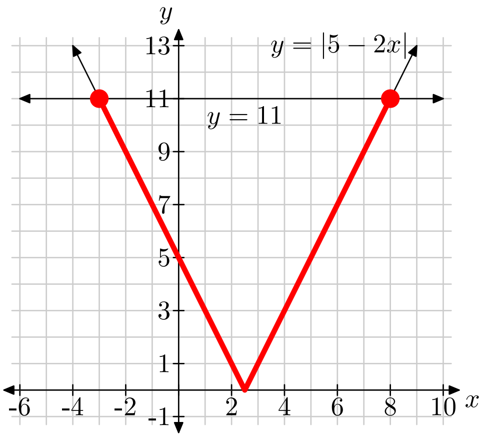
$-(2x - 3) > 5$

$2x - 3 < -5$

$2x < -2$

$x < -1$

$x < -1$ or $x > 4$

8. 

Algebraically:

For $5 - 2x \geq 0$:

$5 - 2x \leq 11$

$-2x \leq 6$

$x \geq -3$

For $5 - 2x \leq 0$:

$-(5 - 2x) \leq 11$

$-5 + 2x \leq 11$

$2x \leq 16$

$x \leq 8$

$-3 \leq x \leq 8$

9. Centred on 0, no more than 3 units from centre: $|x| \leq 3$
10. Centred on 0, less than 4 units from centre: $|x| < 4$
11. Centred on 0, at least 1 unit from centre: $|x| \geq 1$
12. Centred on 0, more than 2 units from centre: $|x| > 2$
13. Centred on 0, no more than 4 units from centre: $|x| \leq 4$
14. Centred on 0, at least 3 units from centre: $|x| \geq 3$

15. Distance from 3 is greater than distance from 7. Distance is equal at $x = 5$ so possible values are $\{x \in \mathbb{R} : x > 5\}$.

16. Distance from 1 is less than or equal to distance from 8. Distance is equal at $x = 4.5$ so possible values are $\{x \in \mathbb{R} : x \leq 4.5\}$.

17. Distance from -2 is less than distance from 2. Distance is equal at $x = 0$ so possible values are $\{x \in \mathbb{R} : x < 0\}$.

18. Distance from 5 is greater than or equal to distance from -1. Distance is equal at $x = 2$ so possible values are $\{x \in \mathbb{R} : x \leq 2\}$.

19. Distance from 13 is greater than distance from 5. (Note $|5 - x| = |x - 5|$.) Distance is equal at $x = 9$ so possible values are $\{x \in \mathbb{R} : x < 9\}$.

20. Distance from -12 is greater than or equal to distance from 2. Distance is equal at $x = -5$ so possible values are $\{x \in \mathbb{R} : x \geq -5\}$.

21. Centred on 2, no more than 3 units from centre: $|x - 2| \leq 3$

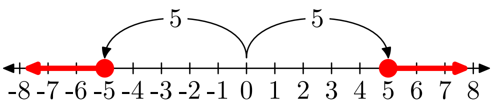
22. Centred on 3, less than 1 unit from centre: $|x - 3| < 1$

23. Centred on 2, at least 2 units from centre: $|x - 2| \geq 2$

24. Centred on 1, more than 2 units from centre: $|x - 1| > 2$

25. Centred on 1, no more than 4 units from centre: $|x - 1| \leq 4$

26. Centred on 1, at least 4 units from centre: $|x - 1| \geq 4$

27. 

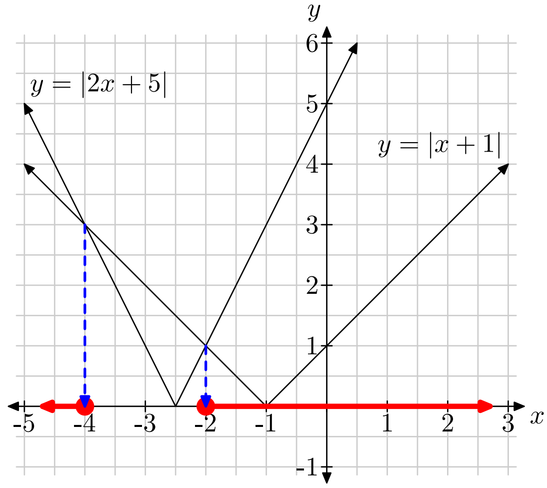
$$x \leq -5 \text{ or } x \geq 5$$

28. For $2x > 0$:
$$2x < 8$$
$$x < 4$$
For $2x < 0$:
$$-2x < 8$$
$$2x > -8$$
$$x > -4$$

$$-4 < x < 4$$

29. $|x| > -3$ is true for all x (since the absolute value is always positive).

30. Distance from 3 is greater than or equal to distance from -5. Distance is equal at -1 so $x \leq -1$.

31. 

Algebraically:
First solve $|x + 1| = |2x + 5|$

$$x + 1 = 2x + 5 \quad \text{or} \quad -(x+1) = 2x + 5$$
$$x = -4 \qquad\qquad -x - 1 = 2x + 5$$
$$-6 = 3x$$
$$x = -2$$

Now consider the three intervals delimited by these two solutions.

- $x < -4$
  Try a value, say -5:
  Is it true that $|-5 + 1| \leq |2(-5) + 5|$?
  Yes $(4 \leq 5)$.
- $-4 < x < -2$
  Try a value, say -3:
  Is it true that $|-3 + 1| \leq |2(-3) + 5|$?
  No $(2 \not\leq 1)$.
- $x > -2$
  Try a value, say 0:
  Is it true that $|0 + 1| \leq |2(0) + 5|$?
  Yes $(1 \leq 5)$.

Solution set is

$$\{x \in \mathbb{R} : x \leq -4\} \cup \{x \in \mathbb{R} : x \geq -2\}$$

32. No solution (absolute value can not be negative.)

33. First solve $|5x + 1| = |3x + 9|$

$$5x + 1 = 3x + 9 \quad \text{or} \quad -(5x + 1) = 3x + 9$$
$$2x = 8 \qquad\qquad -5x - 1 = 3x + 9$$
$$x = 4 \qquad\qquad -10 = 8x$$
$$x = -1.25$$

Now consider the three intervals delimited by these two solutions.

- $x < -1.25$
  Try a value, say -2:
  Is it true that $|5(-2) + 1| > |3(-2) + 9|$?
  Yes $(9 > 3)$.

- $-1.25 < x < 4$
  Try a value, say 0:
  Is it true that $|5(0) + 1| > |3(0) + 9|$?
  No ($1 \not> 9$).
- $x > 4$
  Try a value, say 5:
  Is it true that $|5(5) + 1| > |3(5) + 9|$?
  Yes ($26 > 24$).

Solution set is

$$\{x \in \mathbb{R} : x < -1.25\} \cup \{x \in \mathbb{R} : x > 4\}$$

34. First solve $|2x + 5| = |3x - 1|$

$$\begin{aligned} 2x + 5 &= 3x - 1 \quad \text{or} \quad & -(2x + 5) &= 3x - 1 \\ x &= 6 & -2x - 5 &= 3x - 1 \\ & & -4 &= 5x \\ & & x &= -0.8 \end{aligned}$$

Now consider the three intervals delimited by these two solutions.

- $x < -0.8$
  Try a value, say -2:
  Is it true that $|2(-2) + 5| \geq |3(-2) - 1|$?
  No ($1 \not\geq 7$).
- $-0.8 < x < 6$
  Try a value, say 0:
  Is it true that $|2(0) + 5| \geq |3(0) - 1|$?
  Yes ($5 \geq -1$).
- $x > 6$
  Try a value, say 7:
  Is it true that $|2(7) + 5| \geq |3(7) - 1|$?
  No ($19 \not\geq 20$).

Solution set is

$$\{x \in \mathbb{R} : -0.8 \leq x \leq 6\}$$

Actually we only need to test one of the three intervals. At each of the two initial solutions we have lines crossing so if the LHS<RHS on one side of the intersection it follows that LHS>RHS on the other side, and vice versa. We'll use this in the next questions.

35. First solve $|6x + 1| = |2x + 5|$

$$\begin{aligned} 6x + 1 &= 2x + 5 \quad \text{or} \quad & -(6x + 1) &= 2x + 5 \\ 4x &= 4 & -6x - 1 &= 2x + 5 \\ x &= 1 & -8x &= 6 \\ & & x &= -0.75 \end{aligned}$$

Now test one of the three intervals delimited by these two solutions.

- $x < -0.75$
  Try a value, say -1:
  Is it true that $|6(-1) + 1| \leq |2(-1) + 5|$?
  No ($5 \not\leq 3$).

Solution set is

$$\{x \in \mathbb{R} : -0.75 \leq x \leq 1\}$$

36. First solve $|3x + 7| = |2x - 4|$

$$\begin{aligned} 3x + 7 &= 2x - 4 \quad \text{or} \quad & -(3x + 7) &= 2x - 4 \\ x &= -11 & -3x - 7 &= 2x - 4 \\ & & -5x &= 3 \\ & & x &= -0.6 \end{aligned}$$

Now test one of the three intervals delimited by these two solutions.

- $x < -11$
  Try a value, say -12:
  Is it true that $|3(-12) + 7| > |2(-12) - 4|$?
  Yes ($29 > 28$).

Solution set is

$$\{x \in \mathbb{R} : x < -11\} \cup \{x \in \mathbb{R} : x > -0.6\}$$

37. This is true for all $x \in \mathbb{R}$ since the absolute value is never negative, and hence always greater than -5.

38. First solve $|x - 1| = |2x + 7|$

$$\begin{aligned} x - 1 &= 2x + 7 \quad \text{or} \quad & -(x - 1) &= 2x + 7 \\ x &= -8 & -x + 1 &= 2x + 7 \\ & & -3x &= 6 \\ & & x &= -2 \end{aligned}$$

Now test one of the three intervals delimited by these two solutions.

- $x < -8$
  Try a value, say -10:
  Is it true that $| - 10 - 1| \leq |2(-10) + 7|$?
  Yes ($11 \leq 13$).

Solution set is

$$\{x \in \mathbb{R} : x < -8\} \cup \{x \in \mathbb{R} : x > -2\}$$

39. Distance from 11 is greater than or equal to distance from -5. 3 is equidistant, so $x \leq 3$

40. First solve $|3x + 7| = |7 - 2x|$

$$\begin{aligned} 3x + 7 &= 7 - 2x \quad \text{or} \quad & -(3x + 7) &= 7 - 2x \\ 5x &= 0 & -3x - 7 &= 7 - 2x \\ x &= 0 & -x &= 14 \\ & & x &= -14 \end{aligned}$$

Now test one of the three intervals delimited by these two solutions.

- $x < -14$
  Try a value, say -20:
  Is it true that $|3(-20) + 7| > |7 - 2(-20)|$?
  Yes ($53 > 47$).

Solution set is

$$\{x \in \mathbb{R} : x < -14\} \cup \{x \in \mathbb{R} : x > 0\}$$

41. No solution (LHS=RHS $\forall x \in \mathbb{R}$)

42. True for all x (LHS=RHS $\forall x \in \mathbb{R}$)

43. We can rewrite this as $3|x+1| \leq |x+1|$ which can only be true at $x+1=0$, i.e. $x=-1$.

44. We can rewrite this as $2|x-3| < 5|x-3|$ which simplifies to $2 < 5$ for all $x \neq 3$, so the solution set is
$$\{x \in \mathbb{R} : x \neq 3\}$$

45. First solve $x = |2x-6|$

$$\begin{aligned} x &= 2x-6 \quad &\text{or} \quad x &= -(2x-6) \\ x &= 6 & x &= -2x+6 \\ & & 3x &= 6 \\ & & x &= 2 \end{aligned}$$

Now test one of the three intervals delimited by these two solutions.

- $x < 2$
  Try a value, say 0:
  Is it true that $0 > |2(0)-6|$?
  No ($0 \not> 6$).

Solution set is

$$\{x \in \mathbb{R} : 2 < x < 6\}$$

46. First solve $|x-3| = 2x$

$$\begin{aligned} x-3 &= 2x \quad &\text{or} \quad -(x-3) &= 2x \\ x &= -3 & -x+3 &= 2x \\ & & 3x &= 3 \\ & & x &= 1 \end{aligned}$$

The first of these is not really a solution, because it was found based on the premise of $x-3$ being positive which is not true for $x=-3$. As a result we really have only one solution. (Graph it on your calculator if you're not sure of this.)

Now test one of the two intervals delimited by this solution.

- $x < 1$
  Try a value, say 0:
  Is it true that $|0-3| \leq 2(0)$?
  No ($3 \not\leq 0$).

Solution set is

$$\{x \in \mathbb{R} : x \geq 1\}$$

47. First solve $2x-2 = |x|$

$$\begin{aligned} 2x-2 &= x \quad &\text{or} \quad 2x-2 &= -x \\ x &= 2 & 3x &= 2 \\ & & x &= \frac{2}{3} \end{aligned}$$

The second of these is not really a solution, because it was found based on the premise of x being negative which is not true for $x = \frac{2}{3}$. As a result we really have only one solution.

Now test one of the two intervals delimited by this solution.

- $x < 2$
  Try a value, say 0:
  Is it true that $2(0)-2 < |0|)$?
  Yes ($-2 < 0$).

Solution set is

$$\{x \in \mathbb{R} : x < 2\}$$

48. First solve $|x|+1 = 2x$. If you sketch the graph of LHS and RHS it should be clear that this will have one solution with positive x:

$$\begin{aligned} x+1 &= 2x \\ x &= 1 \end{aligned}$$

The LHS is clearly greater than the RHS for negative x so we can conclude that the solution set is
$$\{x \in \mathbb{R} : x \leq 1\}$$

49. Apart from having a $>$ instead of $\geq$ this problem can be rearranged to be identical to the previous one, so it will have a corresponding solution set:

$$\{x \in \mathbb{R} : x < 1\}$$

50. First solve $|x+4| = x+2$

$$\begin{aligned} x+4 &= x+2 \quad &\text{or} \quad -(x+4) &= x+2 \\ &\text{No Solution} & -x-4 &= x+2 \\ & & -2x &= 6 \\ & & x &= -3 \end{aligned}$$

The second of these is not really a solution, because it was found based on the premise of $x+4$ being negative which is not true for $x=-3$. As a result we have no solution. Graphically, the graphs of the LHS and RHS never intersect, so the inequality is either always true or never true. Test a value to determine which:
Try a value, say 0:
Is it true that $|(0)+4| > 0+2$?
Yes ($4 > 2$).

Solution set is $\mathbb{R}$.

51. "*" must be $>$ because we are including all values of x greater than some distance from the central point.

At the value $x = 3$ we must have

$$\begin{aligned} |2x+5| &= a \\ |2\times 3+5| &= a \\ a &= 11 \end{aligned}$$

Then at $x = b$

$$\begin{aligned} -(2b+5) &= 11 \\ -2b-5 &= 11 \\ -2b &= 16 \\ b &= -8 \end{aligned}$$

52. Since 3 is a member of the solution set, resulting in the LHS being zero, the smallest possible absolute value, the inequality must be either $<$ or $\leq$. Since we have a filled circle at the starting point we can conclude that "*" is $\leq$.

    Point $x = 5$ is equidistant between 3 and a (i.e. $|x-3| = |x-a|$ at $x = 5$ so we may conclude that $a = 7$.

53. First solve $|2x+5| = |x+a|$

    $2x+5 = x+a$ or $-(2x+5) = x+a$

    $x = a-5$

    $$\begin{aligned} -2x-5 &= x+a \\ -3x &= a+5 \\ x &= -\frac{a+5}{3} \end{aligned}$$

    This gives us either

    - $a-5 = -2$ and $-\frac{a+5}{3} = -4$; or
    - $a-5 = -4$ and $-\frac{a+5}{3} = -2$

    Only the second of these works, and we have $a = 1$

    The open endpoints exclude $\leq$ and $\geq$ and all that remains is to test a value between -2 and -4 to decide between $<$ and $>$.

    $$\begin{aligned} |2(-3)+5| \quad &* \quad |(-3)+1| \\ 1 \quad &* \quad 2 \end{aligned}$$

    and we see that "*" is $<$.

54. (a) False. This equation only holds when x and y are either both positive or both negative. For example, consider $x = 1$, $y = -2$: $|x+y| = 1$ but $|x|+|y| = 3$.

    (b) False. This equation only holds when x and y are not both positive or both negative, and further when $|x| \geq |y|$. For example, if $x = 1$ and $y = 2$, $|x+y| = 3$ but $|x|-|y| = -1$.

    (c) False. For example, consider $x = 1$, $y = -2$: $|x+y| = 1$ but $|x|+|y| = 3$.

    (d) True for all real values of x and y.

Miscellaneous Exercise 1

1. $$\begin{aligned} \text{distance} &= \sqrt{(-3-2)^2+(7--5)^2} \\ &= \sqrt{25+144} \\ &= 13 \end{aligned}$$

2. (a) $$\begin{aligned} f(2) &= 5(2)-3 \\ &= 7 \end{aligned}$$

   (b) $$\begin{aligned} f(-5) &= 5(-5)-3 \\ &= -28 \end{aligned}$$

   (c) $$\begin{aligned} f(1.5) &= 5(1.5)-3 \\ &= 4.5 \end{aligned}$$

   (d) $f(p) = 5p-3$

   (e) $$\begin{aligned} f(q) &= -18 \\ 5q-3 &= -18 \\ 5q &= -15 \\ q &= -3 \end{aligned}$$

3. (a) $8 = 2^3$

   (b) $64 = 8^2 = (2^3)^2 = 2^6$

   (c) $2^3 \times 2^7 = 2^{3+7} = 2^{10}$

   (d) $2^5 \times 16 = 2^5 \times 2^4 = 2^9$

   (e) $2^{10} \div 2^7 = 2^{10-7} = 2^3$

   (f) $2^7 \div 8 = 2^7 \div 2^3 = 2^4$

   (g) $256 \times 64 = 2^8 \times 2^6 = 2^{14}$

   (h) $1 = 2^0$

4. (a) $$\begin{aligned} 5^6 \times 5^x &= 5^{10} \\ 5^{6+x} &= 5^{10} \\ 6+x &= 10 \\ x &= 4 \end{aligned}$$

   (b) $$\begin{aligned} 27 \times 3^x &= 3^7 \\ 3^3 \times 3^x &= 3^7 \\ 3^{3+x} &= 3^7 \\ 3+x &= 7 \\ x &= 4 \end{aligned}$$

(c) $$\begin{aligned} 1\,000\,000 &= 10^x \\ 10^6 &= 10^x \\ x &= 6 \end{aligned}$$

(d) $$\begin{aligned} 12^9 \div 12^x &= 144 \\ 12^{9-x} &= 12^2 \\ 9 - x &= 2 \\ x &= 7 \end{aligned}$$

(e) $$\begin{aligned} 2^3 \times 8 \times 2^x &= 2^{10} \\ 2^3 \times 2^3 \times 2^x &= 2^{10} \\ 2^{3+3+x} &= 2^{10} \\ 6 + x &= 10 \\ x &= 4 \end{aligned}$$

(f) $$\begin{aligned} 0.1 &= 10^x \\ 10^{-1} &= 10^x \\ x &= -1 \end{aligned}$$

5. (a) $-5 < x < 5$
   (b) True for all x (An absolute value is always greater than any negative number.)
   (c) $-6 \leq 2x \leq 6$ so $-3 \leq x \leq 3$
   (d) No value of x satisfies this since an absolute value cannot be less than zero.
   (e) True for points on the number line having a distance from 3 less than their distance from 9, i.e. points nearer 3 than 9. The midpoint of 3 and 9 is 6 so the values of x that satisfy the inequality are $x < 6$.
   (f) True for points on the number line nearer -1 than 5. The midpoint is 2, so $x < 2$.

6. Refer to Sadler's solutions for the sketches. These comments briefly describe the operations that have been enacted to produce these sketches.
   (a) Vertical reflection in the x-axis
   (b) Horizontal reflection in the y-axis
   (c) That part of the curve lying below the x-axis is vertically reflected in the x-axis.
   (d) That part of the curve lying to the left of the y-axis is replaced with a mirror image of the part lying to the right of the axis.

7. Each function is of the form $y = |a(x-b)|$ where a represents the gradient of the positive slope and b where it meets the x-axis. (It may be necessary to expand brackets if comparing these answers with Sadler's.)
   (a) Gradient 1, x-intercept -3: $y = |x + 3|$
   (b) Gradient 1, x-intercept 3: $y = |x - 3|$
   (c) Gradient 3, x-intercept 2: $y = |3(x - 2)|$
   (d) Gradient 2, x-intercept -2: $y = |2(x + 2)|$

8. (a) $\mathrm{f}(3) = 3(3) - 2 = 7$
   (b) $\mathrm{f}(-3) = 3(-3) - 2 = -11$
   (c) $\mathrm{g}(3) = \mathrm{f}(|3|) = \mathrm{f}(3) = 7$
   (d) $\mathrm{g}(-3) = \mathrm{f}(|-3|) = \mathrm{f}(3) = 7$
   (e) $\mathrm{f}(5) = 3(5) - 2 = 13$
   (f) $\mathrm{g}(-5) = \mathrm{f}(|-5|) = \mathrm{f}(5) = 13$
   (g) The graph of $\mathrm{f}(x)$ is a line with gradient 3 and y-intercept -2. The graph of $\mathrm{g}(x)$ is identical to that of $\mathrm{f}(x)$ for $x \geq 0$. For $x < 0$ the graph is a reflection in the y-axis of the graph for positive x.

9. (a) The line lies above the curve for x between b and e (but not including the extremes): $b < x < e$.
   (b) As for the previous question, but including the extremes: $b \leq x \leq e$.
   (c) The line is below the x-axis for $x < a$.
   (d) The line is above or on the x-axis for $x \geq a$.
   (e) The quadratic is above or on the x-axis for $x \leq c$ or $x \geq d$.
   (f) The quadratic is above the x-axis for $x \leq b$ or $x \geq e$.

10. Because a is positive the sign of ax is the same as the sign of x and hence $|ax| = a|x|$. Similarly $|bx| = b|x|$.

$$\begin{aligned} |bx| &> |ax| \\ b|x| &> a|x| \end{aligned}$$

Because $|x|$ is positive we can divide both sides by $|x|$ without being concerned about the inequality changing direction. This is, of course, only valid for $x \neq 0$

$$.b > a$$

which is true for all x so we can conclude that the original inequality is true for all $x \neq 0$.

Chapter 2

Exercise 2A

1. (a) 035° (read directly from the diagram)

   (b) $35 + 45 = 080°$

   (c) $35 + 45 + 30 = 110°$

   (d) $180 - 35 = 145°$

   (e) $180 + 20 = 200°$

   (f) $360 - 60 = 300°$

   (g) Back bearings:
   $35 + 180 = 215°$

   (h) $80 + 180 = 260°$

   (i) $110 + 180 = 290°$

   (j) $145 + 180 = 325°$

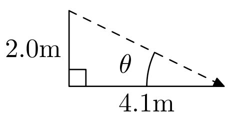
   (k) $200 - 180 = 020°$

   (l) $300 - 180 = 120°$

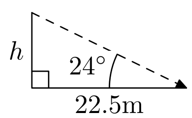
2. No working required. Refer to the answers in Sadler.

3. $$\begin{aligned}\tan 28° &= \frac{h}{22.4}\\ h &= 22.4\tan 28°\\ &= 11.9\text{m}\end{aligned}$$

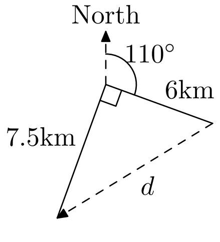
4. $$\begin{aligned}\tan\theta &= \frac{2}{4.1}\\ \theta &= \tan^{-1}\frac{2}{4.1}\\ &= 26°\end{aligned}$$



5. $$\begin{aligned}\tan 24° &= \frac{h}{22.5}\\ h &= 22.5\tan 24°\\ &= 10.0\text{m}\end{aligned}$$

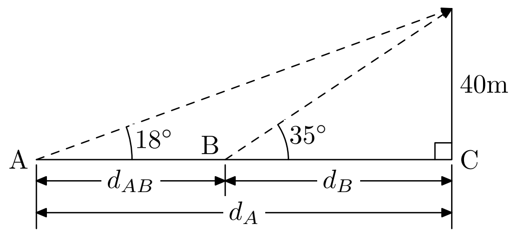


6. After one and a half hours, the first ship has travelled 6km and the second 7.5km.



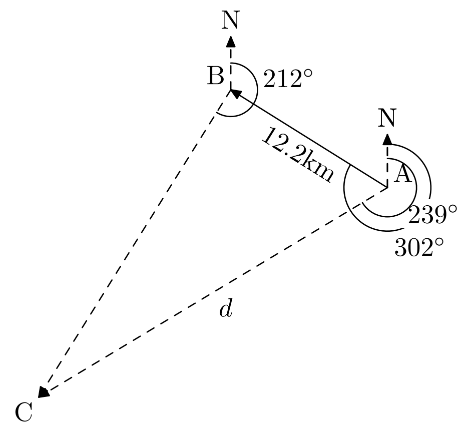
$$\begin{aligned}d^2 &= 6^2 + 7.5^2\\ d &= 9.6\text{km}\\ &\approx 10\text{km}\end{aligned}$$

7.



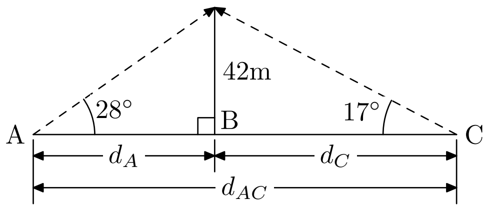
$$\begin{aligned}\tan 18° &= \frac{40}{d_A}\\ d_A &= \frac{40}{\tan 18°}\\ \tan 35° &= \frac{40}{d_B}\\ d_B &= \frac{40}{\tan 35°}\\ d_{AB} &= \frac{40}{\tan 18°} - \frac{40}{\tan 35°}\\ &= 66\text{m}\end{aligned}$$

8.



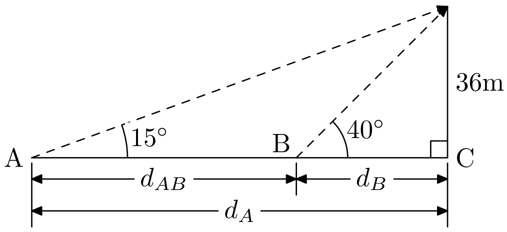
$$\begin{aligned}\angle\text{CAB} &= 302 - 239\\ &= 63°\\ \angle\text{CBA} &= 212 - (302 - 180)\\ &= 90°\\ \cos 63° &= \frac{12.2}{d}\\ d &= \frac{12.2}{\cos 63°}\\ &= 26.9\text{km}\end{aligned}$$

9.



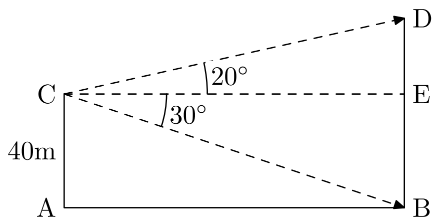
$$\begin{aligned}
\tan 28^\circ &= \frac{42}{d_A} \\
d_A &= \frac{42}{\tan 28^\circ} \\
\tan 17^\circ &= \frac{42}{d_C} \\
d_C &= \frac{42}{\tan 17^\circ} \\
d_{AC} &= \frac{42}{\tan 28^\circ} + \frac{42}{\tan 17^\circ} \\
&= 216\text{m}
\end{aligned}$$

10.



$$\begin{aligned}
\tan 15^\circ &= \frac{36}{d_A} \\
d_A &= \frac{36}{\tan 15^\circ} \\
\tan 40^\circ &= \frac{36}{d_B} \\
d_B &= \frac{36}{\tan 40^\circ} \\
d_{AB} &= \frac{36}{\tan 15^\circ} - \frac{36}{\tan 40^\circ} \\
&= 91\text{m}
\end{aligned}$$

11.



Distance between towers:

$$\begin{aligned}
\tan 30^\circ &= \frac{40}{\text{AB}} \\
\text{AB} &= \frac{40}{\tan 30^\circ} \\
&= 40\sqrt{3}
\end{aligned}$$

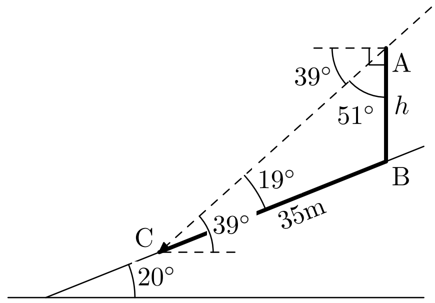
Additional height of second tower:

$$\begin{aligned}
\tan 20^\circ &= \frac{\text{DE}}{40\sqrt{3}} \\
\text{DE} &= 40\sqrt{3}\tan 20 \\
&= 25.22\text{m}
\end{aligned}$$

Total height of second tower:

$$\begin{aligned}
\text{DB} &= 25.22 + 40 \\
&\approx 65.2\text{m}
\end{aligned}$$

12.



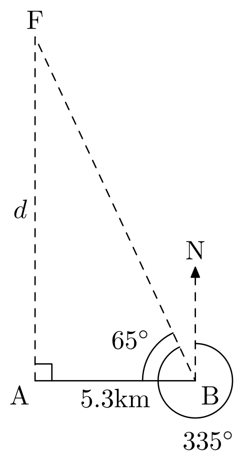
First determine the angles in the triangle made by the tree, the hillslope and the sun's ray.

$$\begin{aligned}
\angle\text{ACB} &= 39 - 20 \\
&= 19^\circ \\
\angle\text{CAB} &= 90 - 39 \\
&= 51^\circ
\end{aligned}$$

Now use the sine rule:

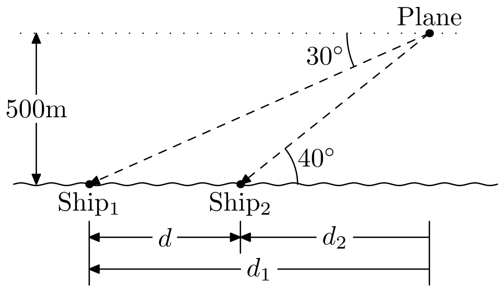
$$\begin{aligned}
\frac{h}{\sin 19^\circ} &= \frac{35}{\sin 51^\circ} \\
h &= \frac{35\sin 19^\circ}{\sin 51^\circ} \\
&= 14.7\text{m}
\end{aligned}$$

13.



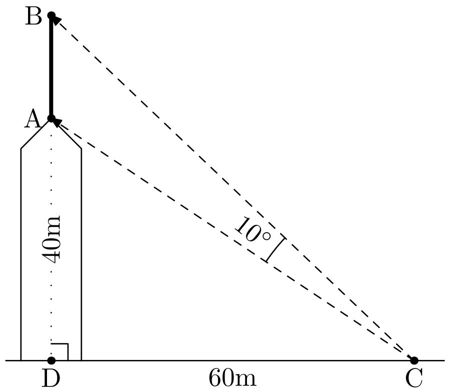
$$\begin{aligned}
\angle\text{ABF} &= 335 - 270 \\
&= 65^\circ \\
\tan 65^\circ &= \frac{d}{5.3} \\
d &= 5.3\tan 65^\circ \\
&= 11.4\text{km}
\end{aligned}$$

14.



$$\begin{aligned}
\tan 30^\circ &= \frac{500}{d_1} \\
d_1 &= \frac{500}{\tan 30^\circ} \\
&= 866\text{m} \\
\tan 40^\circ &= \frac{500}{d_2} \\
d_2 &= \frac{500}{\tan 40^\circ} \\
&= 596\text{m} \\
d &= d_1 - d_2 \\
&= 270\text{m}
\end{aligned}$$

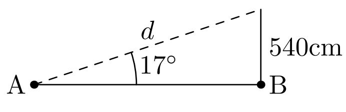
15.



$$\begin{aligned}
\text{AC} &= \sqrt{60^2 + 40^2} \\
&= 20\sqrt{13} \\
\tan \angle\text{DAC} &= \frac{60}{40} \\
\angle\text{DAC} &= 56.3^\circ \\
\angle\text{BAC} &= 180 - 56.3 \\
&= 123.7^\circ \\
\angle\text{ABC} &= 180 - 123.7 - 10 \\
&= 46.3^\circ \\
\frac{\text{AB}}{\sin 10^\circ} &= \frac{20\sqrt{13}}{\sin 46.3^\circ} \\
\text{AB} &= \frac{20\sqrt{13}\sin 10^\circ}{\sin 46.3^\circ} \\
&= 17.3\text{m}
\end{aligned}$$

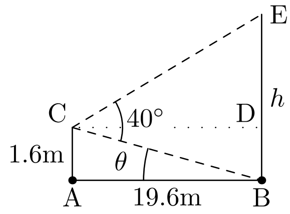
16.

$$\begin{aligned}
\sin 17^\circ &= \frac{540}{d} \\
d &= \frac{540}{\sin 17^\circ} \\
&= 1847\text{cm}
\end{aligned}$$



Rounded up to the next metre this is 19m.

17.

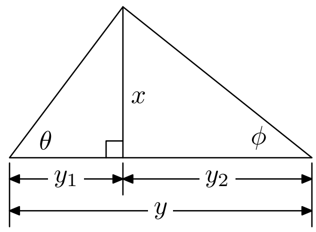


$$\begin{aligned}
\tan\theta &= \frac{1.6}{19.6} \\
\theta &= 4.7^\circ \\
\angle\text{DCE} &= 40 - 4.7 \\
&= 35.3^\circ \\
\tan 35.3^\circ &= \frac{\text{DE}}{19.6} \\
\text{DE} &= 19.6\tan 35.3^\circ \\
&= 13.9\text{m} \\
h &= 13.9 + 1.6 \\
&= 15.5\text{m}
\end{aligned}$$

18. Let the height of the flagpole be h and the distance from the base be d. Let θ be the angle of elevation of the point $\frac{3}{4}$ of the way up the flagpole.

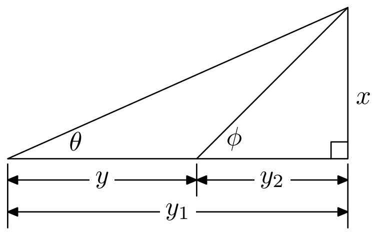
$$\begin{aligned}
\tan 40^\circ &= \frac{h}{d} \\
\tan\theta &= \frac{0.75h}{d} \\
&= 0.75\frac{h}{d} \\
&= 0.75\tan 40^\circ \\
\theta &= \tan^{-1}(0.75\tan 40^\circ) \\
&= 32^\circ
\end{aligned}$$

19.



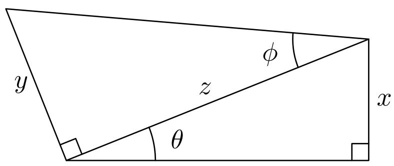
$$\begin{aligned}
\tan\theta &= \frac{x}{y_1} \\
y_1 &= \frac{x}{\tan\theta} \\
\tan\phi &= \frac{x}{y_2} \\
y_2 &= \frac{x}{\tan\phi} \\
y &= y_1 + y_2 \\
&= \frac{x}{\tan\theta} + \frac{x}{\tan\phi} \\
&= x\left(\frac{1}{\tan\theta} + \frac{1}{\tan\phi}\right) \\
&= x\left(\frac{\tan\phi}{\tan\theta\tan\phi} + \frac{\tan\theta}{\tan\theta\tan\phi}\right) \\
&= x\left(\frac{\tan\theta + \tan\phi}{\tan\theta\tan\phi}\right)
\end{aligned}$$

□

20. 

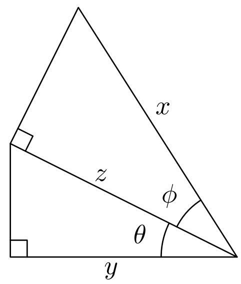
$$
\begin{aligned}
\tan\theta &= \frac{x}{y_1}\\
y_1 &= \frac{x}{\tan\theta}\\
\tan\phi &= \frac{x}{y_2}\\
y_2 &= \frac{x}{\tan\phi}\\
y &= y_1 - y_2\\
&= \frac{x}{\tan\theta} - \frac{x}{\tan\phi}\\
&= x\left(\frac{1}{\tan\theta} - \frac{1}{\tan\phi}\right)\\
&= x\left(\frac{\tan\phi}{\tan\theta\tan\phi} - \frac{\tan\theta}{\tan\theta\tan\phi}\right)\\
&= x\left(\frac{\tan\phi - \tan\theta}{\tan\theta\tan\phi}\right)
\end{aligned}
$$

□

21. 

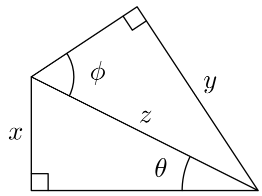
$$
\begin{aligned}
\sin\theta &= \frac{x}{z}\\
z &= \frac{x}{\sin\theta}\\
\tan\phi &= \frac{y}{z}\\
y &= z\tan\phi\\
&= \left(\frac{x}{\sin\theta}\right)\tan\phi\\
&= \frac{x\tan\phi}{\sin\theta}
\end{aligned}
$$

□

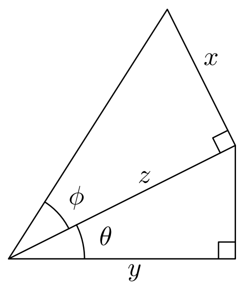
22. 

$$
\begin{aligned}
\cos\phi &= \frac{z}{x}\\
z &= \frac{x}{\cos\phi}\\
\cos\theta &= \frac{y}{z}\\
y &= z\cos\theta\\
&= (x\cos\phi)\cos\theta\\
&= x\cos\phi\cos\theta
\end{aligned}
$$

□

23. (a) 

$$
\begin{aligned}
\sin\theta &= \frac{x}{z}\\
z &= \frac{x}{\sin\theta}\\
\sin\phi &= \frac{y}{z}\\
y &= z\sin\phi\\
&= \left(\frac{x}{\sin\theta}\right)\sin\phi\\
&= \frac{x\sin\phi}{\sin\theta}
\end{aligned}
$$

(b) 

$$
\begin{aligned}
\tan\phi &= \frac{x}{z}\\
z &= \frac{x}{\tan\phi}\\
\cos\theta &= \frac{y}{z}\\
y &= z\cos\theta\\
&= \left(\frac{x}{\tan\phi}\right)\cos\theta\\
&= \frac{x\cos\theta}{\tan\phi}
\end{aligned}
$$

Exercise 2B

1. (a) $\text{AM} = \frac{1}{2}\text{AC}$
$= \frac{1}{2}\sqrt{\text{AB}^2 + \text{BC}^2}$
$= \frac{1}{2}\sqrt{5.2^2 + 5.2^2}$
$= 3.68\text{cm}$

(b) $\tan\angle\text{EAM} = \frac{\text{EM}}{\text{AM}}$
$\angle\text{EAM} = \tan^{-1}\frac{\text{EM}}{\text{AM}}$
$= \tan^{-1}\frac{6.3}{3.68}$
$= 59.7°$

(c) $\angle\text{DEM} = \angle\text{AEM}$
$= 180 - 90 - \angle\text{EAM}$
$= 90 - 59.7$
$= 30.3°$

(d) Let F be the midpoint of AB. The angle between the face EAB and the base ABCD is $\angle$EFM.
$\text{FM} = \frac{1}{2}\text{AB}$
$= 2.6\text{cm}$
$\tan\angle\text{EFM} = \frac{\text{EM}}{\text{FM}}$
$\angle\text{EFM} = \tan^{-1}\frac{\text{EM}}{\text{FM}}$
$= \tan^{-1}\frac{6.3}{2.6}$
$= 67.6°$

2. (a) $\tan\angle\text{GDC} = \frac{\text{GC}}{\text{DC}}$
$\angle\text{GDC} = \tan^{-1}\frac{\text{GC}}{\text{DC}}$
$= \tan^{-1}\frac{35}{62}$
$= 29.4°$

(b) $\tan\angle\text{GBC} = \frac{\text{GC}}{\text{BC}}$
$\angle\text{GBC} = \tan^{-1}\frac{\text{GC}}{\text{BC}}$
$= \tan^{-1}\frac{35}{38}$
$= 42.6°$

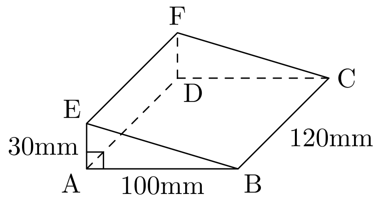
(c) $\tan\angle\text{GAC} = \frac{\text{GC}}{\text{AC}}$
$\angle\text{GAC} = \tan^{-1}\frac{\text{GC}}{\text{AC}}$
$= \tan^{-1}\frac{35}{\sqrt{62^2 + 38^2}}$
$= 25.7°$

(d) $\text{AG} = \sqrt{\text{AC}^2 + \text{GC}^2}$
$= \sqrt{62^2 + 38^2 + 35^2}$
$= 80.7\text{mm}$

(e) The angle between the plane FADG and the base ABCD is equal to $\angle\text{GDC} = 29.4°$.

(f) The angle between skew lines DB and HE is equal to $\angle$ADB.
$\tan\angle\text{ADB} = \frac{\text{AB}}{\text{AD}}$
$\angle\text{ADB} = \tan^{-1}\frac{\text{AB}}{\text{AD}}$
$= \tan^{-1}\frac{62}{38}$
$= 58.5°$

3. The key to this problem and others like it is a clear diagram that captures the information given.



(a) $\text{BF}^2 = \text{BE}^2 + \text{EF}^2$
$= (\text{AB}^2 + \text{AE}^2) + \text{EF}^2$
$= 100^2 + 30^2 + 120^2$
$\text{BF} = \sqrt{25300}$
$= 159\text{mm}$

(b) $\sin\angle\text{FBD} = \frac{\text{DF}}{\text{BF}}$
$= \frac{30}{159}$
$\angle\text{FBD} = \sin^{-1}\frac{30}{159}$
$= 10.9°$

4. (a) $\tan 50° = \frac{186}{\text{AC}}$
$\text{AC} = \frac{186}{\tan 50°}$
$= 156\text{cm}$

(b) $\tan 24° = \frac{186}{\text{AB}}$
$\text{AB} = \frac{186}{\tan 24°}$
$= 418\text{cm}$
$\text{BC} = \sqrt{\text{AB}^2 + \text{AC}^2}$
$= \sqrt{156^2 + 418^2}$
$= 446\text{cm}$

(c) $\tan\angle ACB = \frac{AB}{AC}$

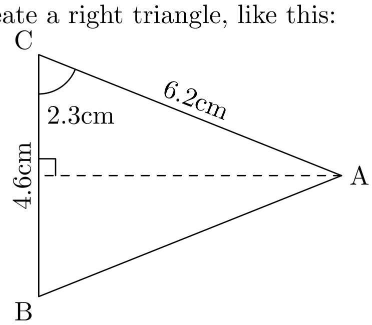
$$\angle ACB = \tan^{-1}\frac{418}{156} = 69.5^\circ$$

5. (a) $\triangle DCA \cong \triangle DBA$ (SAS) so $\angle DCA \cong \angle DBA$ and $\angle DBA = 50^\circ$

(b) $\angle DBA = \frac{DA}{AB}$

$$AB = \frac{DA}{\tan\angle DBA} = \frac{7.4}{\tan 50^\circ} = 6.2\text{cm}$$

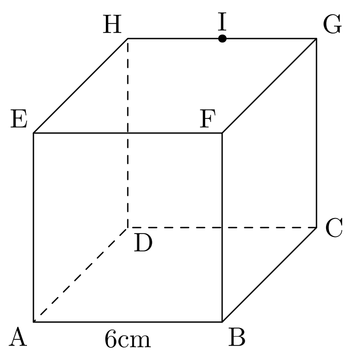
(c) There are a couple of ways this could be done. Since we now know all three sides of triangle ABC we could use the cosine rule to find ∠ACB. Alternatively, since we have an isosceles triangle, we can divide it in half to create a right triangle, like this:



$$\cos\angle ACB = \frac{2.3}{6.2}$$

$$\angle ACB = \cos^{-1}\frac{2.3}{6.2} = 68^\circ$$

6.



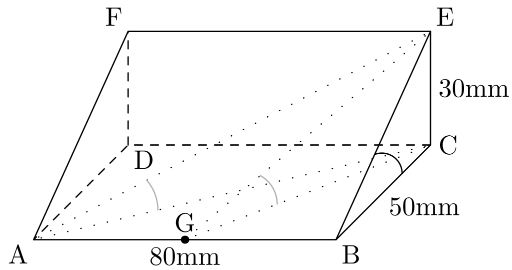
(a) $BI = \sqrt{BF^2 + FI^2} = \sqrt{BF^2 + FG^2 + GI^2} = \sqrt{6^2 + 6^2 + 3^2} = 9$

(b) $\cos\angle IBF = \frac{BF}{FI} = \frac{6}{9} = \frac{2}{3}$

$$\angle IBF = \cos^{-1}\frac{2}{3} = 48^\circ$$

(c) The angle between △IAB and the base ABCD is the same as the angle between rectangle ABGH and the base ABCD (since the triangle and the rectangle are coplanar). This is the same as ∠GBC: 45°.

7.



(a) $\tan\angle EBC = \frac{CE}{BC}$

$$\angle EBC = \tan^{-1}\frac{30}{50} = 31^\circ$$

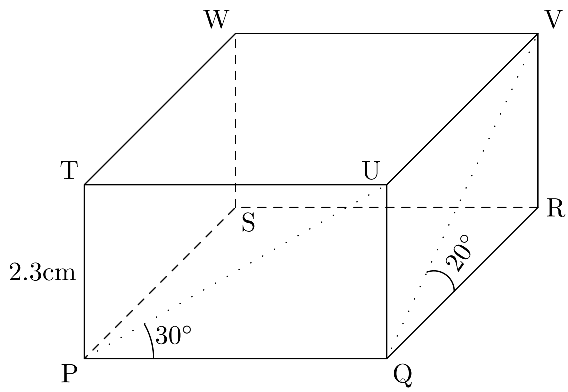
(b) $\tan\angle EGC = \frac{CE}{GC}$

$$\angle EBC = \tan^{-1}\frac{30}{\sqrt{40^2 + 50^2}} = 25^\circ$$

(c) $\tan\angle EAC = \frac{CE}{AC}$

$$\angle EBC = \tan^{-1}\frac{30}{\sqrt{80^2 + 50^2}} = 18^\circ$$

8.



(a) $\tan 30^\circ = \frac{QU}{PQ}$ $\quad$ $\tan 20^\circ = \frac{VR}{QR}$

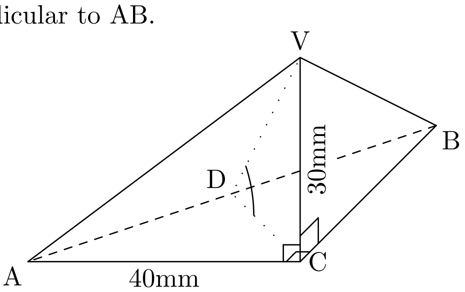
$PQ = \frac{2.3}{\tan 30^\circ} = 3.98\text{cm}$ $\quad$ $QR = \frac{2.3}{\tan 20^\circ} = 6.32\text{cm}$

$$\text{Volume} = 2.3 \times 3.98 \times 6.32 = 57.9\text{cm}^3$$

(b) $$\begin{aligned}\text{PV} &= \sqrt{\text{PQ}^2 + \text{QR}^2 + \text{RV}^2}\\ &= \sqrt{3.98^2 + 6.32^2 + 2.3^2}\\ &= 7.8\text{cm}\end{aligned}$$

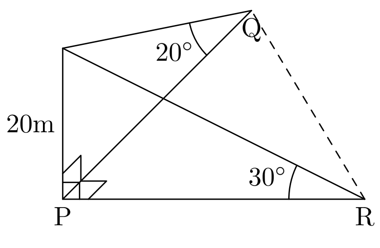
(c) $$\begin{aligned}\tan \angle\text{USW} &= \frac{\text{UW}}{\text{SW}}\\ \angle\text{USW} &= \tan^{-1} \frac{\sqrt{3.98^2 + 6.32^2}}{2.3}\\ &= 73^\circ\end{aligned}$$

9. The angle between plane VAB and plane ABC is equal to the angle between lines that are both perpendicular to AB. Consider point D the mid-point of AB such that VD and VC are both perpendicular to AB.



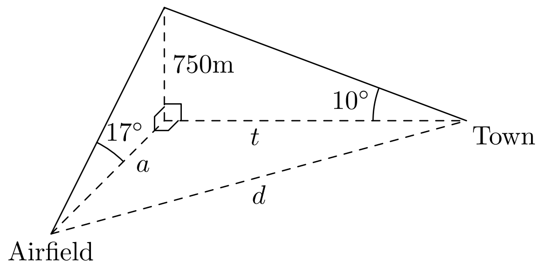
$$\begin{aligned}\sin 45^\circ &= \frac{\text{DC}}{\text{AC}}\\ \text{DC} &= 40 \sin 45^\circ\\ &= 28.28\text{mm}\\ \tan \angle\text{VDC} &= \frac{\text{VC}}{\text{DC}}\\ \angle\text{VDC} &= \tan^{-1} \frac{30}{28.28}\\ &= 47^\circ\end{aligned}$$

10.



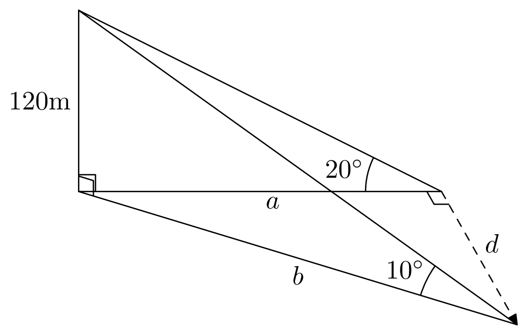
$$\begin{aligned}\text{PQ} &= \frac{20}{\tan 20^\circ}\\ &= 54.9\text{m}\\ \text{PR} &= \frac{20}{\tan 30^\circ}\\ &= 34.6\text{m}\\ \text{QR} &= \sqrt{\text{PQ}^2 + \text{PR}^2}\\ &= 65\text{m}\end{aligned}$$

11.

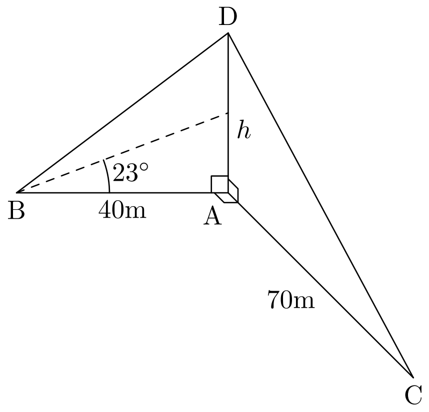


$$\begin{aligned}a &= \frac{750}{\tan 17^\circ}\\ &= 2453\text{m}\\ t &= \frac{750}{\tan 30^\circ}\\ &= 4253\text{m}\\ d &= \sqrt{a^2 + t^2}\\ &= 4910\text{m}\\ &\approx 5\text{km}\end{aligned}$$

12.



$$\begin{aligned}a &= \frac{120}{\tan 20^\circ}\\ &= 330\text{m}\\ b &= \frac{120}{\tan 30^\circ}\\ &= 681\text{m}\\ d &= \sqrt{b^2 - a^2}\\ &= 595\text{m}\\ \text{Speed} &= \frac{595}{10}\\ &= 59.5\text{m/min}\\ &= 59.5 \times 60\text{m/hr}\\ &= 3572\text{m/hr}\\ &= 3.6\text{km/hr}\end{aligned}$$

13. 

(a)
$$\frac{h}{2} = 40\tan 23^\circ$$
$$h = 33.96\text{m}$$
$$\angle\text{ABD} = \tan^{-1}\frac{h}{40}$$
$$= 40^\circ$$

(b)
$$\angle\text{ACD} = \tan^{-1}\frac{h}{70}$$
$$= 26^\circ$$

14.
$$\text{BD} = h$$
$$\text{AB} = \frac{h}{\sin 28^\circ}$$
$$\cos 35^\circ = \frac{\text{AB}}{\text{AC}}$$
$$\text{AC} = \frac{\text{AB}}{\cos 35^\circ}$$
$$= \frac{\frac{h}{\sin 28^\circ}}{\cos 35^\circ}$$
$$= \frac{h}{\sin 28^\circ \cos 35^\circ}$$
$$\sin\theta = \frac{h}{\text{AC}}$$
$$= \frac{h}{1} \times \frac{\sin 28^\circ \cos 35^\circ}{h}$$
$$= \sin 28^\circ \cos 35^\circ$$
$$\theta = \sin^{-1}(\sin 28^\circ \cos 35^\circ)$$
$$= 23^\circ$$

Exercise 2C

1. (a)
$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$10.2^2 = x^2 + 6.9^2 - 2 \times x \times 6.9 \times \cos 50^\circ$$
$$x = -4.29 \text{ or } x = 13.16$$
Reject the negative solution and round to 1d.p.: $x = 13.2$cm.

$$B = 180 - A - C$$
$$= 180 - 50 - 31.2$$
$$= 98.8^\circ \quad \frac{x}{\sin 98.8} = \frac{10.2}{\sin 50}$$
$$x = \frac{10.2\sin 98.8}{\sin 50}$$
$$= 13.2\text{cm}$$

(b)
$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\sin A = \frac{a\sin C}{c}$$
$$A = \sin^{-1}\frac{a\sin C}{c}$$
$$= \sin^{-1}\frac{6.9\sin 50^\circ}{10.2}$$
$$= 31.2^\circ$$
$$\text{or } A = 180 - 31.2$$
$$= 148.8$$
Reject the obtuse solution since it results in an internal angle sum greater than 180°.

2.
$$\frac{\sin x}{11.2} = \frac{\sin 50^\circ}{12.1}$$
$$x = \sin^{-1}\frac{11.2\sin 50}{12.1}$$
$$= 45^\circ$$
No need to consider the obtuse solution since the opposite side is not the longest in the triangle (x must be less than 50°).

3.
$$x^2 = 6.8^2 + 14.3^2 - 2 \times 6.8 \times 14.3 \times \cos 20^\circ$$
$$x = \sqrt{6.8^2 + 14.3^2 - 2 \times 6.8 \times 14.3 \times \cos 20^\circ}$$
$$= 8.2\text{cm}$$

4. $19.7^2 = 9.8^2 + 14.3^2 - 2 \times 9.8 \times 14.3 \times \cos x$

$$\cos x = \frac{9.8^2 + 14.3^2 - 19.7^2}{2 \times 9.8 \times 14.3}$$
$$x = \cos^{-1} \frac{9.8^2 + 14.3^2 - 19.7^2}{2 \times 9.8 \times 14.3}$$
$$= 108°$$

5. $$\frac{x}{\sin(180 - 105 - 25)} = \frac{11.8}{\sin 105}$$
$$x = \frac{11.8 \sin 50}{\sin 105}$$
$$= 9.4\text{cm}$$

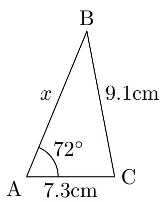
6. $$\frac{\sin x}{7.2} = \frac{\sin 40°}{4.8}$$
$$x = \sin^{-1} \frac{7.2 \sin 40°}{4.8}$$
$$= 75° \quad \text{or } x = 180 - 75 = 105°$$

7. $c^2 = a^2 + b^2 - 2ab \cos C$
$11.8^2 = x^2 + 8.7^2 - 2 \times x \times 8.7 \times \cos 80°$
$x = 9.6$
(rejecting the negative solution)

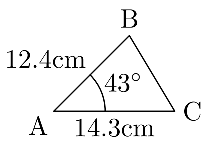
8. The smallest angle is opposite the shortest side, so

$$27^2 = 33^3 + 55^2 - 2 \times 33 \times 55 \times \cos \theta$$
$$\theta = \cos^{-1} \frac{33^2 + 55^2 - 27^2}{2 \times 33 \times 55}$$
$$= 21°$$

9. $a^2 = b^2 + c^2 - 2bc \cos A$
$9.1^2 = 7.3^2 + x^2 - 2 \times 7.3x \cos 72°$
$x = 8.1$
(rejecting the negative solution)
AB=8.1cm



10.

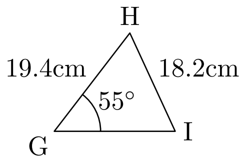


$a^2 = b^2 + c^2 - 2bc \cos A$
$a = \sqrt{12.4^2 + 14.3^2 - 2 \times 12.4 \times 14.3 \cos 43°}$
$a = 9.9\text{cm}$

$$\frac{\sin C}{12.4} = \frac{\sin 43°}{a}$$
$$C = \sin^{-1} \frac{12.4 \sin 43°}{9.9}$$
$$= 58°$$
(Cannot be obtuse because c is not the longest side.)

$B = 180 - 43 - 58$
$= 79°$

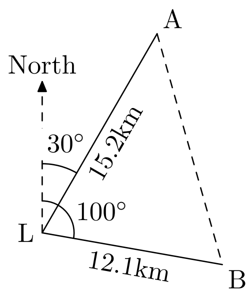
11.



$$\frac{\sin I}{19.4} = \frac{\sin 55°}{18.2}$$
$$I = \sin^{-1} \frac{19.4 \sin 55°}{18.2}$$
$= 61°$ or $119°$
$H = 180 - 55 - 61$ or $180 - 55 - 119$
$= 64°$ or $=6°$

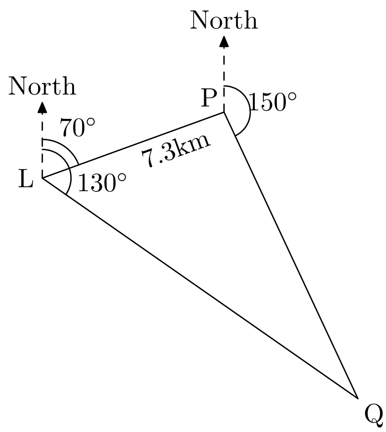
$$\frac{h}{\sin H} = \frac{g}{\sin G}$$
$$h = \frac{g \sin H}{\sin G}$$
$$= \frac{18.2 \sin 64°}{\sin 55°} \quad \text{or} \quad \frac{18.2 \sin 6°}{\sin 55°}$$
$= 20.0\text{cm}$ or $=2.3\text{cm}$

12.



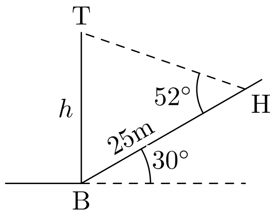
$\angle \text{ALB} = 100 - 30$
$= 70°$
$\text{AB} = \sqrt{15.2^2 + 12.1^2 - 2 \times 15.2 \times 12.1 \cos 70°}$
$= 15.9\text{km}$

13.

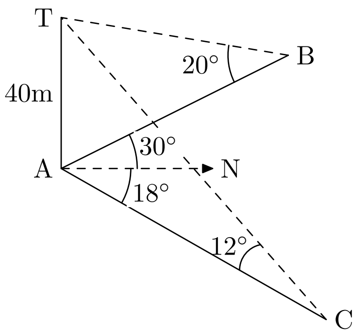


$\angle \text{PQL} = 150 - 130$
$= 20°$
$\angle \text{PLQ} = 130 - 70$
$= 60°$
$\angle \text{LPQ} = 180 - 20 - 60$
$= 100°$

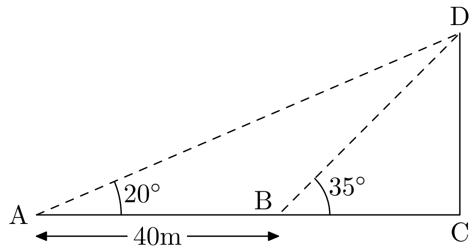
$$\begin{aligned}\frac{\text{LQ}}{\sin\angle\text{LPQ}} &= \frac{\text{LP}}{\sin\angle\text{PQL}}\\ \text{LQ} &= \frac{\text{LP}\sin\angle\text{LPQ}}{\sin\angle\text{PQL}}\\ &= \frac{7.3\sin 100^\circ}{\sin 20^\circ}\\ &= 21.0\text{km}\end{aligned}$$

14. 

$$\begin{aligned}\angle\text{HBT} &= 90-30\\ &= 60^\circ\\ \angle\text{BTH} &= 180-60-52\\ &= 68^\circ\\ \frac{h}{\sin 52^\circ} &= \frac{25}{\sin 68^\circ}\\ h &= \frac{25\sin 52^\circ}{\sin 68^\circ}\\ &= 21\text{m}\end{aligned}$$

15. 

$$\begin{aligned}\tan 20^\circ &= \frac{40}{\text{AB}}\\ \text{AB} &= \frac{40}{\tan 20^\circ}\\ &= 109.9\text{m}\\ \tan 12^\circ &= \frac{40}{\text{AC}}\\ \text{AC} &= \frac{40}{\tan 12^\circ}\\ &= 188.2\text{m}\\ \angle\text{BAC} &= 30+18\\ &= 48^\circ\\ \text{BC}^2 &= \text{AB}^2+\text{AC}^2-2\text{AB}\times\text{AC}\cos\angle\text{BAC}\\ \text{BC} &= \sqrt{109.9^2+188.2^2-2\times 109.9\times 188.2\cos 48^\circ}\\ &= 141\text{m}\end{aligned}$$

16. 

$$\begin{aligned}\angle\text{ADB} &= 35-20\\ &= 15^\circ\\ \frac{\text{BD}}{\sin 20^\circ} &= \frac{40}{\sin 15^\circ}\\ \text{BD} &= \frac{40\sin 20^\circ}{\sin 15^\circ}\\ &= 52.9\text{m}\\ \sin\angle\text{DBC} &= \frac{\text{DC}}{\text{BD}}\\ \text{DC} &= \text{BD}\sin\angle\text{DBC}\\ &= 52.9\sin 35\\ &= 30\text{m}\end{aligned}$$

17. There are a couple of ways you could approach this problem. You could use the cosine rule to determine an angle, then use the formula Area= $\frac{1}{2}ab\sin C$. Alternatively you could use Heron's formula:

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = \frac{a+b+c}{2}$ and determine the area without resort to trigonometry at all. I'll use trigonometry for the first block, and Heron's formula for the second.
First block—I'll start by finding the largest angle:

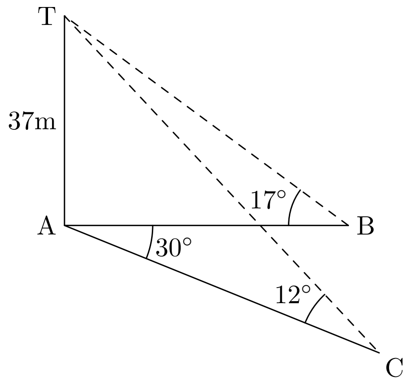
$$\begin{aligned}\theta &= \cos^{-1}\frac{25^2+48^2-53^2}{2\times 25\times 48}\\ &= 87.1^\circ\\ \text{Area} &= \frac{1}{2}ab\sin\theta\\ &= \frac{1}{2}\times 25\times 48\sin 87.1\\ &= 599.2\text{m}^2\end{aligned}$$

Second block:

$$\begin{aligned}s &= \frac{33+38+45}{2}\\ &= 58\\ \text{Area} &= \sqrt{58(58-33)(58-38)(58-45)}\\ &= 614.0\text{m}^2\end{aligned}$$

The second block is larger by 15m^2.

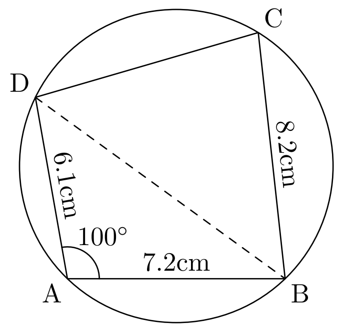
18.



$$\tan 17° = \frac{37}{\text{AB}}$$
$$\text{AB} = \frac{37}{\tan 17°} = 121.0\text{m}$$

$$\tan 12° = \frac{37}{\text{AC}}$$
$$\text{AC} = \frac{37}{\tan 12°} = 174.0\text{m}$$
$$\text{BC}^2 = \text{AB}^2 + \text{AC}^2 - 2 \times \text{AB} \times \text{AC} \cos 30°$$
$$\text{BC} = \sqrt{121.0^2 + 174.0^2 - 2 \times 121.0 \times 174.0 \cos 30°} = 92.0\text{m}$$

19.



(a) $\angle\text{BCD} = 180 - 100 = 80°$

(b) $\text{BD}^2 = 6.1^2 + 7.2^2 - 2 \times 6.1 \times 7.2 \cos 100° = 104.3$

$\text{BD} = 10.2\text{cm}$

$$\frac{\sin \angle\text{ADB}}{7.2} = \frac{\sin 100°}{10.2}$$
$$\angle\text{ADB} = \sin^{-1} \frac{7.2 \sin 100°}{10.2} = 44.0°$$
$$\frac{\sin \angle\text{CDB}}{8.2} = \frac{\sin 80°}{10.2}$$
$$\angle\text{CDB} = \sin^{-1} \frac{8.2 \sin 100°}{10.2} = 52.3°$$
$$\angle\text{ADC} = 44.0 + 52.3 = 96°$$

(c) $\text{BD}^2 = \text{BC}^2 + \text{CD}^2 - 2 \times \text{BC} \times \text{CD} \cos 80°$

$104.3 = 8.2^2 + \text{CD}^2 - 2 \times 8.2 \times \text{CD} \cos 80°$

$\text{CD} = 7.7\text{cm}$

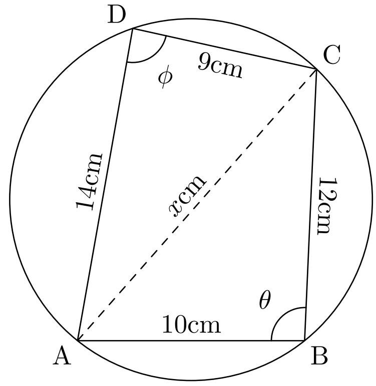
$P = 7.2 + 8.2 + 7.7 + 6.1 = 29.2\text{cm}$

(d) $A_{\triangle ABD} = \frac{1}{2} \times 6.1 \times 7.2 \times \sin 100° = 21.6\text{cm}^2$

$A_{\triangle CBD} = \frac{1}{2} \times 8.2 \times 7.7 \times \sin 80° = 31.1\text{cm}^2$

$A_{ABCD} = 21.6 + 31.1 = 52.7\text{cm}^2$

20.



(a) $x^2 = 10^2 + 12^2 - 2 \times 10 \times 12 \cos\theta = 100 + 144 - 240\cos\theta = 244 - 240\cos\theta$

(b) $x^2 = 14^2 + 9^2 - 2 \times 14 \times 9 \cos\phi = 196 + 81 - 252\cos\phi = 277 - 252\cos\phi$

(c)
$$\phi = 180° - \theta$$
$$\cos\phi = \cos(180° - \theta) = -\cos\theta$$
$$244 - 240\cos\theta = 277 - 252\cos\phi = 277 + 252\cos\theta$$
$$-240\cos\theta = 33 + 252\cos\theta$$
$$-492\cos\theta = 33$$
$$\cos\theta = -\frac{33}{492}$$
$$\theta = 94°$$

Exercise 2D

Questions 1–15 are single step problems. No worked solutions necessary.

Note: My exact values are given with rational denominators. I write $\frac{\sqrt{2}}{2}$ rather than $\frac{1}{\sqrt{2}}$. Your answers may appear different without being wrong.

16. 120° makes an angle of 60° with the x-axis and is in quadrant II (where sine is positive) so $\sin 120° = \sin 60° = \frac{\sqrt{3}}{2}$.
17. 135° makes an angle of 45° with the x-axis and is in quadrant II (where cosine is negative) so $\cos 135° = -\cos 45° = -\frac{\sqrt{2}}{2}$.
18. 150° makes an angle of 30° with the x-axis and is in quadrant II (where cosine is negative) so $\cos 150° = -\cos 30° = -\frac{\sqrt{3}}{2}$.
19. 120° makes an angle of 60° with the x-axis and is in quadrant II (where cosine is negative) so $\cos 120° = -\cos 60° = -\frac{1}{2}$.
20. 180° makes an angle of 0° with the x-axis and is on the negative x-axis (where cosine is negative) so $\cos 180° = -\cos 0° = -1$.
21. 135° makes an angle of 45° with the x-axis and is in quadrant II (where tangent is negative) so $\tan 135° = -\tan 45° = -1$.
22. 120° makes an angle of 60° with the x-axis and is in quadrant II (where tangent is negative) so $\tan 120° = -\tan 60° = -\sqrt{3}$.
23. 150° makes an angle of 30° with the x-axis and is in quadrant II (where tangent is negative) so $\tan 150° = -\tan 30° = \frac{\sqrt{3}}{3}$.
24. 180° lies on the negative x-axis (where tangent is zero) so $\tan 180° = 0$.
25. 180° lies on the negative x-axis (where sine is zero) so $\sin 180° = 0$.
26. 150° makes an angle of 30° with the x-axis and is in quadrant II (where sine is positive) so $\sin 150° = \sin 30° = \frac{1}{2}$.
27. 135° makes an angle of 45° with the x-axis and is in quadrant II (where sine is positive) so $\sin 135° = \sin 45° = \frac{\sqrt{2}}{2}$.
28. $\sqrt{20} = \sqrt{4 \times 5} = \sqrt{4}\sqrt{5} = 2\sqrt{5}$
29. $\sqrt{45} = \sqrt{9 \times 5} = \sqrt{9}\sqrt{5} = 3\sqrt{5}$
30. $\sqrt{32} = \sqrt{16 \times 2} = \sqrt{16}\sqrt{2} = 4\sqrt{2}$
31. $\sqrt{72} = \sqrt{36 \times 2} = \sqrt{36}\sqrt{2} = 6\sqrt{2}$
32. $\sqrt{50} = \sqrt{25 \times 2} = \sqrt{25}\sqrt{2} = 5\sqrt{2}$
33. $\sqrt{200} = \sqrt{100 \times 2} = \sqrt{100}\sqrt{2} = 10\sqrt{2}$
34. $\sqrt{2} \times \sqrt{3} = \sqrt{2 \times 3} = \sqrt{6}$
35. $\sqrt{5} \times \sqrt{3} = \sqrt{5 \times 3} = \sqrt{15}$
36. $\sqrt{5} \times \sqrt{5} = (\sqrt{5})^2 = 5$
37. $\sqrt{15} \times \sqrt{3} = \sqrt{15 \times 3} = \sqrt{9 \times 5} = \sqrt{9}\sqrt{5} = 3\sqrt{5}$
38. $\sqrt{8} \times \sqrt{6} = \sqrt{8 \times 6} = \sqrt{16 \times 3} = \sqrt{16}\sqrt{3} = 4\sqrt{3}$
39. $3\sqrt{2} \times 4\sqrt{2} = 12\sqrt{2}\sqrt{2} = 12(\sqrt{2})^2 = 12 \times 2 = 24$
40. $(5\sqrt{2})(3\sqrt{8}) = 15\sqrt{2}\sqrt{8} = 15\sqrt{2 \times 8} = 15\sqrt{16} = 15 \times 4 = 60$
41. $(6\sqrt{3})(\sqrt{12}) = 6\sqrt{3 \times 12} = 6\sqrt{36} = 6 \times 6 = 36$
42. $(3\sqrt{5})(7\sqrt{2}) = 21\sqrt{5 \times 2} = 21\sqrt{10}$
43. $(5\sqrt{2}) \div (\sqrt{8}) = 5\sqrt{2} \div \sqrt{4 \times 2} = 5\sqrt{2} \div (2\sqrt{2}) = 5 \div 2 = 2.5$
44. $(5\sqrt{3})^2 = 5^2 \times (\sqrt{3})^2 = 25 \times 3 = 75$
45. $(3\sqrt{2})^2 = 3^2 \times (\sqrt{2})^2 = 9 \times 2 = 18$
46. $\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
47. $\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
48. $\frac{1}{\sqrt{5}} = \frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$
49. $\frac{3}{\sqrt{2}} = \frac{3}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$
50. $\frac{2}{\sqrt{7}} = \frac{2}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{2\sqrt{7}}{7}$
51. $\frac{6}{\sqrt{3}} = \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$
52. $\frac{1}{3+\sqrt{5}} = \frac{1}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}} = \frac{3-\sqrt{5}}{9-5} = \frac{3-\sqrt{5}}{4}$
53. $\frac{1}{3-\sqrt{2}} = \frac{1}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}} = \frac{3+\sqrt{2}}{9-2} = \frac{3+\sqrt{2}}{7}$
54. $\frac{1}{3+\sqrt{2}} = \frac{1}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}} = \frac{3-\sqrt{2}}{9-2} = \frac{3-\sqrt{2}}{7}$
55. $\frac{2}{\sqrt{3}+\sqrt{2}} = \frac{2}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{2(\sqrt{3}-\sqrt{2})}{3-2} = 2\sqrt{3} - 2\sqrt{2}$
56. $\frac{3}{\sqrt{3}-\sqrt{2}} = \frac{3}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{3(\sqrt{3}+\sqrt{2})}{3-2} = 3\sqrt{3} + 3\sqrt{2}$
57. $\frac{6}{\sqrt{5}+\sqrt{2}} = \frac{6}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}} = \frac{6(\sqrt{5}-\sqrt{2})}{5-2} = \frac{6(\sqrt{5}-\sqrt{2})}{3} = 2\sqrt{5} - 2\sqrt{2}$

58. $$\begin{aligned}\sin 60° &= \frac{9}{x}\\ \frac{\sqrt{3}}{2} &= \frac{9}{x}\\ \sqrt{3}x &= 18\\ x &= \frac{18}{\sqrt{3}}\\ &= \frac{18}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}\\ &= \frac{18\sqrt{3}}{3}\\ &= 6\sqrt{3}\end{aligned}$$

59. $$\begin{aligned}x^2 + 3^2 &= 7^2\\ x^2 + 9 &= 49\\ x^2 &= 40\\ x &= \sqrt{40}\\ &= \sqrt{4 \times 10}\\ &= \sqrt{4} \times \sqrt{10}\\ &= 2\sqrt{10}\end{aligned}$$

60. Label the vertical in the diagram as y, then

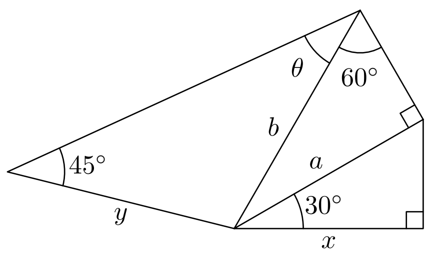
$$\begin{aligned}\sin 45° &= \frac{y}{10}\\ \frac{\sqrt{2}}{2} &= \frac{y}{10}\\ y &= 5\sqrt{2}\\ \sin 60° &= \frac{x}{y}\\ \frac{\sqrt{3}}{2} &= \frac{x}{5\sqrt{2}}\\ x &= \frac{5\sqrt{2}}{1} \times \frac{\sqrt{3}}{2}\\ &= \frac{5\sqrt{3}\sqrt{2}}{2}\\ &= \frac{5\sqrt{6}}{2}\end{aligned}$$

61. Use the cosine rule:

$$\begin{aligned}x^2 &= 4^2 + (2\sqrt{3})^2 - 2 \times 4 \times 2\sqrt{3} \times \cos 150°\\ &= 16 + 2^2 \times (\sqrt{3})^2 - 16\sqrt{3} \times (-\cos 30°)\\ &= 16 + 4 \times 3 - 16\sqrt{3} \times \left(-\frac{\sqrt{3}}{2}\right)\\ &= 16 + 12 + \frac{16\sqrt{3} \times \sqrt{3}}{2}\\ &= 28 + 8 \times 3\\ &= 52\\ x &= \sqrt{52}\\ &= \sqrt{4 \times 13}\\ &= 2\sqrt{13}\end{aligned}$$

62. Label the diagonal in the diagram as y, then

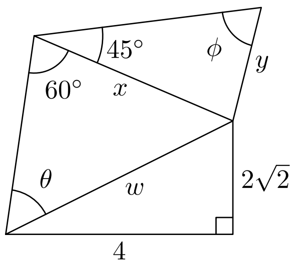
$$\begin{aligned}\frac{y}{\sin 60°} &= \frac{10}{\sin 45°}\\ y &= \frac{10 \sin 60°}{\sin 45°}\\ &= 10 \times \frac{\sqrt{3}}{2} \div \frac{1}{\sqrt{2}}\\ &= \frac{5\sqrt{3}}{1} \times \frac{\sqrt{2}}{1}\\ &= 5\sqrt{3}\sqrt{2}\\ &= 5\sqrt{6}\\ \tan 30° &= \frac{x}{y}\\ x &= y \tan 30°\\ &= 5\sqrt{6} \times \frac{1}{\sqrt{3}}\\ &= \frac{5\sqrt{3}\sqrt{2}}{\sqrt{3}}\\ &= 5\sqrt{2}\end{aligned}$$

63. 

$$\begin{aligned}\cos 30° &= \frac{x}{a}\\ \frac{\sqrt{3}}{2} &= \frac{x}{a}\\ \sqrt{3}a &= 2x\\ a &= \frac{2x}{\sqrt{3}}\\ \sin 60° &= \frac{a}{b}\\ \frac{\sqrt{3}}{2} &= \frac{a}{b}\\ \sqrt{3}b &= 2a\\ &= 2 \times \frac{2x}{\sqrt{3}}\\ &= \frac{4x}{\sqrt{3}}\\ b &= \frac{4x}{\sqrt{3}} \times \frac{1}{\sqrt{3}}\\ &= \frac{4x}{3}\end{aligned}$$

$$\begin{aligned}\frac{y}{\sin\theta} &= \frac{b}{\sin 45^\circ}\\ y &= b\times\sin\theta\div\sin 45^\circ\\ &= b\sin\theta\div\frac{1}{\sqrt{2}}\\ &= b\sin\theta\times\frac{\sqrt{2}}{1}\\ &= \sqrt{2}b\sin\theta\\ &= \sqrt{2}\times\frac{4x}{3}\times\sin\theta\\ &= \frac{4\sqrt{2}x\sin\theta}{3}\end{aligned}$$

□

64. 

$$\begin{aligned}w^2 &= 4^2 + (2\sqrt{2})^2\\ &= 16 + 4\times 2\\ &= 24\\ w &= \sqrt{24}\\ &= \sqrt{4\times 6}\\ &= 2\sqrt{6}\end{aligned}$$

$$\begin{aligned}\frac{x}{\sin\theta} &= \frac{w}{\sin 60^\circ}\\ x &= \frac{w\sin\theta}{\sin 60^\circ}\\ &= \frac{2\sqrt{6}\sin\theta}{\frac{\sqrt{3}}{2}}\\ &= \frac{2\sqrt{6}\sin\theta}{1}\times\frac{2}{\sqrt{3}}\\ &= \frac{4\sqrt{3}\sqrt{2}\sin\theta}{\sqrt{3}}\\ &= 4\sqrt{2}\sin\theta\\ \frac{y}{\sin 45^\circ} &= \frac{x}{\sin\phi}\\ y &= \frac{x\sin 45^\circ}{\sin\phi}\\ &= \frac{x\times\frac{1}{\sqrt{2}}}{\sin\phi}\\ &= \frac{4\sqrt{2}\sin\theta\times\frac{1}{\sqrt{2}}}{\sin\phi}\\ &= \frac{4\sin\theta}{\sin\phi}\end{aligned}$$

□

Exercise 2E

1. $43 - 19 = 24^\circ$

$$\begin{aligned}d &= \frac{24}{360}\times 2\pi\times 6350\\ &= 2660\text{km}\end{aligned}$$

2. $32 - 21 = 11^\circ$

$$\begin{aligned}d &= \frac{11}{360}\times 2\pi\times 6350\\ &= 1219\text{km}\end{aligned}$$

3. $39 - (-32) = 71^\circ$

$$\begin{aligned}d &= \frac{71}{360}\times 2\pi\times 6350\\ &= 7869\text{km}\end{aligned}$$

4. $51.5 - 5 = 46.5^\circ$

$$\begin{aligned}d &= \frac{46.5}{360}\times 2\pi\times 6350\\ &= 5154\text{km}\end{aligned}$$

5. $41 - 4 = 37^\circ$

$$\begin{aligned}d &= \frac{37}{360}\times 2\pi\times 6350\\ &= 4101\text{km}\end{aligned}$$

6. $134 - 114 = 20^\circ$

$$\begin{aligned}d &= \frac{20}{360}\times 2\pi\times 6350\cos 25^\circ\\ &= 2009\text{km}\end{aligned}$$

7. $119 - 77 = 42°$
$$\begin{aligned} d &= \frac{42}{360} \times 2\pi \times 6350 \cos 39° \\ &= 3617\text{km} \end{aligned}$$

8. $105 - 75 = 30°$
$$\begin{aligned} d &= \frac{30}{360} \times 2\pi \times 6350 \cos 40° \\ &= 2547\text{km} \end{aligned}$$

9. $122 - 117 = 5°$
$$\begin{aligned} d &= \frac{5}{360} \times 2\pi \times 6350 \cos 34° \\ &= 459\text{km} \end{aligned}$$

10. $175 - (-73) = 248°$
Longitude difference is greater than 180° so it is shorter to go the other way and cross the date line.
$360 - 248 = 112°$
$$\begin{aligned} d &= \frac{112}{360} \times 2\pi \times 6350 \cos 40° \\ &= 9509\text{km} \end{aligned}$$

11.
$$\begin{aligned} \frac{\theta}{360} &= \frac{555}{2\pi \times 6350} \\ \theta &= \frac{555}{2\pi \times 6350} \times 360 \\ &= 5° \\ \text{latitude} &= 29 + 5 \\ &= 34°\text{S} \end{aligned}$$
Augusta: 34°S, 115°E

12.
$$\begin{aligned} \frac{\theta}{360} &= \frac{3300}{2\pi \times 6350 \cos 34°} \\ \theta &= \frac{3300}{2\pi \times 6350 \cos 34°} \times 360 \\ &= 36° \\ \text{longitude} &= 115 + 36 \\ &= 151°\text{E} \end{aligned}$$
Sydney: 34°S, 151°E

13.
$$\begin{aligned} \frac{\theta}{360} &= \frac{7870}{2\pi \times 6350} \\ \theta &= \frac{7870}{2\pi \times 6350} \times 360 \\ &= 71° \\ \text{latitude} &= 71 - 36 \\ &= 35°\text{S} \end{aligned}$$
Adelaide: 35°S, 138°E

14.
$$\begin{aligned} \frac{\theta}{360} &= \frac{9600}{2\pi \times 6350 \cos 35°} \\ \theta &= \frac{9600}{2\pi \times 6350 \cos 35°} \times 360 \\ &= 106° \\ \text{longitude} &= 135 + 106 \\ &= 241°\text{E} \\ &= 360 - 241 \\ &= 119°\text{W} \end{aligned}$$
Bakersfield: 35°N, 119°W

15.
$$\begin{aligned} \frac{\theta}{360} &= \frac{820}{2\pi \times 6350 \cos 35°} \\ \theta &= \frac{820}{2\pi \times 6350 \cos 35°} \times 360 \\ &= 9° \\ \text{longitude} &= 135 - 9 \\ &= 126°\text{W} \\ \frac{\alpha}{360} &= \frac{2000}{2\pi \times 6350} \\ \alpha &= \frac{2000}{2\pi \times 6350} \times 360 \\ &= 18° \\ \text{latitude} &= 35 + 18 \\ &= 53°\text{S} \end{aligned}$$
New position: 53°S, 126°W

If the ship first heads south, the new latitude remains 53°S.
$$\begin{aligned} \frac{\theta}{360} &= \frac{820}{2\pi \times 6350 \cos 53°} \\ \theta &= \frac{820}{2\pi \times 6350 \cos 53°} \times 360 \\ &= 12° \\ \text{longitude} &= 135 - 12 \\ &= 123°\text{W} \end{aligned}$$
New position: 53°S, 123°W

16. First find the length of the chord LS from Los Angeles to Shimoneski through the earth using the angle subtended at the middle of the latitude circle:
$$\begin{aligned} r &= 6350 \cos 34° \\ &= 5264\text{km} \\ \theta &= 360 - (131 + 118) \\ &= 111° \\ \sin\frac{\theta}{2} &= \frac{0.5\text{LS}}{r} \\ 0.5\text{LS} &= 5264 \sin 55.5° \\ \text{LS} &= 2 \times 5264 \sin 55.5° \\ &= 8677\text{km} \end{aligned}$$

Now consider the angle that same chord subtends at the centre of the earth (i.e. the centre of the great circle passing through the two points). Let's call this angle α.
$$\begin{aligned} \sin\frac{\alpha}{2} &= \frac{0.5\text{LS}}{R} \\ &= \frac{0.5 \times 8677}{6350} \\ \frac{\alpha}{2} &= 43° \\ \alpha &= 86° \end{aligned}$$

Now use this angle to determine the arc length along this great circle:
$$\begin{aligned} d &= \frac{86}{360} \times 2\pi \times 6350 \\ &= 9553\text{km} \end{aligned}$$

Miscellaneous Exercise 2

1. See the answer in Sadler.

2. (a) $\tan 20° = \dfrac{15}{\text{AC}}$

$$\text{AC} = \frac{15}{\tan 20°} = 41.2\text{m}$$

(b) $\tan 30° = \dfrac{15}{\text{AB}}$

$$\text{AB} = \frac{15}{\tan 30°} = 26.0\text{m}$$

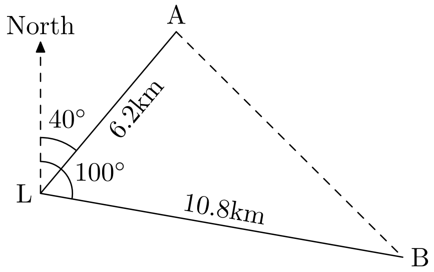
(c) $\text{BC}^2 = \text{AC}^2 + \text{AB}^2$

$$\text{BC} = \sqrt{41.2^2 + 26.0^2} = 48.7\text{m}$$

(d) $\tan \angle\text{ABC} = \dfrac{\text{AC}}{\text{AB}}$

$$\angle\text{ABC} = \tan^{-1}\frac{41.2}{26.0} = 58°$$

$$\text{bearing} = 270 + 58 = 328°$$

3. 

$$\text{AB} = \sqrt{6.2^2 + 10.8^2 - 2 \times 6.2 \times 10.8 \times \cos 60°} = 9.4\text{km}$$

$$\frac{\sin \angle\text{LBA}}{6.2} = \frac{\sin 60°}{9.4}$$

$$\angle\text{LBA} = \sin^{-1}\frac{6.2 \sin 60°}{9.4} = 35°$$

$$\text{bearing} = (100 + 180) + 35 = 315°$$

4. Let l be the length of the ladder.

$$\cos 75° = \frac{a}{l}$$

$$l = \frac{a}{\cos 75°}$$

$$\cos\theta = \frac{\frac{5a}{4}}{l} = \frac{5a}{4} \times \frac{1}{l} = \frac{5a}{4} \times \frac{\cos 75°}{a} = \frac{5\cos 75°}{4}$$

$$\theta = \cos^{-1}\frac{5\cos 75°}{4} = 71°$$

5. $$\frac{3-\sqrt{6}}{5+2\sqrt{6}} = \frac{3-\sqrt{6}}{5+2\sqrt{6}} \times \frac{5-2\sqrt{6}}{5-2\sqrt{6}} = \frac{(3-\sqrt{6})(5-2\sqrt{6})}{(5+2\sqrt{6})(5-2\sqrt{6})} = \frac{15-6\sqrt{6}-5\sqrt{6}+12}{25-24} = \frac{27-11\sqrt{6}}{1} = 27-11\sqrt{6}$$

6. (a) Read the question as "distance from 3 is less than distance from -11". The midpoint between -11 and 3 is -4, so the solution is $x > -4$.

(b) Read the question as "distance from 0 is less than distance from 6". The midpoint between 0 and 6 is 3, so the solution is $x < 3$.

(c) First solve the equation $|3x - 17| = |x - 3|$

$$\begin{aligned} 3x - 17 &= x - 3 & \text{or} \quad 3x - 17 &= -(x-3) \\ 2x &= 14 & 3x - 17 &= -x + 3 \\ x &= 7 & 4x &= 20 \\ & & x &= 5 \end{aligned}$$

Now test a value for x, say $x = 6$, to determine whether the inequality holds at that point.

Is it true that $\quad |3(6) - 17| \geq |(6) - 3|$

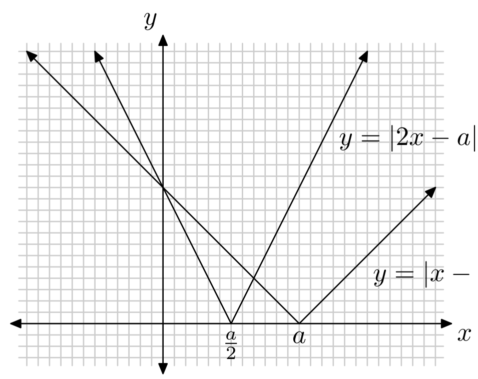
$$1 \ngeq 3 \quad : \text{no.}$$

Conclude that the solution lies outside the interval 5–7:

$$\{x \in \mathbb{R} : x \leq 5\} \cup \{x \in \mathbb{R} : x \geq 7\}$$

(d) This is the complementary case to the previous question, so it will have the complementary solution:

$$\{x \in \mathbb{R} : 5 < x < 7\}$$

7. 

From the graph it appears that $|2x - a| \le |x - a|$ is true for $0 \le x \le \frac{2a}{3}$. (You should confirm that these are the interval endpoints by substitution.)

8. (a)
$$\begin{aligned} \text{AH} &= \sqrt{\text{AG}^2 + \text{GH}^2} \\ &= \sqrt{4^2 + \left(\frac{12-6}{2}\right)^2} \\ &= 5\text{m} \end{aligned}$$

(b)
$$\begin{aligned} \text{EH} &= \sqrt{\text{AE}^2 - \text{AH}^2} \\ &= \sqrt{8^2 - 5^2} \\ &= \sqrt{39}\text{m} \\ &\approx 6.2\text{m} \end{aligned}$$

(c)
$$\begin{aligned} \cos\angle\text{EAH} &= \frac{\text{AH}}{\text{AE}} \\ &= \frac{5}{8} \\ \angle\text{EAH} &= 51^\circ \end{aligned}$$

(d)
$$\begin{aligned} \tan\angle\text{EGH} &= \frac{\text{EH}}{\text{GH}} \\ &= \frac{6.2}{3} \\ \angle\text{EGH} &= 64^\circ \end{aligned}$$

(e)
$$\begin{aligned} \tan\theta &= \frac{\text{EH}}{\text{GB}} \\ &= \frac{6.2}{4} \\ \theta &= 57^\circ \end{aligned}$$

9.
$$\begin{aligned} \frac{\theta}{360} &= \frac{440}{2\pi \times 6350\cos 37^\circ} \\ \theta &= \frac{440}{2\pi \times 6350\cos 37^\circ} \times 360 \\ &= 5^\circ \\ \text{longitude} &= 126 + 5 \\ &= 131^\circ\text{E} \\ \frac{\alpha}{360} &= \frac{330}{2\pi \times 6350} \\ \alpha &= \frac{330}{2\pi \times 6350} \times 360 \\ &= 3^\circ \\ \text{latitude} &= 37 - 3 \\ &= 34^\circ\text{S} \end{aligned}$$

New position: 34°S, 131°W

10. For the triangle to have an obtuse angle, the longest side must be longer than the hypotenuse if it were right-angled, i.e. $c^2 > a^2 + b^2$. This yields two possibilities.

If x is the longest side, then

$$\begin{aligned} x^2 &> 5^2 + 9^2 \\ x &> \sqrt{106} \end{aligned}$$

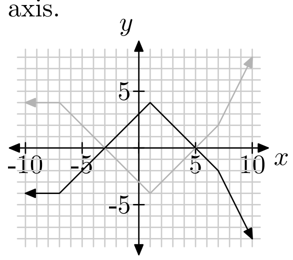
Since it must also satisfy the triangle inequality x must be less than the sum of the other two sides. The solution in this case is $\sqrt{106} < x < 14$.

If x is not the longest side, then

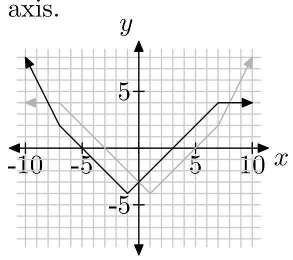
$$\begin{aligned} 9^2 &> 5^2 + x^2 \\ x &< \sqrt{56} \\ x &< 2\sqrt{14} \end{aligned}$$

Since it must also satisfy the triangle inequality x must be greater than the difference between the other two sides. The solution in this case is $4 < x < 2\sqrt{14}$.

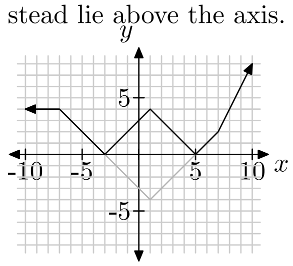
11. (a) $y = -\text{f}(x)$ represents a reflection in the x-axis.



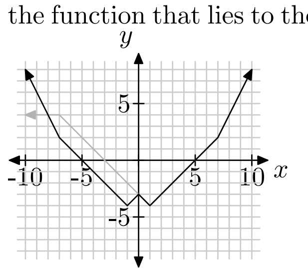
(b) $y = \text{f}(-x)$ represents a reflection in the y-axis.



(c) $y = |\text{f}(x)|$ signifies that any part of $\text{f}(x)$ that falls below the x-axis will be reflected to instead lie above the axis.



(d) $y = \text{f}(|x|)$ signifies that any part of $\text{f}(x)$ that falls left of the y-axis will be discarded and replaced with a mirror image of the part of the function that lies to the right of the axis.



Chapter 3

Exercise 3A

1. (a)
$$\begin{aligned}\angle ABN &= 180 - 50\\ &= 130^\circ\\ \angle ABC &= 360 - 90 - 130\\ &= 140^\circ\end{aligned}$$
$$\begin{aligned}AC &= \sqrt{5.8^2 + 6.4^2 - 2 \times 5.8 \times 6.4 \cos 140^\circ}\\ &= 11.5\text{km}\end{aligned}$$
$$\begin{aligned}\frac{\sin \angle BAC}{6.4} &= \frac{\sin 140^\circ}{11.5}\\ \angle BAC &= \sin^{-1} \frac{6.4 \sin 140^\circ}{11.5}\\ &= 21^\circ\\ 50 + 21 &= 71^\circ\end{aligned}$$
C is 11.5km on a bearing of 071° from A.

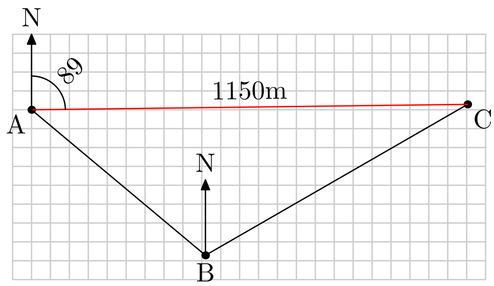
   (b) $71 + 180 = 251^\circ$
   A has a bearing of 251° from C.

2. (a) Bearing of A from B is $300 - 180 = 120^\circ$.
$$\begin{aligned}\angle ABC &= 120 - 70\\ &= 50^\circ\end{aligned}$$
$$\begin{aligned}AC &= \sqrt{4.9^2 + 7.2^2 - 2 \times 4.9 \times 7.2 \cos 50^\circ}\\ &= 5.5\text{km}\end{aligned}$$
We'll initially find ∠BCA rather than ∠BAC because the sine rule is ambiguous for ∠BAC but ∠BCA can not be obtuse (because it is opposite a smaller side).
$$\begin{aligned}\frac{\sin \angle BCA}{4.9} &= \frac{\sin 50^\circ}{5.5}\\ \angle BCA &= \sin^{-1} \frac{4.9 \sin 50^\circ}{5.5}\\ &= 43^\circ\\ \angle BAC &= 180 - 50 - 43\\ &= 87^\circ\\ 300 + 87 &= 387\\ 387 - 360 &= 027^\circ\end{aligned}$$
C is 8.5km on a bearing of 027° from A.

   (b) $27 + 180 = 207^\circ$
   A has a bearing of 207° from C.

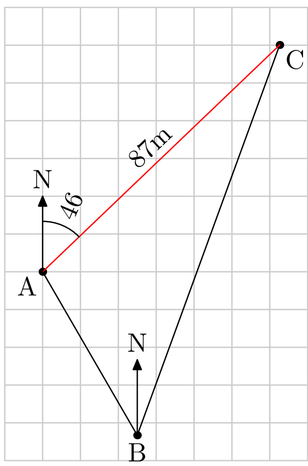
3. (a) Bearing of A from B is $40 + 180 = 220^\circ$.
   Bearing of C from B is $360 - 100 = 260^\circ$.
$$\begin{aligned}\angle ABC &= 260 - 220\\ &= 40^\circ\end{aligned}$$
$$\begin{aligned}AC &= \sqrt{73^2 + 51^2 - 2 \times 73 \times 51 \cos 40^\circ}\\ &= 47\text{km}\end{aligned}$$
We'll initially find ∠BCA rather than ∠BAC because the sine rule is ambiguous for ∠BAC but ∠BCA can not be obtuse (because it is opposite a smaller side).
$$\begin{aligned}\frac{\sin \angle BCA}{51} &= \frac{\sin 40^\circ}{47}\\ \angle BCA &= \sin^{-1} \frac{51 \sin 40^\circ}{47}\\ &= 44^\circ\\ \angle BAC &= 180 - 40 - 44\\ &= 96^\circ\\ 40 - 96 &= -56\\ -56 + 360 &= 304^\circ\end{aligned}$$
C is 47km on a bearing of 304° from A.

   (b) $304 - 180 = 124^\circ$
   A has a bearing of 124° from C.

4. (a)


Scale=1:20000

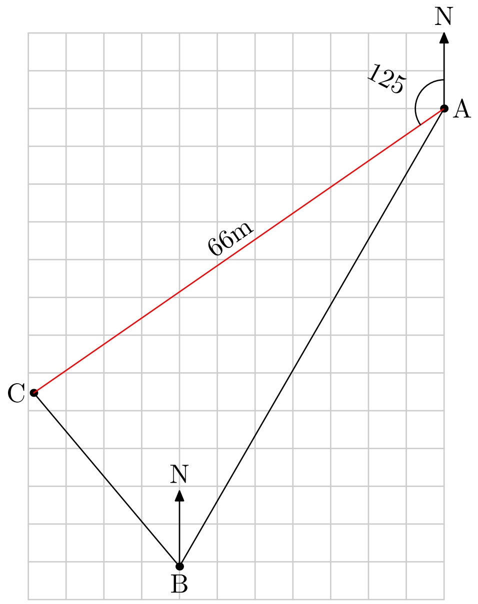
   (b) Bearing of A from C is $89 + 180 = 269^\circ$

5. (a)


Scale=1:2000

   (b) Bearing of A from C is $46 + 180 = 226^\circ$

6. (a)

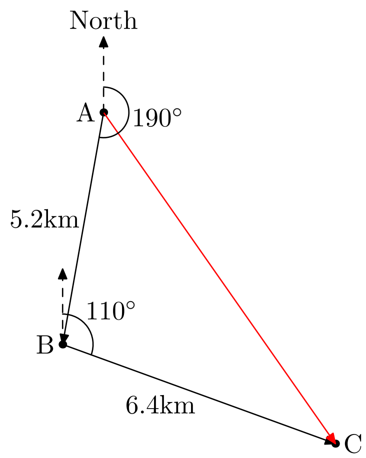


Scale=1:1000

Bearing of C from A is $360 - 125 = 235°$.

(b) Bearing of A from C is $215 - 180 = 055°$

7.



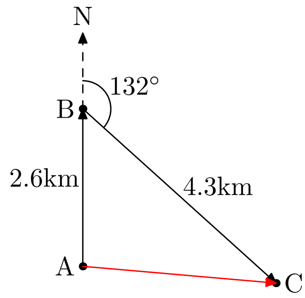
$$\begin{aligned}\angle ABC &= 110 - 10\\ &= 100°\end{aligned}$$

$$\begin{aligned}AC &= \sqrt{5.2^2 + 6.4^2 - 2 \times 5.2 \times 6.4 \cos 100°}\\ &= 8.9\text{km}\end{aligned}$$

$$\begin{aligned}\frac{\sin \angle BAC}{6.4} &= \frac{\sin 100°}{8.9}\\ \angle BAC &= \sin^{-1}\frac{6.4 \sin 100°}{8.9}\\ &= 45°\\ 190 - 45 &= 145°\end{aligned}$$

Final position is 8.9km on a bearing of 145° from initial position.

8.



$$\begin{aligned}\angle ABC &= 180 - 132\\ &= 48°\end{aligned}$$

$$\begin{aligned}AC &= \sqrt{2.6^2 + 4.3^2 - 2 \times 2.6 \times 4.3 \cos 48°}\\ &= 3.2\text{km}\end{aligned}$$

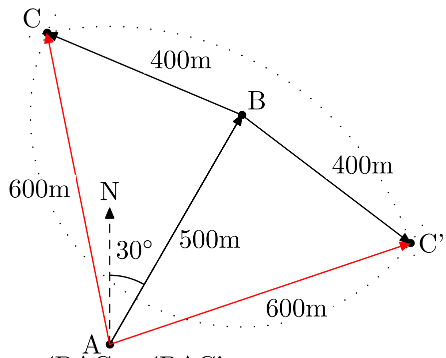
We'll initially find ∠BCA rather than ∠BAC because the sine rule is ambiguous for ∠BAC but ∠BCA can not be obtuse (because it is opposite a smaller side).

$$\begin{aligned}\frac{\sin \angle BCA}{2.6} &= \frac{\sin 48°}{3.2}\\ \angle BCA &= \sin^{-1}\frac{2.6 \sin 48°}{3.2}\\ &= 37°\\ \angle BCA &= 180 - 48 - 41\\ &= 95°\end{aligned}$$

Final position is 3.2km on a bearing of 095° from initial position.

9. $$\begin{aligned}d &= \sqrt{30^2 + 20^2 - 2 \times 30 \times 20 \cos 110}\\ &= 41\text{m}\end{aligned}$$

10.



Let $\theta = \angle$BAC$= \angle$BAC'

$$\begin{aligned}400^2 &= 600^2 + 500^2 - 2 \times 600 \times 500 \cos\theta\\ \cos\theta &= \frac{600^2 + 500^2 - 400^2}{2 \times 600 \times 500}\\ \theta &= \cos^{-1}\frac{600^2 + 500^2 - 400^2}{2 \times 600 \times 500}\\ &= 41°\end{aligned}$$

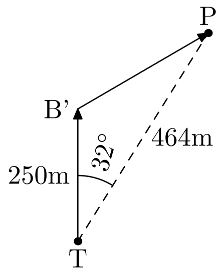
The bearing of the second checkpoint from the start is either: $(30-41)+360 = 349°$ or $30+41 = 071°$.

11. First, determine the bearing and distance from tee to pin. The angle at the bend is 180 − (50 − 20) = 150°. Call the bend point B and tee and pin T and P respectively.

$$\begin{aligned}TP &= \sqrt{280^2 + 200^2 - 2 \times 280 \times 200 \cos 150°} \\ &= 464\text{m}\end{aligned}$$

$$\begin{aligned}\frac{\sin \angle BTP}{200} &= \frac{\sin 150°}{464} \\ \angle BTP &= \sin^{-1} \frac{200 \sin 150°}{464} \\ &= 12°\end{aligned}$$

So the pin is 464m from the tee on a bearing of 20 + 12 = 032°. Now consider the result of the mis-hit:



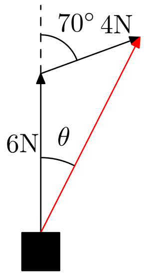
$$\begin{aligned}B'P &= \sqrt{250^2 + 464^2 - 2 \times 250 \times 464 \cos 32} \\ &= 286\text{m}\end{aligned}$$

We now need to find obtuse angle TB'P:

$$\begin{aligned}\frac{\sin \angle TB'P}{464} &= \frac{\sin 32°}{286} \\ \angle TB'P &= 180 - \sin^{-1} \frac{464 \sin 32°}{286} \\ &= 180 - 60°\end{aligned}$$

Hence the pin P is 286m from B' on a bearing of 060°.

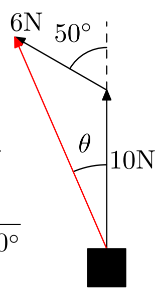
Exercise 3B



1. Let m be the magnitude of the resultant and θ the angle.

$$\begin{aligned}m &= \sqrt{6^2 + 4^2 - 2 \times 6 \times 4 \cos 110°} \\ &= 8.3\end{aligned}$$

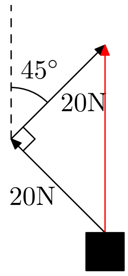
$$\begin{aligned}\frac{\sin \theta}{4} &= \frac{\sin 110°}{8.3} \\ \theta &= \sin^{-1} \frac{4 \sin 110°}{8.3} \\ &= 27°\end{aligned}$$



2. Let m be the magnitude of the resultant and θ the angle.

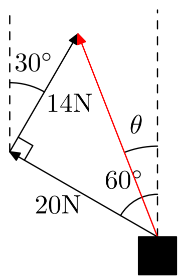
$$\begin{aligned}m &= \sqrt{10^2 + 8^2 - 2 \times 10 \times 8 \cos 130°} \\ &= 16.3\end{aligned}$$

$$\begin{aligned}\frac{\sin \theta}{6} &= \frac{\sin 130°}{16.3} \\ \theta &= \sin^{-1} \frac{6 \sin 130°}{16.3} \\ &= 22°\end{aligned}$$



3. Let m be the magnitude of the resultant and θ the angle.

$$\begin{aligned}m &= \sqrt{20^2 + 20^2} \\ &= 28.3 \\ \theta &= 0\end{aligned}$$

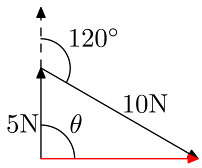


4. Let m be the magnitude of the resultant and θ the angle.

$$\begin{aligned}m &= \sqrt{14^2 + 20^2} \\ &= 24.4\end{aligned}$$

$$\begin{aligned}\tan(60 - \theta) &= \frac{14}{20} \\ 60 - \theta &= 35 \\ \theta &= 25°\end{aligned}$$

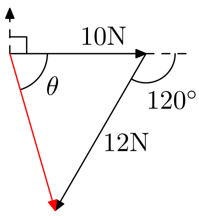
5. Let m be the magnitude of the resultant and θ the angle.



$$m = \sqrt{5^2 + 10^2 - 2 \times 5 \times 10 \cos 60^\circ}$$
$$= \sqrt{25 + 100 - 100 \times \frac{1}{2}}$$
$$= \sqrt{75}$$
$$= 5\sqrt{3}$$
$$\theta = 090^\circ$$

(We recognise it as a right angle triangle from our knowledge of exact trig ratios.)

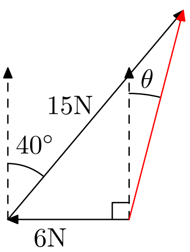
6. Let m be the magnitude of the resultant and θ as shown.



$$m = \sqrt{12^2 + 10^2 - 2 \times 12 \times 10 \cos 60^\circ}$$
$$= \sqrt{144 + 100 - 240 \times \frac{1}{2}}$$
$$= \sqrt{124}$$
$$= 2\sqrt{31}$$
$$\frac{\sin\theta}{12} = \frac{\sin 60}{2\sqrt{31}}$$
$$\theta = \sin^{-1}\frac{12\sin 60}{2\sqrt{31}}$$
$$= 69^\circ$$

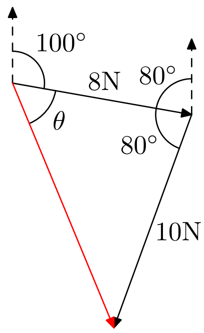
Bearing$=90 + 69 = 159^\circ$

7. Let m be the magnitude of the resultant and θ as shown.



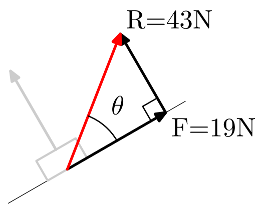
$$m = \sqrt{6^2 + 15^2 - 2 \times 6 \times 15 \cos 50^\circ}$$
$$= 12.1\text{N}$$
$$\frac{\sin(\phi)}{6} = \frac{\sin 50}{12.1}$$
$$\phi = \sin^{-1}\frac{6\sin 50}{12.1}$$
$$= 22^\circ$$
$$\theta = 180 - 90 - 50 - 22^\circ$$
$$= 018^\circ$$

8. Let m be the magnitude of the resultant and θ as shown.



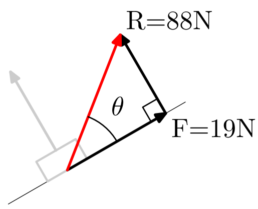
$$m = \sqrt{8^2 + 10^2 - 2 \times 8 \times 10 \cos 80^\circ}$$
$$= 11.7\text{N}$$
$$\frac{\sin\theta}{10} = \frac{\sin 80}{11.7}$$
$$\theta = \sin^{-1}\frac{10\sin 80}{11.7}$$
$$= 58^\circ$$
$$\text{bearing} = 100 + 58^\circ$$
$$= 158^\circ$$

9.



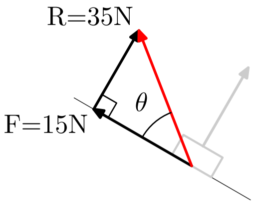
$$\text{magnitude} = \sqrt{R^2 + F^2}$$
$$= \sqrt{43^2 + 19^2}$$
$$= 47\text{N}$$
$$\tan\theta = \frac{R}{F}$$
$$\theta = \tan^{-1}\frac{R}{F}$$
$$= \tan^{-1}\frac{43}{19}$$
$$= 66^\circ$$

10.



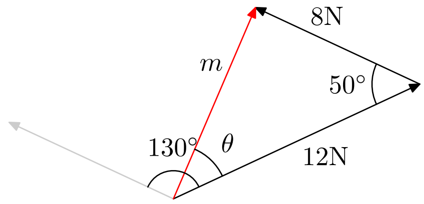
$$\text{magnitude} = \sqrt{R^2 + F^2}$$
$$= \sqrt{88^2 + 19^2}$$
$$= 90\text{N}$$
$$\tan\theta = \frac{R}{F}$$
$$\theta = \tan^{-1}\frac{R}{F}$$
$$= \tan^{-1}\frac{88}{19}$$
$$= 78^\circ$$

11.



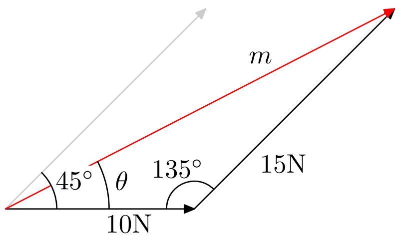
$$\begin{aligned}\text{magnitude} &= \sqrt{R^2+F^2}\\ &= \sqrt{35^2+15^2}\\ &= 38\text{N}\\ \tan\theta &= \frac{R}{F}\\ \theta &= \tan^{-1}\frac{R}{F}\\ &= \tan^{-1}\frac{35}{15}\\ &= 67^\circ\end{aligned}$$

12.



$$\begin{aligned}m &= \sqrt{8^2+12^2-2\times 8\times 12\cos 50^\circ}\\ &= 9.2\text{N}\\ \frac{\sin\theta}{8} &= \frac{\sin 50^\circ}{9.2}\\ \theta &= \sin^{-1}\frac{8\sin 50^\circ}{9.2}\\ &= 42^\circ\end{aligned}$$

13.



$$\begin{aligned}m &= \sqrt{10^2+15^2-2\times 10\times 15\cos 135^\circ}\\ &= 23.2\text{N}\\ \frac{\sin\theta}{15} &= \frac{\sin 135^\circ}{23.2}\\ \theta &= \sin^{-1}\frac{15\sin 135^\circ}{23.2}\\ &= 27^\circ\end{aligned}$$

Exercise 3C

1. $$\begin{aligned}m &= \sqrt{2^2+4^2}\\ &= 4.5\text{m/s}\\ \tan\theta &= \frac{4}{2}\\ \theta &= 63^\circ\end{aligned}$$

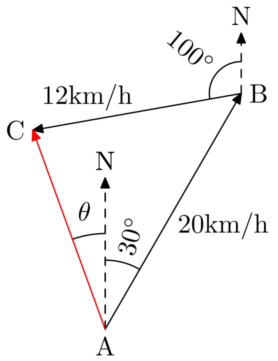
2. The angle formed where the vectors meet head to tail is $90-25=65^\circ$.

$$\begin{aligned}m &= \sqrt{2^2+4^2-2\times 4\times 2\cos 65}\\ &= 3.6\text{m/s}\\ \frac{\sin\theta}{4} &= \frac{\sin 65^\circ}{3.6}\\ \theta &= \sin^{-1}\frac{4\sin 65^\circ}{3.6}\\ &= 85^\circ\end{aligned}$$

3. The angle formed where the vectors meet head to tail is $180-50=130^\circ$.

$$\begin{aligned}m &= \sqrt{2^2+4^2-2\times 4\times 2\cos 130}\\ &= 5.5\text{m/s}\\ \frac{\sin\theta}{4} &= \frac{\sin 130^\circ}{5.5}\\ \theta &= \sin^{-1}\frac{4\sin 130^\circ}{5.5}\\ &= 34^\circ\end{aligned}$$

4.

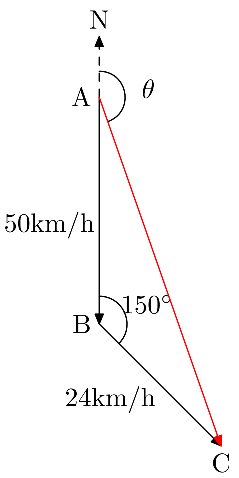


$$\begin{aligned}\angle \text{ABC} &= 180 - 30 - 100\\ &= 50°\\ \text{AC} &= \sqrt{20^2 + 12^2 - 2 \times 20 \times 12 \cos 50°}\\ &= 15.3\text{km/h}\end{aligned}$$

$$\begin{aligned}\frac{\sin(\theta + 30°)}{12} &= \frac{\sin 50°}{15.3}\\ \theta + 30 &= \sin^{-1} \frac{12 \sin 50°}{15.3}\\ &= 37°\\ \theta &= 7°\end{aligned}$$

The boat travels on a bearing of 353° 15.3km in one hour.

5. Wind blowing *from* 330° is blowing *toward* 330 − 180 = 150°.

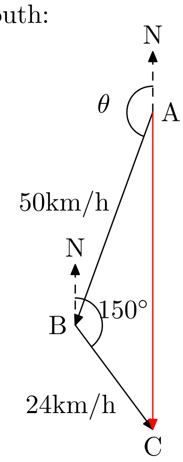


$$\begin{aligned}\text{AC} &= \sqrt{50^2 + 24^2 - 2 \times 50 \times 24 \cos 150°}\\ &= 71.8\text{km/h}\end{aligned}$$

$$\begin{aligned}\frac{\sin(180° - \theta)}{24} &= \frac{\sin 150°}{71.8}\\ 180 - \theta &= \sin^{-1} \frac{24 \sin 150°}{71.8}\\ &= 10°\\ \theta &= 170°\end{aligned}$$

The bird travels on a bearing of 170° at 71.8km/h.

To travel due south:

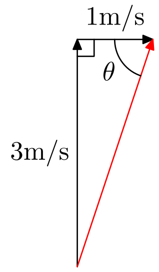


$$\begin{aligned}\angle \text{ACB} &= 180 - 150\\ &= 30°\\ \frac{\sin(180° - \theta)}{24} &= \frac{\sin 30°}{50}\\ 180 - \theta &= \sin^{-1} \frac{24 \sin 30°}{50}\\ &= 14°\\ \theta &= 166°\end{aligned}$$

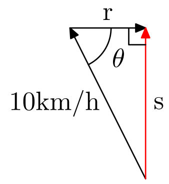
6. (a) $h = 3 \times 60$
$= 180\text{m}$

(b) $s = \sqrt{3^2 + 1^2}$
$= \sqrt{10}\,\text{m/s}$
$\approx 3.2\,\text{m/s}$

(c) $\tan\theta = \dfrac{3}{1}$
$\theta = 72°$



7. The angle can be determined using cosine. If r is the speed of the river current and θ is the angle with the bank, then $\cos\theta = \frac{r}{10}$.



The speed of the boat's movement across the river (s) can be determined using Pythagoras: $s = \sqrt{10^2 - r^2}$.

Then the time taken to cross the river is

$$t = \frac{0.08}{s} \times 3600 = \frac{288}{s} \text{ seconds.}$$

(a) $\theta = \cos^{-1} \dfrac{3}{10}$
$= 73°$
$s = \sqrt{10^2 - 3^2}$
$= 9.5\text{km/h}$
$t = \dfrac{288}{9.5}$
$= 30\,\text{s}$

(b) $\theta = \cos^{-1} \dfrac{4}{10}$
$= 66°$
$s = \sqrt{10^2 - 4^2}$
$= 9.2\text{km/h}$
$t = \dfrac{288}{9.2}$
$= 31\,\text{s}$

(c) $\theta = \cos^{-1}\dfrac{6}{10}$

$= 53°$

$s = \sqrt{10^2 - 6^2}$

$= 8\text{km/h}$

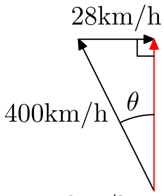
$t = \dfrac{288}{8}$

$= 36\,\text{s}$

8. $\sin\theta = \dfrac{28}{400}$

$\theta = 4°$

The plane should set a heading of N4°W or 356°T.

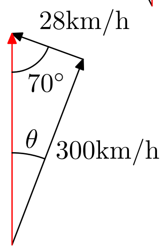


9. $\dfrac{\sin\theta}{28} = \dfrac{\sin 70°}{300}$

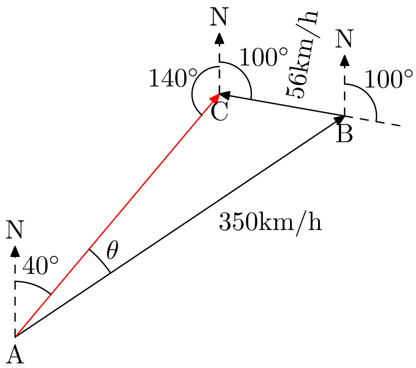
$\theta = \sin^{-1}\dfrac{28\sin 70°}{300}$

$= 5°$

The plane should set a heading of N5°E or 005°T.



10.



$$\angle ACB = 360 - 100 - 140$$
$$= 120°$$
$$\frac{\sin\theta}{56} = \frac{\sin 120°}{350}$$
$$\theta = \sin^{-1}\frac{56\sin 120°}{350}$$
$$= 8°$$

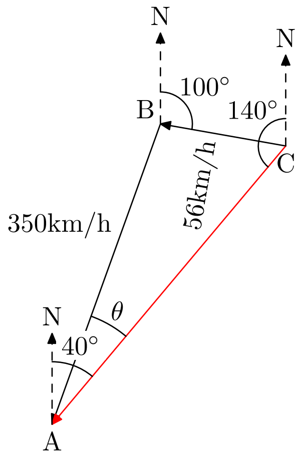
The plane should fly on a bearing of 048°.

$$\angle ABC = 180 - 120 - 8$$
$$= 52°$$
$$\frac{AC}{\sin 52°} = \frac{350}{\sin 120°}$$
$$AC = \frac{350\sin 52°}{\sin 120°}$$
$$= 319\text{km/h}$$

Time required for the flight:

$$t = \frac{500}{319} \times 60 = 94 \text{ minutes}$$

For the return flight:



$$\angle ACB = 140 - 80$$
$$= 60°$$
$$\frac{\sin\theta}{56} = \frac{\sin 60°}{350}$$
$$\theta = \sin^{-1}\frac{56\sin 60°}{350}$$
$$= 8°$$

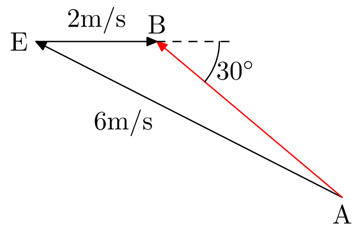
The plane should fly on a bearing of $180 + (40 - 8) = 212°$.

$$\angle ABC = 180 - 60 - 8$$
$$= 112°$$
$$\frac{AC}{\sin 112°} = \frac{350}{\sin 60°}$$
$$AC = \frac{350\sin 112°}{\sin 60°}$$
$$= 374\text{km/h}$$

Time required for the return flight:

$$t = \frac{500}{374} \times 60 = 80 \text{ minutes}$$

11.



$$\angle \text{B} = 180 - 30 = 150°$$

$$\frac{\sin \angle A}{2} = \frac{\sin 150°}{6}$$

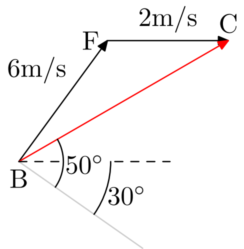
$$\angle A = \sin^{-1} \frac{2\sin 150°}{6} = 9.6°$$

$$\angle E = 180 - 150 - 9.6 = 20.4°$$

$$\frac{\text{AB}}{\sin 20.4°} = \frac{6}{\sin 150°}$$

$$\text{AB} = \frac{6\sin 20.4}{\sin 150°} = 4.2\text{m/s}$$

$$t_{\text{AB}} = \frac{80}{4.2} = 19.12\text{s}$$



$$\angle \text{C} = 50 - 30 = 20°$$

$$\frac{\sin \angle B}{2} = \frac{\sin 20°}{6}$$

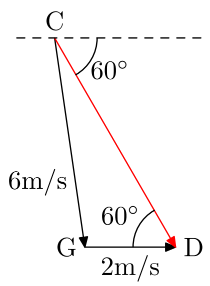
$$\angle B = \sin^{-1} \frac{2\sin 20°}{6} = 6.5°$$

$$\angle F = 180 - 20 - 6.5 = 153.5°$$

$$\frac{\text{BC}}{\sin 153.5°} = \frac{6}{\sin 20°}$$

$$\text{BC} = \frac{6\sin 153.5}{\sin 20°} = 7.8\text{m/s}$$

$$t_{\text{BC}} = \frac{110}{7.8} = 14.03\text{s}$$



$$\frac{\sin \angle C}{2} = \frac{\sin 60°}{6}$$

$$\angle C = \sin^{-1} \frac{2\sin 60°}{6} = 16.8°$$

$$\angle G = 180 - 60 - 16.8 = 103.2°$$

$$\frac{\text{BC}}{\sin 103.2°} = \frac{6}{\sin 60°}$$

$$\text{BC} = \frac{6\sin 103.2}{\sin 60°} = 6.7\text{m/s}$$

Perpendicular width of river:

$$w_{\text{AB}} = 80\sin 30° = 40\text{m}$$

$$w_{\text{BC}} = 110\sin 20° = 37.6\text{m}$$

$$w = 40 + 37.6 = 77.6\text{m}$$

$$\text{CD} = \frac{77.6}{\sin 60°} = 89.6\text{m}$$

$$t_{\text{CD}} = \frac{89.6}{6.7} = 13.29\text{s}$$

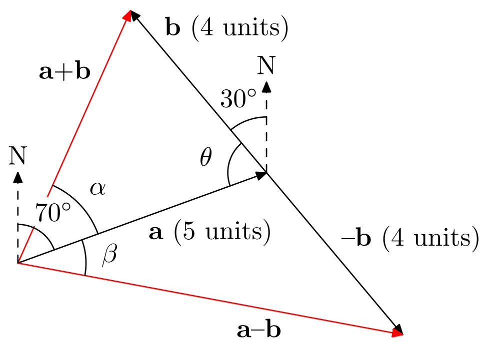
Total time:

$$t = 19.12 + 14.03 + 13.29 \approx 46\text{s}$$

Exercise 3D

No working is needed for questions 1–7. Refer to the answers in Sadler.

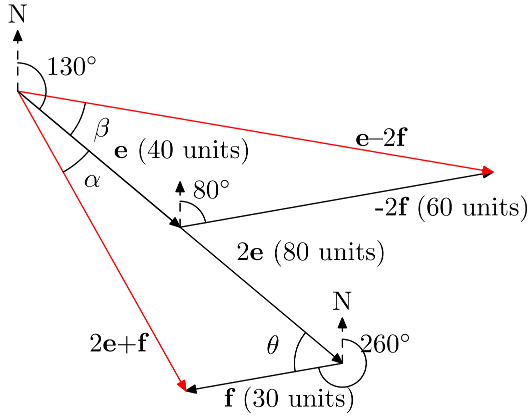
8.



(a)
$$\begin{aligned}
\theta + 30 &= 180 - 70\\
\theta &= 80^\circ\\
|\mathbf{a}+\mathbf{b}| &= \sqrt{5^2 + 4^2 - 2 \times 5 \times 4 \cos 80^\circ}\\
&= 5.8 \text{ units}\\
\frac{\sin\alpha}{4} &= \frac{\sin 80^\circ}{5.8}\\
\alpha &= \sin^{-1}\frac{4\sin 80^\circ}{5.8}\\
&= 42^\circ\\
70 - \alpha &= 28^\circ
\end{aligned}$$

(b)
$$\begin{aligned}
180 - \theta &= 180 - 80\\
&= 100^\circ\\
|\mathbf{a}-\mathbf{b}| &= \sqrt{5^2 + 4^2 - 2 \times 5 \times 4 \cos 100^\circ}\\
&= 6.9 \text{ units}\\
\frac{\sin\beta}{4} &= \frac{\sin 100^\circ}{6.9}\\
\beta &= \sin^{-1}\frac{4\sin 100^\circ}{6.9}\\
&= 35^\circ\\
70 + \beta &= 105^\circ
\end{aligned}$$

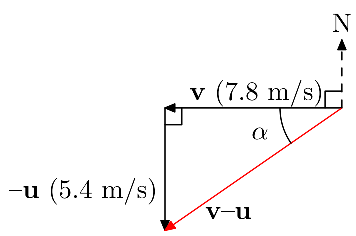
9.



(a)
$$\begin{aligned}
\theta &= 360 - 260 - (180 - 130)\\
&= 50^\circ\\
|2\mathbf{e}+\mathbf{f}| &= \sqrt{80^2 + 30^2 - 2 \times 80 \times 30 \cos 50^\circ}\\
&= 65 \text{ units}\\
\frac{\sin\alpha}{30} &= \frac{\sin 50^\circ}{65}\\
\alpha &= \sin^{-1}\frac{30\sin 50^\circ}{65}\\
&= 21^\circ\\
130 + \alpha &= 151^\circ
\end{aligned}$$

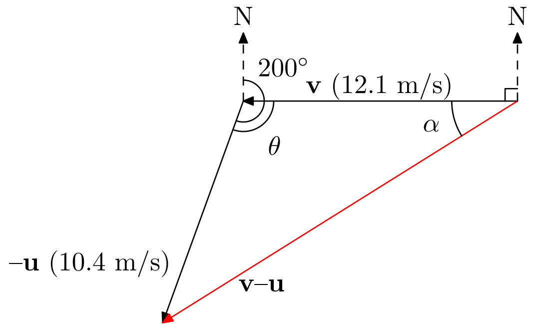
(b)
$$\begin{aligned}
180 - \theta &= 180 - 50\\
&= 130^\circ\\
|\mathbf{e}-2\mathbf{f}| &= \sqrt{40^2 + 60^2 - 2 \times 40 \times 60 \cos 130^\circ}\\
&= 91 \text{ units}\\
\frac{\sin\beta}{60} &= \frac{\sin 130^\circ}{91}\\
\beta &= \sin^{-1}\frac{60\sin 130^\circ}{91}\\
&= 30^\circ\\
130 - \beta &= 100^\circ
\end{aligned}$$

10.



$$\begin{aligned}
|\mathbf{v}-\mathbf{u}| &= \sqrt{5.4^2 + 7.8^2}\\
&= 9.5 \text{ m/s}\\
\tan\alpha &= \frac{5.4}{7.8}\\
\alpha &= \tan^{-1}\frac{5.4}{7.8}\\
&= 35^\circ\\
270 - \alpha &= 235^\circ\\
\mathbf{a} &= \frac{\mathbf{v}-\mathbf{u}}{t}\\
&= \frac{9.5\angle 235^\circ}{5}\\
&= 1.9\,\text{m/s}^2 \text{ on a bearing of } 235^\circ
\end{aligned}$$

11.



$$\begin{aligned}\theta &= 200 - 90\\ &= 110^\circ\end{aligned}$$

$$\begin{aligned}|\mathbf{v}-\mathbf{u}| &= \sqrt{10.4^2 + 12.1^2 - 2\times 10.4\times 12.1\cos 110^\circ}\\ &= 18.5 \text{ m/s}\end{aligned}$$

$$\begin{aligned}\frac{\sin\alpha}{10.4} &= \frac{\sin 110^\circ}{18.5}\\ \alpha &= \sin^{-1}\frac{10.4\sin 110^\circ}{18.5}\\ &= 32^\circ\\ 270-\alpha &= 238^\circ\\ \mathbf{a} &= \frac{\mathbf{v}-\mathbf{u}}{t}\\ &= \frac{18.5\angle 238^\circ}{4}\\ &= 4.6\,\text{m/s}^2 \text{ on a bearing of } 238^\circ\end{aligned}$$

12. (a) $\lambda = \mu = 0$

(b) $\lambda = \mu = 0$

(c) $\lambda - 3 = 0 \quad \mu + 4 = 0$

$\lambda = 3 \quad \mu = -4$

(d) $(\lambda - 2)\mathbf{a} = (5-\mu)\mathbf{b}$

$\lambda - 2 = 0 \quad 5 - \mu = 0$

$\lambda = 2 \quad \mu = 5$

(e)
$$\begin{aligned}\lambda\mathbf{a} - 2\mathbf{b} &= \mu\mathbf{b} + 5\mathbf{a}\\ \lambda\mathbf{a} - 5\mathbf{a} &= \mu\mathbf{b} + 2\mathbf{b}\\ (\lambda-5)\mathbf{a} &= (\mu+2)\mathbf{b}\end{aligned}$$

$\lambda - 5 = 0 \quad \mu + 2 = 0$

$\lambda = 5 \quad \mu = -2$

(f) $(\lambda + \mu - 4)\mathbf{a} = (\mu - 3\lambda)\mathbf{b}$

$\lambda + \mu - 4 = 0 \quad \mu - 3\lambda = 0$

$\mu = 4 - \lambda \quad \mu = 3\lambda$

$$\begin{aligned}4 - \lambda &= 3\lambda\\ 4 &= 4\lambda\\ \lambda &= 1\\ \mu &= 3\lambda\\ \mu &= 3\end{aligned}$$

(g)
$$\begin{aligned}2\mathbf{a} + 3\mathbf{b} + \mu\mathbf{b} &= 2\mathbf{b} + \lambda\mathbf{a}\\ (2-\lambda)\mathbf{a} &= (2 - 3 - \mu)\mathbf{b}\end{aligned}$$

$2 - \lambda = 0 \quad -1 - \mu = 0$

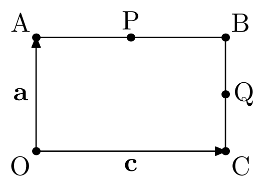
$\lambda = 2 \quad \mu = -1$

(h)
$$\begin{aligned}\lambda\mathbf{a} + \mu\mathbf{b} + 2\lambda\mathbf{b} &= 5\mathbf{a} + 4\mathbf{b} + \mu\mathbf{a}\\ (\lambda - 5 - \mu)\mathbf{a} &= (4 - \mu - 2\lambda)\mathbf{b}\\ \lambda - 5 - \mu &= 0\\ \mu &= \lambda - 5\\ 4 - \mu - 2\lambda &= 0\\ 4 - (\lambda - 5) - 2\lambda &= 0\\ 4 - \lambda + 5 - 2\lambda &= 0\\ 9 - 3\lambda &= 0\\ \lambda &= 3\\ \mu &= \lambda - 5\\ \mu &= -2\end{aligned}$$

(i)
$$\begin{aligned}\lambda\mathbf{a} - \mathbf{b} + \mu\mathbf{b} &= 4\mathbf{a} + \mu\mathbf{a} - 4\lambda\mathbf{b}\\ (\lambda - 4 - \mu)\mathbf{a} &= (-4\lambda + 1 - \mu)\mathbf{b}\\ \lambda - 4 - \mu &= 0\\ \mu &= \lambda - 4\\ -4\lambda + 1 - \mu &= 0\\ -4\lambda + 1 - (\lambda - 4) &= 0\\ -4\lambda + 1 - \lambda + 4 &= 0\\ -5\lambda + 5 &= 0\\ \lambda &= 1\\ \mu &= \lambda - 4\\ \mu &= -3\end{aligned}$$

(j)
$$\begin{aligned}2\lambda\mathbf{a} + 3\mu\mathbf{a} - \mu\mathbf{b} + 2\mathbf{b} &= \lambda\mathbf{b} + 2\mathbf{a}\\ (2\lambda + 3\mu - 2)\mathbf{a} &= (\lambda + \mu - 2)\mathbf{b}\\ 2\lambda + 3\mu - 2 &= 0\\ \lambda + \mu - 2 &= 0\\ 2\lambda + 2\mu - 4 &= 0\\ \mu + 2 &= 0\\ \mu &= -2\\ \lambda + \mu - 2 &= 0\\ \lambda - 2 - 2 &= 0\\ \lambda &= 4\end{aligned}$$

13.



(a) $\overrightarrow{CB} = \mathbf{a}$

(b) $\overrightarrow{BC} = -\overrightarrow{CB} = -\mathbf{a}$

(c) $\overrightarrow{AB} = \mathbf{c}$

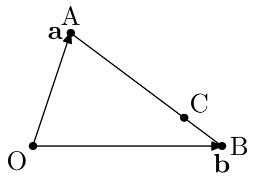
(d) $\overrightarrow{BA} = -\overrightarrow{AB} = -\mathbf{c}$

(e) $\overrightarrow{AP} = 0.5\overrightarrow{AB} = 0.5\mathbf{c}$

(f)
$$\begin{aligned}\overrightarrow{OQ} &= \overrightarrow{OC} + \overrightarrow{CQ}\\ &= \mathbf{c} + 0.5\mathbf{a}\end{aligned}$$

(g)
$$\begin{aligned}\overrightarrow{OP} &= \overrightarrow{OA} + \overrightarrow{AP}\\ &= \mathbf{a} + 0.5\mathbf{c}\end{aligned}$$

(h)
$$\begin{aligned}\overrightarrow{PQ} &= \overrightarrow{PB} + \overrightarrow{BQ}\\ &= 0.5\mathbf{c} - 0.5\mathbf{a}\end{aligned}$$

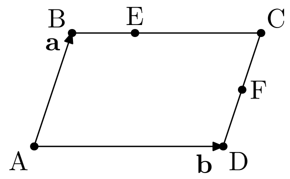
14. 

(a) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -\mathbf{a} + \mathbf{b}$

(b) $\overrightarrow{AC} = 0.75\overrightarrow{AB} = -0.75\mathbf{a} + 0.75\mathbf{b}$

(c) $\overrightarrow{CB} = 0.25\overrightarrow{AB} = -0.25\mathbf{a} + 0.25\mathbf{b}$

(d) $$\begin{aligned}\overrightarrow{OC} &= \overrightarrow{OA} + \overrightarrow{AC} \\ &= \mathbf{a} - 0.75\mathbf{a} + 0.75\mathbf{b} \\ &= 0.25\mathbf{a} + 0.75\mathbf{b}\end{aligned}$$

15. 

(a) $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \mathbf{a} + \mathbf{b}$

(b) $\overrightarrow{BE} = \frac{1}{3}\overrightarrow{BC} = \frac{1}{3}\mathbf{b}$

(c) $\overrightarrow{DF} = \frac{1}{2}\overrightarrow{DC} = \frac{1}{2}\mathbf{a}$

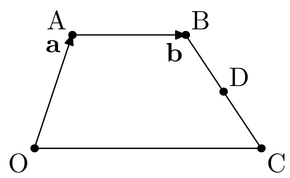
(d) $\overrightarrow{AE} = \overrightarrow{AB} + \overrightarrow{BE} = \mathbf{a} + \frac{1}{3}\mathbf{b}$

(e) $\overrightarrow{AF} = \overrightarrow{AD} + \overrightarrow{DF} = \mathbf{b} + \frac{1}{2}\mathbf{a}$

(f) $$\begin{aligned}\overrightarrow{BF} &= \overrightarrow{BA} + \overrightarrow{AF} \\ &= -\mathbf{a} + \mathbf{b} + \frac{1}{2}\mathbf{a} \\ &= \mathbf{b} - \frac{1}{2}\mathbf{a}\end{aligned}$$

(g) $$\begin{aligned}\overrightarrow{DE} &= \overrightarrow{DA} + \overrightarrow{AE} \\ &= -\mathbf{b} + \mathbf{a} + \frac{1}{3}\mathbf{b} \\ &= \mathbf{a} - \frac{2}{3}\mathbf{b}\end{aligned}$$

(h) $$\begin{aligned}\overrightarrow{EF} &= \overrightarrow{EA} + \overrightarrow{AF} \\ &= -\overrightarrow{AE} + \overrightarrow{AF} \\ &= -(\mathbf{a} + \frac{1}{3}\mathbf{b}) + \mathbf{b} + \frac{1}{2}\mathbf{a} \\ &= -\frac{1}{2}\mathbf{a} + \frac{2}{3}\mathbf{b}\end{aligned}$$

16. 

(a) $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \mathbf{a} + \mathbf{b}$

(b) $\overrightarrow{OC} = 2\overrightarrow{AB} = 2\mathbf{b}$

(c) $$\begin{aligned}\overrightarrow{BC} &= \overrightarrow{BA} + \overrightarrow{AO} + \overrightarrow{OC} \\ &= -\overrightarrow{AB} - \overrightarrow{OA} + \overrightarrow{OC} \\ &= -\mathbf{b} - \mathbf{a} + 2\mathbf{b} \\ &= -\mathbf{a} + \mathbf{b}\end{aligned}$$

(d) $\overrightarrow{BD} = 0.5\overrightarrow{BC} = -0.5\mathbf{a} + 0.5\mathbf{b}$

(e) $$\begin{aligned}\overrightarrow{OD} &= \overrightarrow{OB} + \overrightarrow{BD} \\ &= \mathbf{a} + \mathbf{b} - 0.5\mathbf{a} + 0.5\mathbf{b} \\ &= 0.5\mathbf{a} + 1.5\mathbf{b}\end{aligned}$$

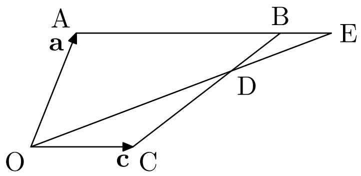
17. (a) $\overrightarrow{OC} = 0.5\overrightarrow{OA} = 0.5\mathbf{a}$

(b) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -\mathbf{a} + \mathbf{b}$

(c) $\overrightarrow{AD} = \frac{2}{3}\overrightarrow{AB} = -\frac{2}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$

(d) $$\begin{aligned}\overrightarrow{CD} &= \overrightarrow{CA} + \overrightarrow{AD} \\ &= \frac{1}{2}\mathbf{a} + (-\frac{2}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}) \\ &= -\frac{1}{6}\mathbf{a} + \frac{2}{3}\mathbf{b}\end{aligned}$$

(e) $$\begin{aligned}\overrightarrow{OC} + \overrightarrow{CE} &= \overrightarrow{OE} \\ \overrightarrow{OC} + h\overrightarrow{CD} &= k\overrightarrow{OB} \\ \frac{1}{2}\mathbf{a} + h(-\frac{1}{6}\mathbf{a} + \frac{2}{3}\mathbf{b}) &= k\mathbf{b} \\ (\frac{1}{2} - \frac{h}{6})\mathbf{a} &= (k - \frac{2h}{3})\mathbf{b} \\ \frac{1}{2} - \frac{h}{6} &= 0 \\ 3 - h &= 0 \\ h &= 3 \\ k - \frac{2h}{3} &= 0 \\ k &= \frac{2h}{3} \\ &= \frac{2 \times 3}{3} \\ &= 2\end{aligned}$$

18. 

$$\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{CD}$$
$$= \mathbf{c} + \frac{2}{3}\overrightarrow{CB}$$
$$= \mathbf{c} + \frac{2}{3}(\overrightarrow{CO} + \overrightarrow{OA} + \overrightarrow{AB})$$
$$= \mathbf{c} + \frac{2}{3}(-\mathbf{c} + \mathbf{a} + 2\mathbf{c})$$
$$= \mathbf{c} + \frac{2}{3}(\mathbf{a} + \mathbf{c})$$
$$= \frac{2}{3}\mathbf{a} + \frac{5}{3}\mathbf{c}$$

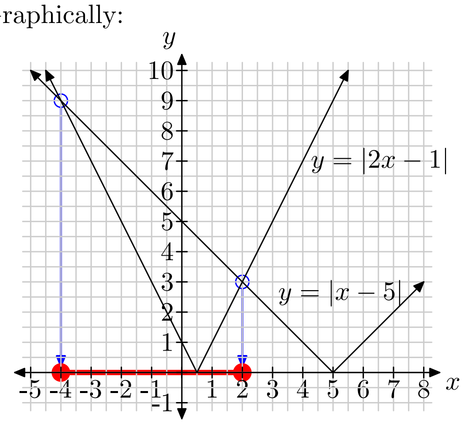
$$\overrightarrow{OE} = \overrightarrow{OA} + \overrightarrow{AE}$$
$$h\overrightarrow{OD} = \overrightarrow{OA} + k\overrightarrow{AB}$$
$$h(\frac{2}{3}\mathbf{a} + \frac{5}{3}\mathbf{c}) = \mathbf{a} + 2k\mathbf{c}$$
$$\frac{2h}{3}\mathbf{a} + \frac{5h}{3}\mathbf{c} = \mathbf{a} + 2k\mathbf{c}$$
$$(\frac{2h}{3} - 1)\mathbf{a} = (2k - \frac{5h}{3})\mathbf{c}$$

$$\frac{2h}{3} - 1 = 0$$
$$\frac{2h}{3} = 1$$
$$2h = 3$$
$$h = \frac{3}{2}$$

$$2k - \frac{5h}{3} = 0$$
$$2k = \frac{5h}{3}$$
$$k = \frac{5h}{6}$$
$$= \frac{5}{6} \times \frac{3}{2}$$
$$= \frac{5}{4}$$

Miscellaneous Exercise 3

1. (a) Graphically:



$$-4 \le x \le 2$$

Algebraically:
First solve $|2x-1| = |x-5|$

$2x - 1 = x - 5$ or $-(2x-1) = x - 5$

$x = -4$ $\quad$ $-2x + 1 = x - 5$

$-3x = -4$

$x = 2$

Now test one of the three intervals delimited by these two solutions. Try a value, say $x = 0$:
Is it true that $|5(0) - 1| \le |(0) - 5|$?
Yes $(1 \le 5)$.

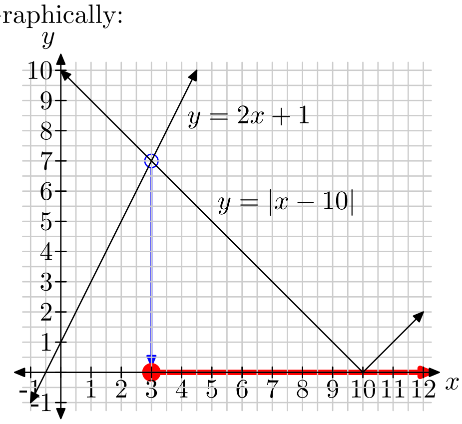
Solution set is

$$\{x \in \mathbb{R} : -4 \le x \le 2\}$$

(b) This is the complementary case to the previous question, so it has the complementary solution:

$$\{x \in \mathbb{R} : x < -4\} \cup \{x \in \mathbb{R} : x > 2\}$$

(c) Graphically:



$$x \geq 3$$

Algebraically:
First solve $|x - 10| = 2x + 1$

$$x - 10 = 2x + 1 \quad \text{or} \quad -(x - 10) = 2x + 1$$
$$x = -11 \qquad\qquad -x + 10 = 2x + 1$$
$$-3x = -9$$
$$x = 3$$

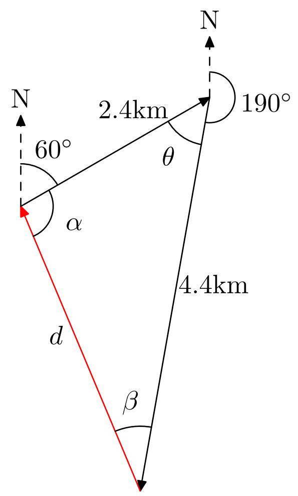
However, $x = -11$ is not actually a solution, as you can see by substituting into the equation, so we are left with two intervals (either side of $x = 3$).

Now test one of these intervals delimited by these two solutions. Try a value, say $x = 0$: Is it true that $|(0) - 10| \leq 2(0) + 1$?
No ($10 \not\leq 1$).

Solution set is

$$\{x \in \mathbb{R} : x \geq 3\}$$

2.

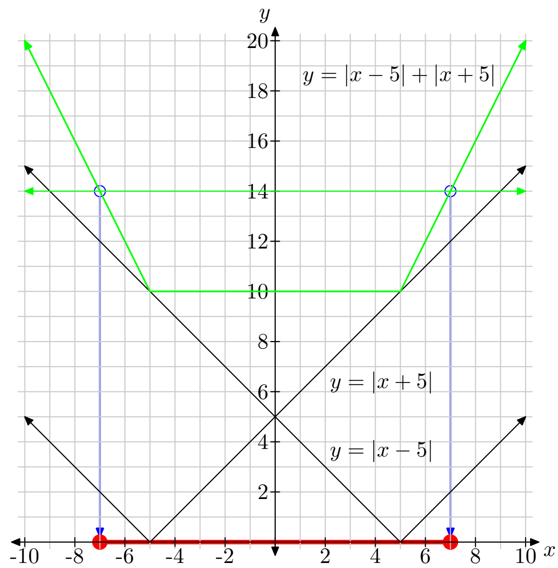


$$\theta = 369 - 190 - (180 - 60)$$
$$= 50°$$
$$d = \sqrt{2.4^2 + 4.4^2 - 2 \times 2.4 \times 4.4 \cos 50°}$$
$$= 3.4\text{km}$$

It's tempting to find angle α using the sine rule, but because it's opposite the longest side of the triangle, it could be either acute or obtuse: it's the ambiguous case. Finding β instead is unambiguous. β can not be obtuse because it is opposite a shorter side.

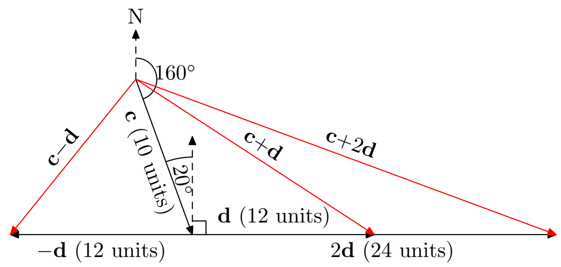
$$\frac{\sin\beta}{2.4} = \frac{\sin 50°}{3.4}$$
$$\beta = \sin^{-1}\frac{2.4\sin 50°}{3.4}$$
$$= 33°$$
$$\text{bearing} = 190 + (180 - 33)$$
$$= 327°$$

3.



$$|x - 5| + |x + 5| \leq 14 \text{ for } \{x \in \mathbb{R} : -7 \leq x \leq 7\}$$

4.



In each case below, let θ be the angle formed between $\mathbf{c}$ and the resultant.

(a) $|\mathbf{c} + \mathbf{d}| = \sqrt{10^2 + 12^2 - 2 \times 10 \times 12 \cos 110°}$
$= 18.1$ units

$$\frac{\sin\theta}{12} = \frac{\sin 110°}{18.1}$$
$$\theta = \sin^{-1}\frac{12\sin 110°}{18.1}$$
$$= 39°$$
$$\text{direction} = 160 - 39$$
$$= 121°$$

(b) $$\begin{aligned}|\mathbf{c}-\mathbf{d}| &= \sqrt{10^2+12^2-2\times 10\times 12\cos 70^\circ}\\ &= 12.7\,\text{units}\end{aligned}$$

$$\begin{aligned}\frac{\sin\theta}{12} &= \frac{\sin 70^\circ}{12.7}\\ \theta &= \sin^{-1}\frac{12\sin 70^\circ}{12.7}\\ &= 62^\circ\\ \text{direction} &= 160+62\\ &= 222^\circ\end{aligned}$$

(c) $$\begin{aligned}|\mathbf{c}+2\mathbf{d}| &= \sqrt{10^2+24^2-2\times 10\times 24\cos 110^\circ}\\ &= 29.0\,\text{units}\end{aligned}$$

$$\begin{aligned}\frac{\sin\theta}{24} &= \frac{\sin 110^\circ}{29.0}\\ \theta &= \sin^{-1}\frac{24\sin 110^\circ}{29.0}\\ &= 51^\circ\\ \text{direction} &= 160-51\\ &= 109^\circ\end{aligned}$$

5. First, rearrange the equation to

$$|x-a|+|x+3|=5$$

and read this as "distance from a plus distance from −3 is equal to 5".

- If the distance between a and -3 is greater than 5 then the equation has no solution.
- If the distance between a and -3 is equal to 5 then every point between a and -3 is a solution.
- If the distance between a and -3 is less than 5 then there will be two solutions, one lying above the interval between -3 and a and one lying below it.

(a) For exactly two solutions,

$$\begin{aligned}|a+3| &< 5\\ -5 < a+3 &< 5\\ -8 < a &< 2\end{aligned}$$

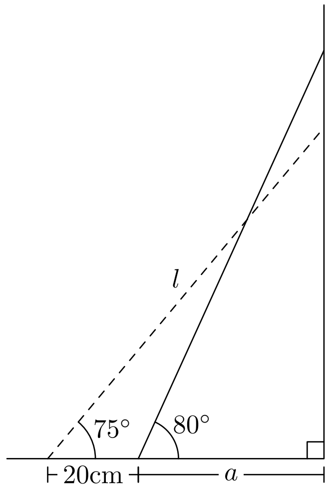
(b) For more than two solutions,

$$|a+3| = 5$$

$$a+3=5 \quad\text{or}\quad a+3=-5$$

$$a=2 \qquad\qquad a=-8$$

6. Let l be the length of the ladder.



$$\begin{aligned}\cos 80^\circ &= \frac{a}{l}\\ a &= l\cos 80^\circ\\ \cos 75^\circ &= \frac{a+20}{l}\\ a+20 &= l\cos 75^\circ\\ a &= l\cos(75^\circ)-20\\ l\cos 80^\circ &= l\cos(75^\circ)-20\\ l\cos(75^\circ)-l\cos 80^\circ &= 20\\ l(\cos(75^\circ)-\cos 80^\circ) &= 20\\ l &= \frac{20}{\cos(75^\circ)-\cos 80^\circ}\\ &= 235\text{cm}\\ a &= l\cos 80^\circ\\ &= 41\text{cm}\end{aligned}$$

7. (a) $h=k=0$

(b) $$\begin{aligned}h\mathbf{a}+\mathbf{b} &= k\mathbf{b}\\ h\mathbf{a} &= k\mathbf{b}-\mathbf{b}\\ &= (k-1)\mathbf{b}\end{aligned}$$

$$h=0 \qquad k-1=0$$

$$k=1$$

(c) $$(h-3)\mathbf{a}=(k+1)\mathbf{b}$$

$$h-3=0 \qquad k+1=0$$

$$h=3 \qquad k=-1$$

(d) $$\begin{aligned}h\mathbf{a}+2\mathbf{a} &= k\mathbf{b}-3\mathbf{a}\\ h\mathbf{a}+5\mathbf{a} &= k\mathbf{b}\\ (h+5)\mathbf{a} &= k\mathbf{b}\end{aligned}$$

$$h+5=0 \qquad k=0$$

$$h=-5$$

(e) $$\begin{aligned}3h\mathbf{a}+k\mathbf{a}+h\mathbf{b}-2k\mathbf{b} &= \mathbf{a}+5\mathbf{b}\\ 3h\mathbf{a}+k\mathbf{a}-\mathbf{a} &= 5\mathbf{b}-h\mathbf{b}+2k\mathbf{b}\\ (3h+k-1)\mathbf{a} &= (5-h+2k)\mathbf{b}\end{aligned}$$

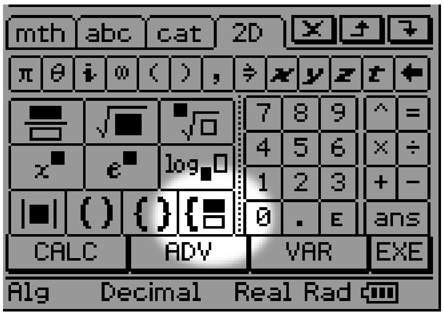
$$3h+k-1=0 \qquad 5-h+2k=0$$

$$3h+k=1 \qquad h-2k=5$$

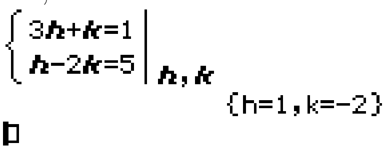
$$h=1$$

$$k=-2$$

(Note: the final step in the solution above is done by solving the simultaneous equations $3h + k = 1$ and $h - 2k = 5$. You should be familiar with doing this by elimination or substitution. (Either would be suitable here.) You should also know how to do it on the ClassPad:



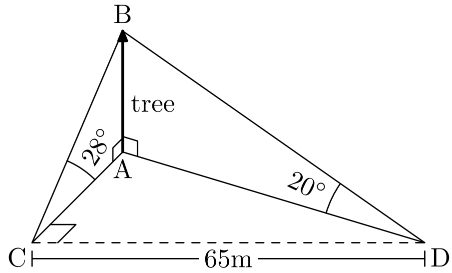
In the Main application, select the simultaneous equations icon in the 2D tab. Enter the two equations to the left of the vertical bar, and the two variables to the right:



(f) $$h(\mathbf{a}+\mathbf{b}) + k(\mathbf{a}-\mathbf{b}) = 3\mathbf{a} + 5\mathbf{b}$$
$$(h+k)\mathbf{a} + (h-k)\mathbf{b} = 3\mathbf{a} + 5\mathbf{b}$$
$$(h+k-3)\mathbf{a} = -(h-k-5)\mathbf{b}$$
$$h+k-3 = 0 \qquad h-k-5 = 0$$
$$h+k = 3 \qquad h-k = 5$$

solving by elimination:

$$2h = 8$$
$$h = 4$$
$$4 + k = 3$$
$$k = -1$$

8. 

Let the height of the tree be h. Let A be the point at the base of the tree and B the point at the apex.

$$\tan 28^\circ = \frac{h}{\text{AC}}$$
$$\text{AC} = \frac{h}{\tan 28^\circ}$$
$$\tan 20^\circ = \frac{h}{\text{AD}}$$
$$\text{AD} = \frac{h}{\tan 20^\circ}$$

$\triangle$ACD is right-angled at C, so

$$\text{AD}^2 = \text{AC}^2 + \text{CD}^2$$
$$\frac{h^2}{\tan^2 20^\circ} = \frac{h^2}{\tan^2 28^\circ} + 65^2$$
$$h^2\left(\frac{1}{\tan^2 20^\circ} - \frac{1}{\tan^2 28^\circ}\right) = 65^2$$

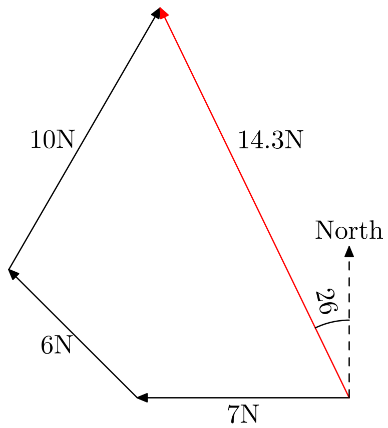
Solving this and discarding the negative root:

$$h = 32.5\text{m}$$
$$\text{AC} = \frac{h}{\tan 28^\circ}$$
$$= 61.0\text{m}$$

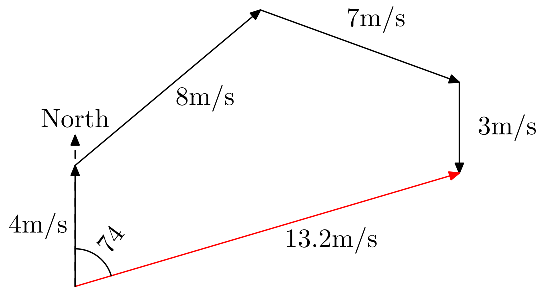
Chapter 4

Exercise 4A

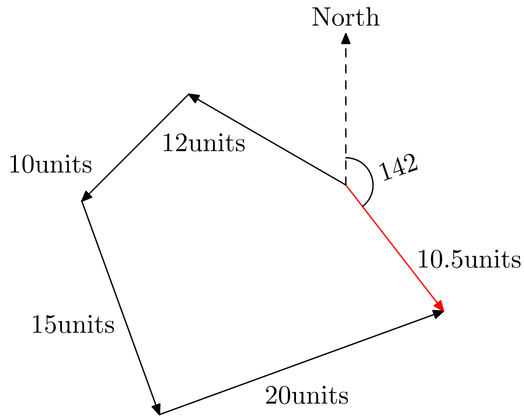
1.



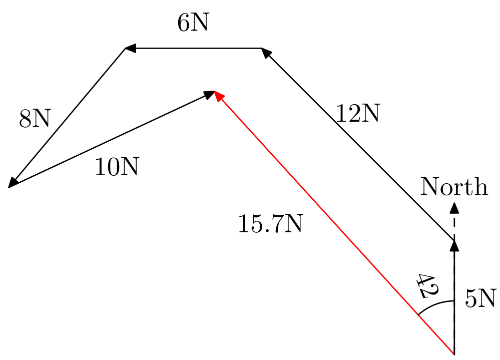
2.



3.



4.



5. $\mathbf{a} = 3\mathbf{i} + 2\mathbf{j}$;
$\mathbf{b} = 3\mathbf{i} + 1\mathbf{j} = 3\mathbf{i} + \mathbf{j}$;
$\mathbf{c} = 2\mathbf{i} + 2\mathbf{j}$;
$\mathbf{d} = -1\mathbf{i} + 3\mathbf{j} = -\mathbf{i} + 3\mathbf{j}$;
$\mathbf{e} = 0\mathbf{i} + 2\mathbf{j} = 2\mathbf{j}$;
$\mathbf{f} = -1\mathbf{i} + 2\mathbf{j} = -\mathbf{i} + 2\mathbf{j}$;
$\mathbf{g} = 1\mathbf{i} - 2\mathbf{j} = \mathbf{i} - 2\mathbf{j}$;
$\mathbf{h} = 4\mathbf{i} + 0\mathbf{j} = 4\mathbf{i}$;
$\mathbf{k} = 2\mathbf{i} - 4\mathbf{j}$;
$\mathbf{l} = 4\mathbf{i} - 1\mathbf{j} = 4\mathbf{i} - \mathbf{j}$;
$\mathbf{m} = -4\mathbf{i} - 1\mathbf{j} = -4\mathbf{i} - \mathbf{j}$;
$\mathbf{n} = 9\mathbf{i} + 2\mathbf{j}$;

6. $|\mathbf{a}| = \sqrt{3^2 + 2^2} = \sqrt{13}$;
$|\mathbf{b}| = \sqrt{3^2 + 1^2} = \sqrt{10}$;
$|\mathbf{c}| = \sqrt{2^2 + 2^2} = 2\sqrt{2}$;
$|\mathbf{d}| = \sqrt{1^2 + 3^2} = \sqrt{10}$;
$|\mathbf{e}| = 2$;
$|\mathbf{f}| = \sqrt{1^2 + 2^2} = \sqrt{5}$;
$|\mathbf{g}| = \sqrt{1^2 + 2^2} = \sqrt{5}$;
$|\mathbf{h}| = 4$;
$|\mathbf{k}| = \sqrt{2^2 + 4^2} = 2\sqrt{5}$;
$|\mathbf{l}| = \sqrt{4^2 + 1^2} = \sqrt{17}$;
$|\mathbf{m}| = \sqrt{4^2 + 1^2} = \sqrt{17}$;
$|\mathbf{n}| = \sqrt{9^2 + 2^2} = \sqrt{85}$;

7. $|(-7\mathbf{i} + 24\mathbf{j})| = \sqrt{7^2 + 24^2} = 25\text{Newtons}$

8. (a) $(5\cos(30°)\mathbf{i} + 5\sin(30°)\mathbf{j})\text{units} \approx (4.3\mathbf{i} + 2.5\mathbf{j})\text{units}$

(b) $(7\cos(60°)\mathbf{i} + 7\sin(60°)\mathbf{j})\text{units} \approx (3.5\mathbf{i} + 6.1\mathbf{j})\text{units}$

(c) $(10\cos(25°)\mathbf{i} + 10\sin(25°)\mathbf{j})\text{units} \approx (9.1\mathbf{i} + 4.2\mathbf{j})\text{units}$

(d) $(7\sin(50°)\mathbf{i} + 7\cos(50°)\mathbf{j})\text{N} \approx (5.4\mathbf{i} + 4.5\mathbf{j})\text{N}$

(e) $(5 - 8\cos(60°)\mathbf{i} + 8\sin(60°)\mathbf{j})\text{m/s} \approx (-4.0\mathbf{i} + 6.9\mathbf{j})\text{m/s}$

(f) $(10\cos(20°)\mathbf{i} - 10\sin(20°)\mathbf{j})\text{N} \approx (9.4\mathbf{i} - 3.4\mathbf{j})\text{N}$

(g) $(-4\cos(50°)\mathbf{i} + 4\sin(50°)\mathbf{j})\text{units} \approx (-2.6\mathbf{i} + 3.1\mathbf{j})\text{units}$

(h) $(8\cos(24°)\mathbf{i} - 8\sin(24°)\mathbf{j})\text{units} \approx (7.3\mathbf{i} - 3.3\mathbf{j})\text{units}$

(i) $(-6\sin(50°)\mathbf{i} - 6\cos(50°)\mathbf{j})\text{units} \approx (-4.6\mathbf{i} - 3.9\mathbf{j})\text{units}$

(j) $(-10\cos(50°)\mathbf{i} + 10\sin(50°)\mathbf{j})\text{m/s} \approx (-6.4\mathbf{i} + 7.7\mathbf{j})\text{m/s}$

(k) $(-8\cos(25°)\mathbf{i} - 8\sin(25°)\mathbf{j})\text{N} \approx (-7.3\mathbf{i} - 3.4\mathbf{j})\text{N}$

(l) $(5\cos(35°)\mathbf{i} + 5\sin(35°)\mathbf{j})\text{m/s} \approx (4.1\mathbf{i} + 2.9\mathbf{j})\text{m/s}$

9. (a) $|\mathbf{a}| = \sqrt{3^2 + 4^2} = 5$

$$\theta = \tan^{-1}\frac{4}{3} \approx 53.1°$$

(b) $|\mathbf{b}| = \sqrt{5^2 + 2^2} = \sqrt{29}$

$$\theta = \tan^{-1}\frac{2}{5} \approx 21.8^\circ$$

(c) $|\mathbf{c}| = \sqrt{2^2 + 3^2} = \sqrt{13}$

$$\theta = 180^\circ - \tan^{-1}\frac{3}{2} \approx 123.7^\circ$$

(d) $|\mathbf{d}| = \sqrt{4^2 + 3^2} = 5$

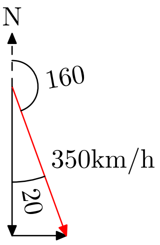
$$\theta = \tan^{-1}\frac{4}{3} \approx 53.1^\circ$$

(e) $|\mathbf{e}| = \sqrt{5^2 + 4^2} = \sqrt{41}$

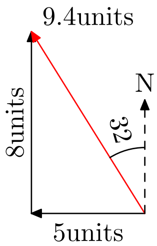
$$\theta = \tan^{-1}\frac{4}{5} \approx 38.7^\circ$$

(f) $|\mathbf{f}| = \sqrt{4^2 + 4^2} = 4\sqrt{2}$

$$\theta = \tan^{-1}\frac{4}{4} = 45.0^\circ$$

10. 

Northerly component $= 350\cos 160^\circ$ (or $-350\cos 20^\circ$) $= -328.9$km/h
Easterly component $= 350\sin 160^\circ$ (or $350\sin 20^\circ$) $= 119.7$km/h

11. 

Magnitude$= \sqrt{5^2 + 8^2} = \sqrt{89} \approx 9.4$units
Direction$= 360 - \tan^{-1}\frac{5}{8} \approx 360 - 32.0 = 328^\circ$T

12. (a) $\mathbf{a}+\mathbf{b} = 2\mathbf{i}+3\mathbf{j}+\mathbf{i}+4\mathbf{j} = (2+1)\mathbf{i}+(3+4)\mathbf{j} = 3\mathbf{i} + 7\mathbf{j}$
(b) $\mathbf{a} - \mathbf{b} = (2-1)\mathbf{i} + (3-4)\mathbf{j} = \mathbf{i} - \mathbf{j}$
(c) $\mathbf{b} - \mathbf{a} = (1-2)\mathbf{i} + (4-3)\mathbf{j} = -\mathbf{i} + \mathbf{j}$
(d) $2\mathbf{a} = 2(2\mathbf{i}) + 2(3\mathbf{j}) = 4\mathbf{i} + 6\mathbf{j}$
(e) $3\mathbf{b} = 3(\mathbf{i}) + 3(4\mathbf{j}) = 3\mathbf{i} + 12\mathbf{j}$
(f) $2\mathbf{a}+3\mathbf{b} = (2\times2+3\times1)\mathbf{i}+(2\times3+3\times4)\mathbf{j} = 7\mathbf{i} + 18\mathbf{j}$
(g) $2\mathbf{a} - 3\mathbf{b} = (4-3)\mathbf{i} + (6-12)\mathbf{j} = \mathbf{i} - 6\mathbf{j}$
(h) $-2\mathbf{a}+3\mathbf{b} = (-4+3)\mathbf{i}+(-6+12)\mathbf{j} = -\mathbf{i}+6\mathbf{j}$
(i) $|\mathbf{a}| = \sqrt{2^2 + 3^2} = \sqrt{13} \approx 3.61$
(j) $|\mathbf{b}| = \sqrt{1^2 + 4^2} = \sqrt{17} \approx 4.12$
(k) $|\mathbf{a}| + |\mathbf{b}| = \sqrt{13} + \sqrt{17} \approx 7.73$
(l) $|\mathbf{a}+\mathbf{b}| = |3\mathbf{i}+7\mathbf{j}| = \sqrt{3^2 + 7^2} = \sqrt{58} \approx 7.62$

13. (a) $2\mathbf{c} + \mathbf{d} = (2+2)\mathbf{i} + (-2+1)\mathbf{j} = 4\mathbf{i} - \mathbf{j}$
(b) $\mathbf{c} - \mathbf{d} = (1-2)\mathbf{i} + (-1-1)\mathbf{j} = -\mathbf{i} - 2\mathbf{j}$
(c) $\mathbf{d} - \mathbf{c} = \mathbf{i} + 2\mathbf{j}$
(d) $5\mathbf{c} = 5\mathbf{i} - 5\mathbf{j}$
(e) $5\mathbf{c} + \mathbf{d} = (5+2)\mathbf{i} + (-5+1)\mathbf{j} = 7\mathbf{i} - 4\mathbf{j}$
(f) $5\mathbf{c} + 2\mathbf{d} = (5+4)\mathbf{i} + (-5+2)\mathbf{j} = 9\mathbf{i} - 3\mathbf{j}$
(g) $2\mathbf{c} + 5\mathbf{d} = (2+10)\mathbf{i} + (-2+5)\mathbf{j} = 12\mathbf{i} + 3\mathbf{j}$
(h) $2\mathbf{c} - \mathbf{d} = (2-2)\mathbf{i} + (-2-1)\mathbf{j} = -3\mathbf{j}$
(i) $|\mathbf{d} - 2\mathbf{c}| = |(2-2)\mathbf{i} + (1- -2)\mathbf{j}| = |3\mathbf{j}| = 3$
(j) $|\mathbf{c}|+|\mathbf{d}| = \sqrt{1^2+1^2}+\sqrt{2^2+1^2} = \sqrt{2}+\sqrt{5} \approx 3.65$
(k) $|\mathbf{c} + \mathbf{d}| = |(1+2)\mathbf{i} + (-1+1)\mathbf{j}| = |3\mathbf{i}| = 3$
(l) $|\mathbf{c}-\mathbf{d}| = |(1-2)\mathbf{i}+(-1-1)\mathbf{j}| = |-1\mathbf{i}-2\mathbf{j}| = \sqrt{5} \approx 2.24$

14. (a) $\mathbf{a} + \mathbf{b} = \langle 5+2, 4+-3\rangle = \langle 7, 1\rangle$
(b) $\mathbf{a} + -\mathbf{b} = \langle 5-2, 4- -3\rangle = \langle 3, 7\rangle$
(c) $2\mathbf{a} = 2\langle 5, 4\rangle = \langle 10, 8\rangle$
(d) $3\mathbf{a} + \mathbf{b} = \langle 3\times5+2, 3\times4+-3\rangle = \langle 17, 9\rangle$
(e) $2\mathbf{b} - \mathbf{a} = \langle 4-5, -6-4\rangle = \langle -1, -10\rangle$
(f) $|\mathbf{a}| = \sqrt{5^2+4^2} = \sqrt{41} \approx 6.40$
(g) $|\mathbf{a}+\mathbf{b}| = \sqrt{7^2+1^2} = \sqrt{50} = 5\sqrt{2} \approx 7.07$
(h) $|\mathbf{a}| + |\mathbf{b}| = \sqrt{41} + \sqrt{2^2+3^2} = \sqrt{41} + \sqrt{13} \approx 10.01$

15. (a) $\begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

(b) $\begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$

(c) $\begin{pmatrix} -1 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} -4 \\ -4 \end{pmatrix}$

(d) $$2\begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$$

(e) $$\begin{pmatrix} 3 \\ 4 \end{pmatrix} + 2\begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} -2 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

(f) $$\begin{pmatrix} 3 \\ 4 \end{pmatrix} - 2\begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} -2 \\ 0 \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$$

(g) $$\left|\begin{pmatrix} 3 \\ 4 \end{pmatrix} - 2\begin{pmatrix} -1 \\ 0 \end{pmatrix}\right| = \left|\begin{pmatrix} 5 \\ 4 \end{pmatrix}\right| = \sqrt{5^2+4^2} = \sqrt{41} \approx 6.40$$

(h) $$\left|2\begin{pmatrix} -1 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ 4 \end{pmatrix}\right| = \left|\begin{pmatrix} -5 \\ -4 \end{pmatrix}\right| = \sqrt{5^2+4^2} = \sqrt{41} \approx 6.40$$

16. (a) $\left|\begin{pmatrix} 2 \\ 7 \end{pmatrix}\right| = \sqrt{2^2+7^2} = \sqrt{53}$

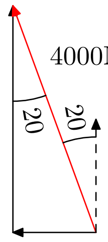
(b) $\left|\begin{pmatrix} -2 \\ 3 \end{pmatrix}\right| = \sqrt{2^2+3^2} = \sqrt{13}$

(c) $\left|2\begin{pmatrix} 2 \\ 7 \end{pmatrix}\right| = \sqrt{4^2+14^2} = \sqrt{212} = 2\sqrt{53}$

(d) $\left|\begin{pmatrix}2\\7\end{pmatrix}+\begin{pmatrix}-2\\3\end{pmatrix}\right|=\left|\begin{pmatrix}0\\10\end{pmatrix}\right|=10$

(e) $\left|\begin{pmatrix}2\\7\end{pmatrix}-\begin{pmatrix}-2\\3\end{pmatrix}\right|=\left|\begin{pmatrix}4\\4\end{pmatrix}\right|$

$$=\sqrt{4^2+4^2}$$
$$=\sqrt{32}$$
$$=4\sqrt{2}$$

17. 

lift$=4000\cos 20° = 3759$N
drag$=4000\sin 20° = 1368$N

18. $(12\cos 50°\mathbf{i}+12\sin 50°\mathbf{j})+10\mathbf{i}\approx(17.7\mathbf{i}+9.2\mathbf{j})$N

19. $(-12\cos 50°\mathbf{i}+12\sin 50°\mathbf{j})+10\mathbf{i}\approx(2.3\mathbf{i}+9.2\mathbf{j})$N

20. $\begin{pmatrix}-8\sin 40°\\8\cos 40°\end{pmatrix}+\begin{pmatrix}5\cos 30°\\5\sin 30°\end{pmatrix}+\begin{pmatrix}10\\0\end{pmatrix}$

$$=\begin{pmatrix}-8\sin 40°+5\cos 30°+10\\8\cos 40°+5\sin 30°\end{pmatrix}$$
$$\approx 9.2\mathbf{i}+8.6\mathbf{j}\text{N}$$

21. $\begin{pmatrix}0+10\cos 30°-8\sin 20°\\6+10\sin 30°-8\cos 20°\end{pmatrix}$

$$=5.9\mathbf{i}+3.5\mathbf{j}\text{m/s}$$

22.
$$\begin{aligned}&0\mathbf{i}+5\mathbf{j}\\+\ &10\cos 30°\mathbf{i}+10\sin 30°\mathbf{j}\\+\ &4\mathbf{i}+0\mathbf{j}\\+\ &7\cos 60°\mathbf{i}-7\sin 60°\mathbf{j}\\\approx\ &(16.2\mathbf{i}+3.9\mathbf{j})\text{N}\end{aligned}$$

23.
$$\begin{aligned}&-10\sin 40°\mathbf{i}+10\cos 40°\mathbf{j}\\+\ &10\cos 30°\mathbf{i}+10\sin 30°\mathbf{j}\\+\ &10\cos 10°\mathbf{i}-10\sin 10°\mathbf{j}\\+\ &-10\sin 10°\mathbf{i}-10\cos 10°\mathbf{j}\\\approx\ &(10.3\mathbf{i}+1.1\mathbf{j})\text{N}\end{aligned}$$

24.
$$\begin{aligned}\mathbf{F}_1+\mathbf{F}_2+\mathbf{F}_3&=(2+4+2)\mathbf{i}+(3+3-4)\mathbf{j}\\&=(8\mathbf{i}+2\mathbf{j})\text{N}\\|\mathbf{F}_1+\mathbf{F}_2+\mathbf{F}_3|&=|8\mathbf{i}+2\mathbf{j}|\\&=\sqrt{8^2+2^2}\\&=2\sqrt{17}\text{N}\end{aligned}$$

25.
$$\begin{aligned}(\mathbf{a}+\mathbf{b})+(\mathbf{a}-\mathbf{b})&=(3\mathbf{i}+\mathbf{j})+(\mathbf{i}-7\mathbf{j})\\2\mathbf{a}&=4\mathbf{i}-6\mathbf{j}\\\mathbf{a}&=2\mathbf{i}-3\mathbf{j}\end{aligned}$$

$$\begin{aligned}(\mathbf{a}+\mathbf{b})-(\mathbf{a}-\mathbf{b})&=(3\mathbf{i}+\mathbf{j})-(\mathbf{i}-7\mathbf{j})\\2\mathbf{b}&=2\mathbf{i}+8\mathbf{j}\\\mathbf{b}&=\mathbf{i}+4\mathbf{j}\end{aligned}$$

26.
$$\begin{aligned}2(2\mathbf{c}+\mathbf{d})-2(\mathbf{c}+\mathbf{d})&=2(-\mathbf{i}+6\mathbf{j})-(2\mathbf{i}-10\mathbf{j})\\2\mathbf{c}&=-4\mathbf{i}+22\mathbf{j}\\\mathbf{c}&=-2\mathbf{i}+11\mathbf{j}\end{aligned}$$

$$\begin{aligned}(2\mathbf{c}+\mathbf{d})-2(\mathbf{c}+\mathbf{d})&=(-\mathbf{i}+6\mathbf{j})-(2\mathbf{i}-10\mathbf{j})\\-\mathbf{d}&=-3\mathbf{i}+16\mathbf{j}\\\mathbf{d}&=3\mathbf{i}-16\mathbf{j}\end{aligned}$$

Exercise 4B

1.
- (a) $\mathbf{a}=4\mathbf{i}+3\mathbf{j}$
  (b) $2\mathbf{a}=8\mathbf{i}+6\mathbf{j}$
  (c) $\frac{\mathbf{a}}{|\mathbf{a}|}=\frac{4\mathbf{i}+3\mathbf{j}}{5}=0.8\mathbf{i}+0.6\mathbf{j}$
  (d) $2\frac{\mathbf{a}}{|\mathbf{a}|}=2(0.8\mathbf{i}+0.6\mathbf{j})=1.6\mathbf{i}+1.2\mathbf{j}$
- (a) $\mathbf{b}=4\mathbf{i}-3\mathbf{j}$
  (b) $2\mathbf{b}=8\mathbf{i}-6\mathbf{j}$
  (c) $\frac{\mathbf{b}}{|\mathbf{b}|}=\frac{4\mathbf{i}-3\mathbf{j}}{5}=0.8\mathbf{i}-0.6\mathbf{j}$
  (d) $2\frac{\mathbf{b}}{|\mathbf{b}|}=2(0.8\mathbf{i}-0.6\mathbf{j})=1.6\mathbf{i}-1.2\mathbf{j}$
- (a) $\mathbf{c}=2\mathbf{i}+2\mathbf{j}$
  (b) $2\mathbf{c}=4\mathbf{i}+4\mathbf{j}$
  (c) $\frac{\mathbf{c}}{|\mathbf{c}|}=\frac{2\mathbf{i}+2\mathbf{j}}{2\sqrt{2}}=\frac{1}{\sqrt{2}}\mathbf{i}+\frac{1}{\sqrt{2}}\mathbf{j}$
  (d) $2\frac{\mathbf{c}}{|\mathbf{c}|}=2(\frac{1}{\sqrt{2}}\mathbf{i}+\frac{1}{\sqrt{2}}\mathbf{j})=\sqrt{2}\mathbf{i}+\sqrt{2}\mathbf{j}$
- (a) $\mathbf{d}=3\mathbf{i}-2\mathbf{j}$
  (b) $2\mathbf{d}=6\mathbf{i}-4\mathbf{j}$
  (c) $\frac{\mathbf{d}}{|\mathbf{d}|}=\frac{3\mathbf{i}-2\mathbf{j}}{\sqrt{13}}=\frac{3}{\sqrt{13}}\mathbf{i}-\frac{2}{\sqrt{13}}\mathbf{j}$
  (d) $2\frac{\mathbf{d}}{|\mathbf{d}|}=2(\frac{3}{\sqrt{13}}\mathbf{i}-\frac{2}{\sqrt{13}}\mathbf{j})=\frac{6}{\sqrt{13}}\mathbf{i}-\frac{4}{\sqrt{13}}\mathbf{j}$

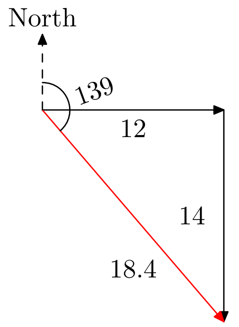
2. (a) $\frac{\mathbf{b}}{|\mathbf{b}|}=\frac{2\mathbf{i}+\mathbf{j}}{\sqrt{5}}=\frac{2}{\sqrt{5}}\mathbf{i}+\frac{1}{\sqrt{5}}\mathbf{j}$
   (b) $|\mathbf{a}|\frac{\mathbf{b}}{|\mathbf{b}|}=5(\frac{2}{\sqrt{5}}\mathbf{i}+\frac{1}{\sqrt{5}}\mathbf{j})=2\sqrt{5}\mathbf{i}+\sqrt{5}\mathbf{j}$
   (c) $|\mathbf{c}|\frac{\mathbf{a}}{|\mathbf{a}|}=\sqrt{13}\frac{-3\mathbf{i}+4\mathbf{j}}{5}=-\frac{3\sqrt{13}}{5}\mathbf{i}+\frac{4\sqrt{13}}{5}\mathbf{j}$
   (d) $\mathbf{a}+\mathbf{b}+\mathbf{c}=2\mathbf{i}+3\mathbf{j}$
   $|\mathbf{a}+\mathbf{b}+\mathbf{c}|=\sqrt{13}$
   $|\mathbf{a}|=5$
   $|\mathbf{a}|\frac{\mathbf{a}+\mathbf{b}+\mathbf{c}}{|\mathbf{a}+\mathbf{b}+\mathbf{c}|}=5\frac{2\mathbf{i}+3\mathbf{j}}{\sqrt{13}}=\frac{10}{\sqrt{13}}\mathbf{i}+\frac{15}{\sqrt{13}}\mathbf{j}$

3. (a) $\mathbf{a}$ and $\mathbf{d}$ are parallel since $\mathbf{a}=2\mathbf{d}$.

(b) $\mathbf{a}+\mathbf{b}+\mathbf{c}+\mathbf{d}+\mathbf{e}$
$= (2+4+1+1+4)\mathbf{i}+(-4+2-8-2-2)\mathbf{j}$
$= 12\mathbf{i}-14\mathbf{j}$

(c) $|\mathbf{a}+\mathbf{b}+\mathbf{c}+\mathbf{d}+\mathbf{e}|$
$= \sqrt{12^2+14^2} = \sqrt{340}$
$= 2\sqrt{85}$

(d)



bearing=$90^\circ + \tan^{-1}\frac{14}{12} \approx 139^\circ$

4. • **a** is of magnitude 5 units and w is negative.

$$\begin{aligned}|\mathbf{a}| &= 5\\ |w\mathbf{i}+3\mathbf{j}| &= 5\\ \sqrt{w^2+3^2} &= 5\\ w^2+9 &= 25\\ w^2 &= 16\\ w &= -4\end{aligned}$$

• **b** is parallel to **a**

$$\begin{aligned}\mathbf{b} &= k\mathbf{a}\\ -\mathbf{i}+x\mathbf{j} &= k(w\mathbf{i}+3\mathbf{j})\\ -\mathbf{i}+x\mathbf{j} &= k(-4\mathbf{i}+3\mathbf{j})\\ -\mathbf{i}+x\mathbf{j} &= -4k\mathbf{i}+3k\mathbf{j}\\ (-1+4k)\mathbf{i} &= (3k-x)\mathbf{j}\\ -1+4k &= 0\\ k &= \frac{1}{4}\\ 3k-x &= 0\\ x &= \frac{3}{4}\end{aligned}$$

• **c** is a unit vector

$$\begin{aligned}|\mathbf{c}| &= 1\\ |0.5\mathbf{i}+y\mathbf{j}| &= 1\\ \sqrt{0.5^2+y^2} &= 1\\ 0.25+y^2 &= 1\\ y^2 &= \frac{3}{4}\\ y &= \pm\frac{\sqrt{3}}{2}\end{aligned}$$

• the resultant of **a** and **d** has a magnitude of 13 units

$$\begin{aligned}|\mathbf{a}+\mathbf{d}| &= 13\\ |(w-1)\mathbf{i}+(3-z)\mathbf{j}| &= 13\\ |(-4-1)\mathbf{i}+(3-z)\mathbf{j}| &= 13\\ \sqrt{5^2+(3-z)^2} &= 13\\ 25+(9-6z+z^2) &= 169\\ z^2-6z+9+25-169 &= 0\\ z^2-6z-135 &= 0\\ (z-15)(z+9) &= 0\\ z &= 15\\ \text{or } z &= -9\end{aligned}$$

$w=-4$; $x=\frac{3}{4}$; $y=\pm\frac{\sqrt{3}}{2}$; $z=15$ or 9.

5. • **p** is a unit vector and a is positive

$$\begin{aligned}|0.6\mathbf{i}-a\mathbf{j} &= 1\\ 0.6^2+a^2 &= 1^2\\ a &= 0.8\end{aligned}$$

• **q** is in the same direction as **p** and five times the magnitude.

$$\begin{aligned}\mathbf{q} &= 5\mathbf{p}\\ b\mathbf{i}+c\mathbf{i} &= 5(0.6\mathbf{i}-0.8\mathbf{j})\\ &= 3\mathbf{i}-4\mathbf{j}\\ b &= 3\\ c &= -4\end{aligned}$$

•

$$\begin{aligned}\mathbf{r}+2\mathbf{q} &= 11\mathbf{i}-20\mathbf{j}\\ (d\mathbf{i}+e\mathbf{j})+2(3\mathbf{i}-4\mathbf{j}) &= 11\mathbf{i}-20\mathbf{j}\\ (d+6)\mathbf{i}+(e-8)\mathbf{j} &= 11\mathbf{i}-20\mathbf{j}\\ d+6 &= 11\\ d &= 5\\ e-8 &= -20\\ e &= -12\end{aligned}$$

• **s** is in the same direction as **r** but equal in magnitude to **q**

$$\begin{aligned}\mathbf{s} &= |\mathbf{q}|\frac{\mathbf{r}}{|\mathbf{r}|}\\ f\mathbf{i}+g\mathbf{j} &= 5\frac{5\mathbf{i}-12\mathbf{j}}{\sqrt{5^2+12^2}}\\ &= \frac{25}{13}\mathbf{i}-\frac{60}{13}\mathbf{j}\\ f &= \frac{25}{13}\\ g &= -\frac{60}{13}\end{aligned}$$

$a=0.8$, $b=3$, $c=-4$, $d=5$, $e=-12$, $f=\frac{25}{13}$ and $g=-\frac{60}{13}$

6. $$\begin{aligned}\mathbf{R} &= \mathbf{a}+\mathbf{b}+\mathbf{c}+\mathbf{d}\\ \mathbf{R} &= 7\cos 30^\circ\mathbf{i} \quad +7\sin 30^\circ\mathbf{j}\\ &\quad +0\mathbf{i} \quad +6\mathbf{j}\\ &\quad +10\cos 45^\circ\mathbf{i} \quad +10\sin 45^\circ\mathbf{j}\\ &\quad +4\cos 145^\circ\mathbf{i} \quad +4\sin 145^\circ\mathbf{j}\\ \mathbf{R} &= 9.9\mathbf{i}+18.9\mathbf{j}\\ |\mathbf{R}| &= \sqrt{9.9^2+18.9^2}\\ &= 21.3\\ \therefore \quad \mathbf{e} &= -9.9\mathbf{i}-18.9\mathbf{j}\end{aligned}$$

7. $P = |(-6\mathbf{i}+5\mathbf{j})| = \sqrt{6^2+5^2} \approx 7.8$
$\theta = \tan^{-1}\frac{6}{5} \approx 50^\circ$

8. Horizontal components:
$$P\sin\theta = 8\sin 50^\circ$$
Vertical components:
$$P\cos\theta = 8\cos 50^\circ + 5$$
Dividing gives:
$$\begin{aligned}\frac{P\sin\theta}{P\cos\theta} &= \frac{8\sin 50^\circ}{8\cos 50^\circ+5}\\ \tan\theta &= \frac{8\sin 50^\circ}{8\cos 50^\circ+5}\\ \theta &= \tan^{-1}\frac{8\sin 50^\circ}{8\cos 50^\circ+5}\\ &\approx 31^\circ\end{aligned}$$
Substituting:
$$\begin{aligned}P\sin\theta &= 8\sin 50^\circ\\ P &= \frac{8\sin 50^\circ}{\sin 31^\circ} \qquad \approx 11.9\end{aligned}$$

9. Horizontal components:
$$P\sin\theta = 12 - 10\sin 40^\circ$$
Vertical components:
$$P\cos\theta = 10\cos 40^\circ$$
Dividing gives:
$$\begin{aligned}\frac{P\sin\theta}{P\cos\theta} &= \frac{12-10\sin 40^\circ}{10\cos 40^\circ}\\ \tan\theta &= \frac{12-10\sin 40^\circ}{10\cos 40^\circ}\\ \theta &= \tan^{-1}\frac{12-10\sin 40^\circ}{10\cos 40^\circ}\\ &\approx 36^\circ\end{aligned}$$
Substituting:
$$\begin{aligned}P\cos\theta &= 10\cos 40^\circ\\ P &= \frac{10\cos 40^\circ}{\cos 36^\circ} \qquad \approx 9.5\end{aligned}$$

10. $$\begin{aligned}-T_1\sin 30^\circ + T_2\sin 30^\circ &= 0\\ \therefore \quad T_1 &= T_2\\ T_1\cos 30^\circ + T_2\cos 30^\circ &= 100\\ T_1\cos 30^\circ &= 50\\ T_1 &= \frac{50}{\cos 30^\circ}\\ &= \frac{100}{\sqrt{3}}\\ \therefore \quad T_1 = T_2 &= \frac{100}{\sqrt{3}}\text{N}\end{aligned}$$

11. $$\begin{aligned}-T_1\sin 30^\circ + T_2\sin 30^\circ &= 0\\ \therefore \quad T_1 &= T_2\\ T_1\cos 60^\circ + T_2\cos 60^\circ &= 100\\ T_1\cos 60^\circ &= 50\\ T_1 &= \frac{50}{\cos 60^\circ}\\ &= 100\\ \therefore \quad T_1 = T_2 &= 100\text{N}\end{aligned}$$

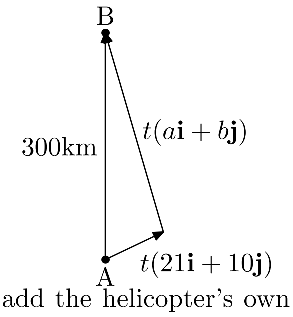
12. First the horizontal components:
$$\begin{aligned}-T_1\sin 30^\circ + T_2\sin 60^\circ &= 0\\ T_1\sin 30^\circ &= T_2\sin 60^\circ\\ \frac{1}{2}T_1 &= \frac{\sqrt{3}}{2}T_2\\ T_1 &= \sqrt{3}T_2\end{aligned}$$
Now the vertical components:
$$\begin{aligned}T_1\cos 30^\circ + T_2\cos 60^\circ &= 100\\ \frac{\sqrt{3}}{2}T_1 + \frac{1}{2}T_2 &= 100\\ \sqrt{3}T_1 + T_2 &= 200\end{aligned}$$
Substituting:
$$\begin{aligned}\sqrt{3}(\sqrt{3}T_2) + T_2 &= 200\\ 3T_2 + T_2 &= 200\\ 4T_2 &= 200\\ T_2 &= 50\text{N}\\ T_1 &= 50\sqrt{3}\text{N}\end{aligned}$$

13. Speed of A is $\sqrt{21^2+17^2} = \sqrt{730}$m/s.
Speed of B is $\sqrt{26^2+2^2} = \sqrt{680}$m/s.
Particle A is moving fastest.

14. Speed=$\sqrt{5^2+2^2} = \sqrt{29}$m/s.
In one minute it will move $60\sqrt{29} \approx 323.1$m.

15. (a) When there is no wind blowing, the pilot flies due North with velocity vector $75\mathbf{j}$m/s for $300000 \div 75 = 4000$ seconds=1hr 6min 40sec.

(b)



We must add the helicopter's own velocity to the wind velocity to produce a resultant headed due North.
Easterly ($\mathbf{i}$) components:

$$\begin{aligned}21+a&=0\\a&=-21\end{aligned}$$

Now find the northerly ($\mathbf{j}$) component to give the correct speed:

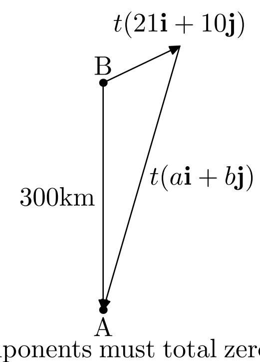
$$\begin{aligned}\sqrt{a^2+b^2}&=75\\21^2+b^2&=75^2\\b&=\pm\sqrt{75^2-21^2}\\&=\pm72\end{aligned}$$

We know we're heading north, so we disregard the negative solution and conclude $b=72$.
To calculate time, we use the total northerly component (i.e. wind plus plane):

$$\begin{aligned}10t+bt&=300000\\82t&=300000\\t&=\frac{300000}{82}\\&\approx 3659\text{s}\\&\approx 61\text{min}\end{aligned}$$

The velocity vector is $(-21\mathbf{i}+72\mathbf{j})$m/s and the trip will take about one hour and one minute.

16.



Easterly components must total zero, so as in the previous question $a=-21$.

Calculation of the northerly component is the same as in the previous question, but this time we are heading southwards, so we reject the *positive* solution and conclude $b=-72$.

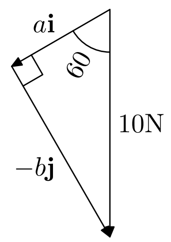
We now have a total southwards speed of $72-10=62$m/s so the time is

$$\begin{aligned}t&=\frac{300000}{62}\\&\approx 4839\text{s}\\&\approx 81\text{min}\end{aligned}$$

The velocity vector is $(-21\mathbf{i}-72\mathbf{j})$m/s and the trip will take about 81 minutes.

17. No working is required for this question. Refer to the answers in Sadler.

18.



$a=10\cos 60°=5$N, $b=-10\sin 60°=-5\sqrt{3}$N. The weight is $(5\mathbf{i}-5\sqrt{3}\mathbf{j})$N.

19. (a)
$$\begin{aligned}x(2\mathbf{i}+3\mathbf{j})+y(\mathbf{i}-\mathbf{j})&=3\mathbf{i}+2\mathbf{j}\\(2x+y)\mathbf{i}+(3x-y)\mathbf{j}&=3\mathbf{i}+2\mathbf{j}\\2x+y&=3\\3x-y&=2\end{aligned}$$
solving simultaneously:
$$\begin{aligned}x&=1\\y&=1\\3\mathbf{i}+2\mathbf{j}&=\mathbf{a}+\mathbf{b}\end{aligned}$$

(b)
$$\begin{aligned}(2x+y)\mathbf{i}+(3x-y)\mathbf{j}&=5\mathbf{i}+5\mathbf{j}\\2x+y&=5\\3x-y&=5\\x&=2\\y&=1\\5\mathbf{i}+5\mathbf{j}&=2\mathbf{a}+\mathbf{b}\end{aligned}$$

(c)
$$\begin{aligned}(2x+y)\mathbf{i}+(3x-y)\mathbf{j}&=\mathbf{i}+9\mathbf{j}\\2x+y&=1\\3x-y&=9\\x&=2\\y&=-3\\\mathbf{i}+9\mathbf{j}&=2\mathbf{a}-3\mathbf{b}\end{aligned}$$

(d)
$$\begin{aligned}(2x+y)\mathbf{i}+(3x-y)\mathbf{j}&=4\mathbf{i}+7\mathbf{j}\\2x+y&=4\\3x-y&=7\\x&=\frac{11}{5}\\y&=-\frac{2}{5}\\4\mathbf{i}+7\mathbf{j}&=\frac{11}{5}\mathbf{a}-\frac{2}{5}\mathbf{b}\end{aligned}$$

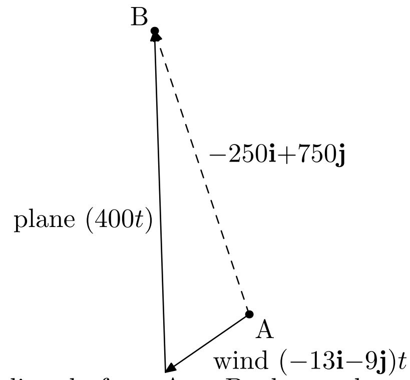
(e) $(2x+y)\mathbf{i}+(3x-y)\mathbf{j} = 3\mathbf{i}-\mathbf{j}$

$$\begin{aligned} 2x+y &= 3\\ 3x-y &= -1\\ x &= \frac{2}{5}\\ y &= \frac{11}{5}\\ 3\mathbf{i}-\mathbf{j} &= \frac{2}{5}\mathbf{a}+\frac{11}{5}\mathbf{b} \end{aligned}$$

(f) $(2x+y)\mathbf{i}+(3x-y)\mathbf{j} = 3\mathbf{i}+7\mathbf{j}$

$$\begin{aligned} 2x+y &= 3\\ 3x-y &= 7\\ x &= 2\\ y &= -1\\ 3\mathbf{i}+7\mathbf{j} &= 2\mathbf{a}-\mathbf{b} \end{aligned}$$

20.



To fly directly from A to B, the resultant of the plane's velocity and the wind must be in the same direction as $\overrightarrow{AB}$. That is,

$$(a\mathbf{i}+b\mathbf{j})+(-13\mathbf{i}-9\mathbf{j}) = (a-13)\mathbf{i}+(b-9)\mathbf{j}$$

in the same direction as

$$-250\mathbf{i}+750\mathbf{j}$$

Let θ represent this direction (an angle measured from the positive $\mathbf{i}$ direction) then

$$\tan\theta = \frac{750}{-250} = -3$$

and

$$\tan\theta = \frac{b-9}{a-13}$$

hence

$$\begin{aligned} \frac{b-9}{a-13} &= -3\\ b-9 &= -3(a-13)\\ &= -3a+39\\ b &= 48-3a \end{aligned}$$

Now consider speed

$$a^2+b^2 = 400^2$$

and substitute for b:

$$\begin{aligned} a^2+(48-3a)^2 &= 160\,000\\ a^2+2304-288a+9a^2 &= 160\,000\\ 10a^2-288a-157696 &= 0 \end{aligned}$$

$$\begin{aligned} a &= -112 & \text{or}\quad a &= 140.8\\ b &= 48-3(-112) & b &= 48-3(140.8)\\ &= 384 & &= -374.4 \end{aligned}$$

It should be clear that the first of these solutions takes us in the correct direction to go from A to B. The pilot should set a vector of $(-112\mathbf{i}+384\mathbf{j})$km/h for the trip from A to B.

For the return trip the same calculations apply (since $\tan(\theta+180°) = \tan\theta$) so we will get the same solutions for a and b, but here we will reject the first and accept the second.

The pilot should set a vector of $(140.8\mathbf{i} - 374.4\mathbf{j})$km/h for the return trip from B to A.

Exercise 4C

1. (a) $\overrightarrow{OA} = 2\mathbf{i}+5\mathbf{j}$

   (b) $\overrightarrow{OB} = -3\mathbf{i}+6\mathbf{j}$

   (c) $\overrightarrow{OC} = 0\mathbf{i}-5\mathbf{j}$

   (d) $\overrightarrow{OD} = 3\mathbf{i}+8\mathbf{j}$

2. (a) $$\begin{aligned} \overrightarrow{AB} &= \overrightarrow{AO}+\overrightarrow{OB}\\ &= -(3\mathbf{i}+\mathbf{j})+(2\mathbf{i}-\mathbf{j})\\ &= -\mathbf{i}-2\mathbf{j} \end{aligned}$$

   (b) $$\begin{aligned} \overrightarrow{BA} &= -\overrightarrow{AB}\\ &= \mathbf{i}+2\mathbf{j} \end{aligned}$$

3. (a) $$\begin{aligned} \overrightarrow{AB} &= -\overrightarrow{OA}+\overrightarrow{OB}\\ &= -(-\mathbf{i}+4\mathbf{j})+(2\mathbf{i}-3\mathbf{j})\\ &= 3\mathbf{i}-7\mathbf{j} \end{aligned}$$

   (b) $$\begin{aligned} \overrightarrow{BC} &= -\overrightarrow{OB}+\overrightarrow{OC}\\ &= -(2\mathbf{i}-3\mathbf{j})+(\mathbf{i}+5\mathbf{j})\\ &= -\mathbf{i}+8\mathbf{j} \end{aligned}$$

(c) $\overrightarrow{CA} = -(\mathbf{i}+5\mathbf{j}) + (-\mathbf{i}+4\mathbf{j})$
$= -2\mathbf{i} - \mathbf{j}$

4. (a) $\overrightarrow{AB} = -(\mathbf{i}+2\mathbf{j}) + (4\mathbf{i}-2\mathbf{j})$
$= 3\mathbf{i} - 4\mathbf{j}$

(b) $\overrightarrow{BC} = -(4\mathbf{i}-2\mathbf{j}) + (-\mathbf{i}+11\mathbf{j})$
$= -5\mathbf{i} + 13\mathbf{j}$

(c) $\overrightarrow{CD} = -(-\mathbf{i}+11\mathbf{j}) + (6\mathbf{i}-13\mathbf{j})$
$= 7\mathbf{i} - 24\mathbf{j}$

(d) $|\overrightarrow{CD}|\dfrac{\overrightarrow{AB}}{|\overrightarrow{AB}|} = 25\dfrac{3\mathbf{i}-4\mathbf{j}}{5}$
$= 15\mathbf{i} - 20\mathbf{j}$

5. (a) $|\overrightarrow{OA}| = |3\mathbf{i}+7\mathbf{j}|$
$= \sqrt{3^2+7^2}$
$= \sqrt{58}$

(b) $|\overrightarrow{OB}| = |-2\mathbf{i}+\mathbf{j}|$
$= \sqrt{2^2+1^2}$
$= \sqrt{5}$

(c) $|\overrightarrow{AB}| = |-(3\mathbf{i}+7\mathbf{j}) + (-2\mathbf{i}+\mathbf{j})|$
$= |-5\mathbf{i}-6\mathbf{j}|$
$= \sqrt{5^2+6^2}$
$= \sqrt{61}$

6. (a) $|\overrightarrow{AB}| = |3\mathbf{i}-4\mathbf{j}| = 5$

(b) $|\overrightarrow{BA}| = |-3\mathbf{i}+4\mathbf{j}| = 5$

(c) $|\overrightarrow{AC}| = |\mathbf{i}+4\mathbf{j}| = \sqrt{17}$

(d) $|\overrightarrow{BC}| = |-2\mathbf{i}+8\mathbf{j}| = \sqrt{68} = 2\sqrt{17}$

7. (a) $\mathsf{OA} = \sqrt{1^2+6^2} = \sqrt{37}$

(b) $\mathsf{OB} = \sqrt{5^2+3^2} = \sqrt{34}$

(c) $\mathsf{BA} = \sqrt{(5--1)^2+(3-6)^2} = \sqrt{45} = 3\sqrt{5}$

8. (a) $\overrightarrow{AB} = (1-2)\mathbf{i} + (2--3)\mathbf{j}$
$= -\mathbf{i} + 5\mathbf{j}$

(b) $\overrightarrow{BC} = (9-1)\mathbf{i} + (21-2)\mathbf{j}$
$= 8\mathbf{i} + 19\mathbf{j}$

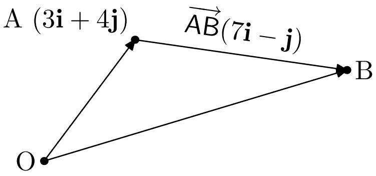
(c) $\overrightarrow{CD} = (6-9)\mathbf{i} + (-2-21)\mathbf{j}$
$= -3\mathbf{i} - 23\mathbf{j}$

(d) $|\overrightarrow{AC}| = |(9-2)\mathbf{i} + (21--3)\mathbf{j}|$
$= \sqrt{7^2+24^2}$
$= 25$

(e) $\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$
$= \mathbf{i} + 2\mathbf{j}$

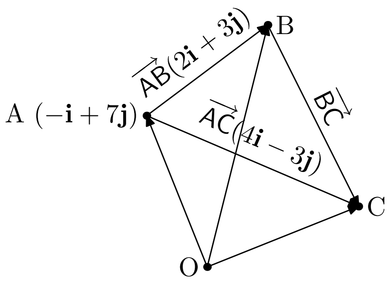
(f) $\overrightarrow{OA} + 2\overrightarrow{AC}$
$= (2\mathbf{i}-3\mathbf{j}) + 2\left((9-2)\mathbf{i} + (21--3)\mathbf{j}\right)$
$= 16\mathbf{i} + 45\mathbf{j}$

9.



$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$
$= (3\mathbf{i}+4\mathbf{j}) + (7\mathbf{i}-\mathbf{j})$
$= 10\mathbf{i} + 3\mathbf{j}$

10.

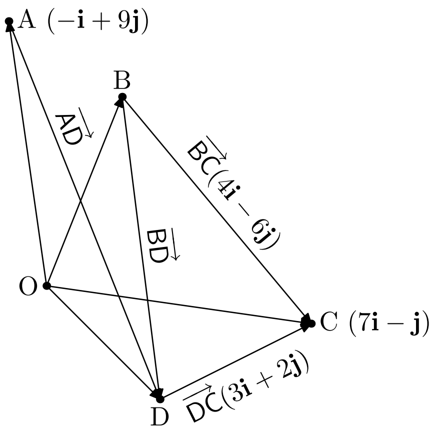


(a) $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$
$= (-\mathbf{i}+7\mathbf{j}) + (2\mathbf{i}+3\mathbf{j})$
$= \mathbf{i} + 10\mathbf{j}$

(b) $\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$
$= (-\mathbf{i}+7\mathbf{j}) + (4\mathbf{i}-3\mathbf{j})$
$= 3\mathbf{i} + 4\mathbf{j}$

(c) $\overrightarrow{BC} = -\overrightarrow{OB} + \overrightarrow{OC}$
$= -(\mathbf{i}+10\mathbf{j}) + (3\mathbf{i}+4\mathbf{j})$
$= 2\mathbf{i} - 6\mathbf{j}$

11.



(a) $\overrightarrow{OB} = \overrightarrow{OC} + -\overrightarrow{BC}$
$= (7\mathbf{i}-\mathbf{j}) - (4\mathbf{i}-6\mathbf{j})$
$= 3\mathbf{i} + 5\mathbf{j}$

(b) $\overrightarrow{OD} = \overrightarrow{OC} + -\overrightarrow{DC}$
$= (7\mathbf{i}-\mathbf{j}) - (3\mathbf{i}+2\mathbf{j})$
$= 4\mathbf{i} - 3\mathbf{j}$

(c) $\overrightarrow{BD} = \overrightarrow{BC} - \overrightarrow{DC}$
$= (4\mathbf{i}-6\mathbf{j}) - (3\mathbf{i}+2\mathbf{j})$
$= \mathbf{i} - 8\mathbf{j}$

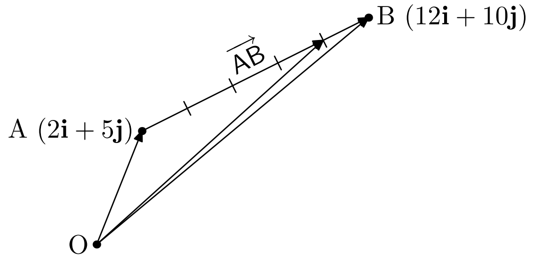
(d) $|\overrightarrow{\mathrm{AD}}| = |-\overrightarrow{\mathrm{OA}} + \overrightarrow{\mathrm{OD}}|$
$= |-(-\mathbf{i} + 9\mathbf{j}) + (4\mathbf{i} - 3\mathbf{j})|$
$= |5\mathbf{i} - 12\mathbf{j}|$
$= 13$

12. (a) $(2\mathbf{i} + 9\mathbf{j}) + (2\mathbf{i} - 5\mathbf{j}) = (4\mathbf{i} + 4\mathbf{j})$m
(b) $(2\mathbf{i} + 9\mathbf{j}) + 2(2\mathbf{i} - 5\mathbf{j}) = (6\mathbf{i} - \mathbf{j})$m
(c) $(2\mathbf{i} + 9\mathbf{j}) + 10(2\mathbf{i} - 5\mathbf{j}) = (22\mathbf{i} - 41\mathbf{j})$m
(d) $|(2\mathbf{i} + 9\mathbf{j}) + 5(2\mathbf{i} - 5\mathbf{j})| = |12\mathbf{i} - 16\mathbf{j}| = 20$m

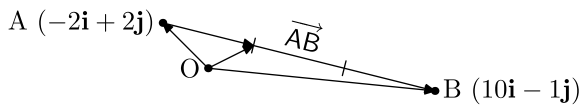
13. (a) $(5\mathbf{i} - 6\mathbf{j}) + 2(\mathbf{i} + 6\mathbf{j}) = (7\mathbf{i} + 6\mathbf{j})$m
(b) $(5\mathbf{i} - 6\mathbf{j}) + 3(\mathbf{i} + 6\mathbf{j}) = (8\mathbf{i} + 12\mathbf{j})$m
(c) $(5\mathbf{i} - 6\mathbf{j}) + 7(\mathbf{i} + 6\mathbf{j}) = (12\mathbf{i} + 36\mathbf{j})$m
(d) $|(5\mathbf{i} - 6\mathbf{j}) + 5(\mathbf{i} + 6\mathbf{j})| = |10\mathbf{i} + 24\mathbf{j}| = 26$m
(e)
$$\begin{aligned}
|(5\mathbf{i} - 6\mathbf{j}) + t(\mathbf{i} + 6\mathbf{j})| &= 50 \\
|(5 + t)\mathbf{i} + (-6 + 6t)\mathbf{j}| &= 50 \\
\sqrt{(5 + t)^2 + (-6 + 6t)^2} &= 50 \\
(5 + t)^2 + (-6 + 6t)^2 &= 2500 \\
25 + 10t + t^2 + 36 - 72t + 36t^2 &= 2500 \\
37t^2 - 62t - 2439 &= 0 \\
t &= 9 \\
\text{or } t &= -\frac{271}{37}
\end{aligned}$$
The particle is 50m from the origin after 9 seconds.

14. If A, B and C are collinear, vectors $\overrightarrow{\mathrm{AB}}$, $\overrightarrow{\mathrm{AC}}$ and $\overrightarrow{\mathrm{BC}}$ will all be parallel; showing that any pair of these are scalar multiples of each other will demonstrate collinearity.
$\overrightarrow{\mathrm{AB}} = -(3\mathbf{i} - \mathbf{j}) + (-\mathbf{i} + 15\mathbf{j}) = -4\mathbf{i} + 16\mathbf{j}$
$\overrightarrow{\mathrm{AC}} = -(3\mathbf{i} - \mathbf{j}) + (9\mathbf{i} - 25\mathbf{j}) = 6\mathbf{i} - 24\mathbf{j}$
$\overrightarrow{\mathrm{AC}} = -\frac{3}{2}\overrightarrow{\mathrm{AB}} \implies$ collinear.

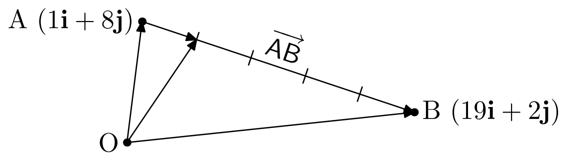
15. $\overrightarrow{\mathrm{DE}} = -(9\mathbf{i} - 7\mathbf{j}) + (-11\mathbf{i} + 8\mathbf{j}) = -20\mathbf{i} + 15\mathbf{j}$
$\overrightarrow{\mathrm{DF}} = -(9\mathbf{i} - 7\mathbf{j}) + (25\mathbf{i} - 19\mathbf{j}) = 16\mathbf{i} - 12\mathbf{j}$
$\overrightarrow{\mathrm{DE}} = -\frac{5}{4}\overrightarrow{\mathrm{DF}} \implies$ collinear.

16. 

$$\begin{aligned}
\overrightarrow{\mathrm{OA}} + \frac{4}{5}\overrightarrow{\mathrm{AB}} &= (2\mathbf{i} + 5\mathbf{j}) + \frac{4}{5}\left(-(2\mathbf{i} + 5\mathbf{j}) + (12\mathbf{i} + 10\mathbf{j})\right) \\
&= (2\mathbf{i} + 5\mathbf{j}) + \frac{4}{5}(10\mathbf{i} + 5\mathbf{j}) \\
&= (2\mathbf{i} + 5\mathbf{j}) + (8\mathbf{i} + 4\mathbf{j}) \\
&= 10\mathbf{i} + 9\mathbf{j}
\end{aligned}$$

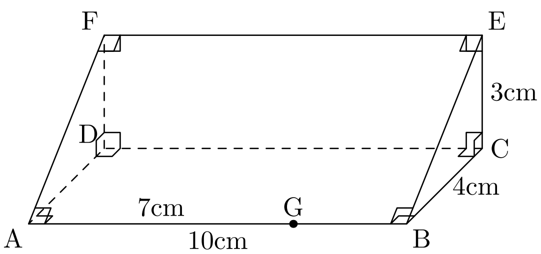
17. 

$$\begin{aligned}
\overrightarrow{\mathrm{OA}} + \frac{1}{3}\overrightarrow{\mathrm{AB}} &= (-2\mathbf{i} + 2\mathbf{j}) + \frac{1}{3}\left(-(-2\mathbf{i} + 2\mathbf{j}) + (10\mathbf{i} - \mathbf{j})\right) \\
&= (-2\mathbf{i} + 2\mathbf{j}) + \frac{1}{3}(12\mathbf{i} - 3\mathbf{j}) \\
&= (-2\mathbf{i} + 2\mathbf{j}) + (4\mathbf{i} - \mathbf{j}) \\
&= 2\mathbf{i} + \mathbf{j}
\end{aligned}$$

18. 

$$\begin{aligned}
\overrightarrow{\mathrm{OA}} + \frac{1}{5}\overrightarrow{\mathrm{AB}} &= (\mathbf{i} + 8\mathbf{j}) + \frac{1}{5}\left(-(\mathbf{i} + 8\mathbf{j}) + (19\mathbf{i} + 2\mathbf{j})\right) \\
&= (\mathbf{i} + 8\mathbf{j}) + \frac{1}{5}(18\mathbf{i} - 6\mathbf{j}) \\
&= (\mathbf{i} + 8\mathbf{j}) + (3.6\mathbf{i} - 1.2\mathbf{j}) \\
&= 4.6\mathbf{i} + 6.8\mathbf{j}
\end{aligned}$$

Miscellaneous Exercise 4

1. 

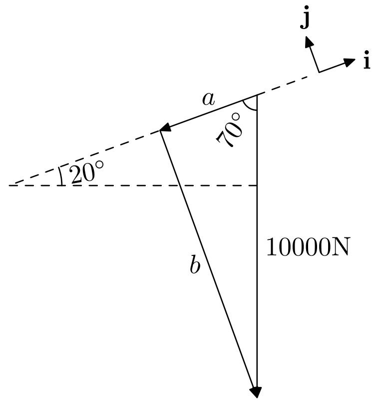
(a) $\tan\angle\mathrm{EBC} = \frac{3}{4}$
$\angle\mathrm{EBC} = \tan^{-1}\frac{3}{4} \approx 36.9°$

(b) To find $\angle$EGC we must first find the length GC.
Consider $\triangle$BCG.
GB $= 10 -$ AG $= 3$cm.
Using Pythagoras GC $= \sqrt{4^2 + 3^2} = 5$cm.
Now in $\triangle$CGE,
$\tan\angle\mathrm{EGC} = \frac{3}{5}$
$\angle\mathrm{EGC} = \tan^{-1}\frac{3}{5} \approx 31.0°$

(c) To find $\angle$EAC we must first find the length AC.

Consider △BCA.
Using Pythagoras AC $= \sqrt{4^2+10^2} = 2\sqrt{29}$cm.
Now in △CAE,
$\tan\angle\text{EAC} = \frac{3}{2\sqrt{29}}$
$\angle\text{EAC} = \tan^{-1}\frac{3}{2\sqrt{29}} \approx 15.6°$

2.



(a) $a = 10000\cos 70° \approx 3400$
$b = 10000\sin 70° \approx 9400$
Weight=$(-3400\mathbf{i} - 9400\mathbf{j})$N.

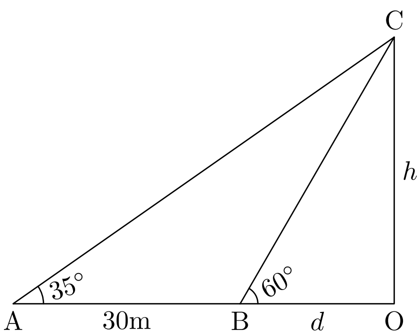
(b) The resistance force the brakes must apply is equal and opposite the **i** component of the weight, that is 3 400 N.

3.
$$\begin{aligned}
\mathbf{c} &= \lambda\mathbf{a} + \mu\mathbf{b}\\
2\mathbf{i}+\mathbf{j} &= \lambda(2\mathbf{i}+3\mathbf{j}) + \mu(3\mathbf{i}-4\mathbf{j})\\
2\mathbf{i}+\mathbf{j} &= 2\lambda\mathbf{i} + 3\lambda\mathbf{j} + 3\mu\mathbf{i} - 4\mu\mathbf{j}\\
(2-2\lambda-3\mu)\mathbf{i} &= (-1+3\lambda-4\mu)\mathbf{j}
\end{aligned}$$

Since **i** and uj are not parallel, LHS and RHS must evaluate to the zero vector:

$$\begin{aligned}
2\lambda + 3\mu &= 2 && ①\\
3\lambda - 4\mu &= 1 && ②\\
17\mu &= 4 && (3\times① - 2\times②)\\
\mu &= \frac{4}{17}\\
3\lambda - \frac{16}{17} &= 1 && (\text{subst. } \mu \text{ into } ②)\\
3\lambda &= \frac{33}{17}\\
\lambda &= \frac{11}{17}
\end{aligned}$$

4.



First consider △BCO:

$$\begin{aligned}
\tan 60° &= \frac{h}{d}\\
\sqrt{3} &= \frac{h}{d}\\
d &= \frac{h}{\sqrt{3}} && \approx 0.577h
\end{aligned}$$

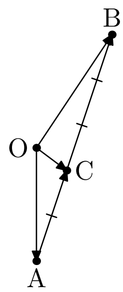
Now consider △ACO:

$$\begin{aligned}
\tan 35° &= \frac{h}{d+30}\\
d+30 &= \frac{h}{\tan 35°}\\
d &= \frac{h}{\tan 35°} - 30\\
&\approx 1.428h - 30
\end{aligned}$$

combining these two results ...

$$\begin{aligned}
\therefore\ 1.428h - 30 &= 0.577h\\
(1.428 - 0.577)h &= 30\\
0.851h &= 30\\
h &= \frac{30}{0.851}\\
&\approx 35\text{m}
\end{aligned}$$

5.



$$\begin{aligned}
\overrightarrow{\text{AC}} &= \frac{2}{3}\overrightarrow{\text{CB}}\\
\overrightarrow{\text{OC}} - \overrightarrow{\text{OA}} &= \frac{2}{3}(\overrightarrow{\text{OB}} - \overrightarrow{\text{OC}})\\
(4\mathbf{i} - 3\mathbf{j}) - (a\mathbf{i} - 15\mathbf{j}) &= \frac{2}{3}\left((10\mathbf{i} + b\mathbf{j}) - (4\mathbf{i} - 3\mathbf{j})\right)\\
(4-a)\mathbf{i} + 12\mathbf{j} &= \frac{2}{3}\left(6\mathbf{i} + (b+3)\mathbf{j}\right)\\
&= 4\mathbf{i} + \frac{2(b+3)}{3}\mathbf{j}
\end{aligned}$$

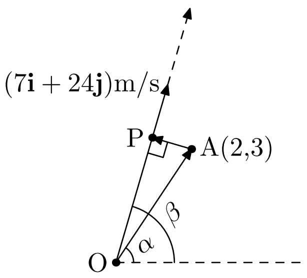
i components:

$$\begin{aligned}
4 - a &= 4\\
a &= 0
\end{aligned}$$

j components:

$$\begin{aligned}
12 &= \frac{2(b+3)}{3}\\
18 &= b+3\\
b &= 15
\end{aligned}$$

6. (a) The ball speed is $|7\mathbf{i} + 24\mathbf{j}| = \sqrt{7^2 + 24^2} = 25$m/s.
The time the ball takes to reach the boundary is $t = \frac{60}{25} = 2.4$s.

(b) 

Let P be the point of closest approach.

$$\begin{aligned}
\tan\alpha &= \frac{3}{2} \\
\alpha &= 56.3^\circ \\
\tan\beta &= \frac{24}{7} \\
\beta &= 73.7^\circ \\
\angle POA &= \beta - \alpha \\
&= 17.4^\circ \\
OA &= \sqrt{2^2 + 3^2} \\
&= \sqrt{13} \\
\sin 17.4^\circ &= \frac{AP}{OA} \\
AP &= OA \sin 17.4 \\
&= \sqrt{13} \sin 17.4 \\
&= 1.08\text{m}
\end{aligned}$$

Chapter 5

Exercise 5A

1. minor arc $AB = \frac{50}{360} \times 2\pi \times 12.4 \approx 10.8$cm
2. major arc $AB = \frac{235}{360} \times 2\pi \times 14.7 \approx 60.3$cm
3. minor arc $AB = \frac{360-290}{360} \times 2\pi \times 6.7 \approx 8.2$cm
4. major arc $AB = \frac{360-120}{360} \times 2\pi \times 8 = \frac{2}{3} \times 16\pi = \frac{32\pi}{3}$cm
5. minor arc $AB = \frac{150}{360} \times 2\pi \times 10 = \frac{25\pi}{3}$cm
6. major arc $AB = \frac{280}{360} \times 2\pi \times 6 = \frac{28\pi}{3}$cm
7. minor sector= $\frac{60}{360}\pi \times 12^2 = 24\pi\text{cm}^2$
8. minor sector= $\frac{110}{360}\pi \times 6^2 = 11\pi\text{cm}^2$
9. major sector= $\frac{360-120}{360}\pi \times 8^2 = \frac{128\pi}{3}\text{cm}^2$
10. minor sector= $\frac{360-205}{360}\pi \times 15.4^2 \approx 321\text{cm}^2$
11. First, from arc length l to angle θ

$$\begin{aligned} l &= \frac{\theta}{360}2\pi r \\ \theta &= \frac{360l}{2\pi r} \end{aligned}$$

Then from angle θ to sector area a

$$\begin{aligned} a &= \frac{\theta}{360}\pi r^2 \\ &= \frac{\frac{360l}{2\pi r}}{360}\pi r^2 \\ &= \frac{360\pi l r^2}{720\pi r} \\ &= \frac{lr}{2} \end{aligned}$$

So for question 11
minor sector= $\frac{12.3\times17.6}{2} \approx 108\text{cm}^2$

12. major sector= $\frac{40\times10}{2} = 200\text{cm}^2$
13. minor segment = minor sector −triangle

$$\begin{aligned} &= \frac{100}{360}\pi \times 15^2 - \frac{15^2 \times \sin 100}{2} \\ &\approx 86\text{cm}^2 \end{aligned}$$

14.

$$\begin{aligned} \theta &= \frac{360l}{2\pi r} \\ &= \frac{288}{\pi} \\ \text{minor segment} &= \frac{\theta}{360}\pi r^2 - \frac{r^2 \sin\theta}{2} \\ &= \frac{288}{360} \times 10^2 - \frac{10^2 \sin\left(\frac{288}{\pi}\right)}{2} \\ &\approx 80.0 - 50.0 \\ &\approx 30\text{cm}^2 \end{aligned}$$

15.

$$\begin{aligned} \theta &= \frac{90}{\pi \times 10^2} \times 360 \\ &= \frac{324}{\pi} \\ \text{minor segment} &= 90 - \frac{10^2 \sin\frac{324}{\pi}}{2} \\ &\approx 41\text{cm}^2 \end{aligned}$$

16.

$$\begin{aligned} \text{minor segment} &= \frac{60}{360}\pi \times 12^2 - \frac{1}{2}12^2 \sin 60^\circ \\ &= 24\pi - 72 \times \frac{\sqrt{3}}{2} \\ &= 24\pi - 36\sqrt{3} \\ &= 12(2\pi - 3\sqrt{3})\text{cm}^2 \end{aligned}$$

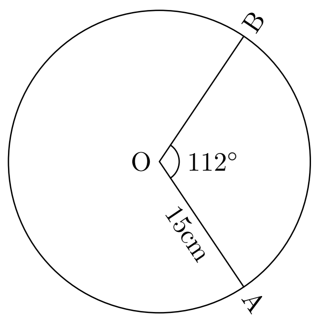
17.

$$\begin{aligned} \text{minor segment} &= \frac{135}{360}\pi \times 6^2 - \frac{1}{2}6^2 \sin 135^\circ \\ &= \frac{27\pi}{2} - 18 \times \frac{\sqrt{2}}{2} \\ &= \frac{27\pi}{2} - 9\sqrt{2} \\ &= 9\left(\frac{3\pi}{2} - \sqrt{2}\right)\text{cm}^2 \end{aligned}$$

18.

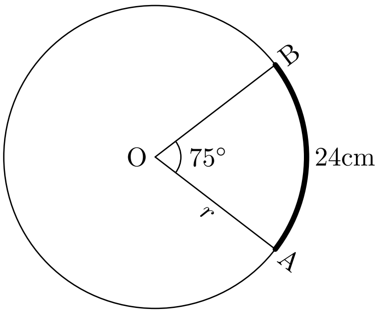
$$\begin{aligned} \theta &= 360 - 210 \\ &= 150^\circ \\ \text{minor segment} &= \frac{150}{360}\pi \times 10^2 - \frac{1}{2}10^2 \sin(150)^\circ \\ &= \frac{125\pi}{3} - 50\sin 150^\circ \\ &= \frac{125\pi}{3} - 25 \\ &= \frac{25}{3}(5\pi - 3)\text{cm}^2 \end{aligned}$$

19.

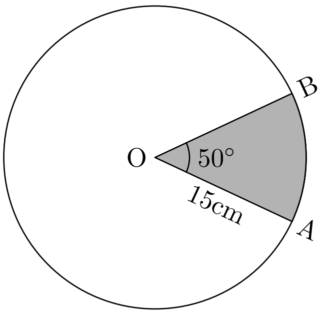


(a) minor arc= $\frac{112}{360} \times 2\pi \times 15 = \frac{28\pi}{3} \approx 29.3$cm

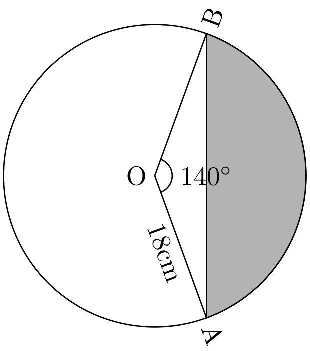
(b) major arc= $\frac{360-112}{360} \times 2\pi \times 15 = \frac{62\pi}{3} \approx 64.9$cm

20. 

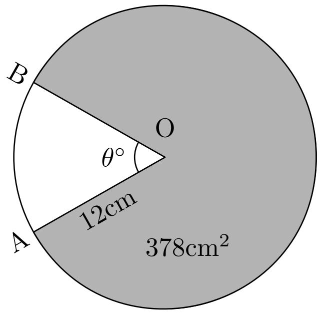
$$\frac{75}{360} \times 2\pi r = 24$$
$$r = \frac{24}{2\pi} \times \frac{360}{75}$$
$$= \frac{288}{5\pi}$$
$$\approx 18.3\text{cm}$$

21. 

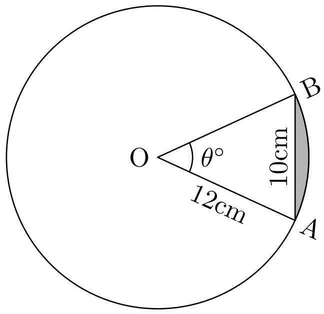
$$a = \frac{50}{360}\pi \times 15^2$$
$$= \frac{125\pi}{4}$$
$$\approx 98.2\text{cm}^2$$

22. 

$$\angle\text{OAB} = 20°$$
$$\implies \quad \text{OAB} = 180 - 2 \times 20$$
$$= 140°$$
$$a = \frac{140}{360}\pi \times 18^2 - \frac{1}{2}18^2 \sin 140°$$
$$\approx 292\text{cm}^2$$

23. 

$$378 = \frac{360 - \theta}{360}\pi \times 12^2$$
$$360 - \theta = \frac{378 \times 360}{144\pi}$$
$$\approx 300.8$$
$$\theta \approx 59°$$

24. 

$$\theta = \cos^{-1}\frac{12^2 + 12^2 - 10^2}{2 \times 12 \times 12}$$
$$\approx 49.2°$$
$$a = \frac{49.2}{360}\pi \times 12^2 - \frac{1}{2}12^2 \sin 49.2°$$
$$\approx 7.3\text{cm}^2$$

25. In half an hour the minute hand sweeps out 180° or half a circle. Its tip travels

$$d = \frac{1}{2} \times 2\pi \times 12$$
$$= 12\pi\text{cm}$$

In half an hour the hour hand sweeps out $\frac{1}{24}$ of a full circle. Its tip travels

$$d = \frac{1}{24} \times 2\pi \times 8$$
$$= \frac{2\pi}{3}\text{cm}$$

26. The ship is travelling through 3 degrees of latitude. $3° = 3 \times 60 = 180'$. The ship travels 180 nautical miles.
One nautical mile in kilometres is

$$d = \frac{\frac{1}{60}}{360} \times 2\pi \times 6350$$
$$\approx 1.85\text{km}$$

27. The circumference of the base of the cone is equal to the arc length of the sector:

$$2\pi r = \frac{240}{360} \times 2\pi \times 10$$
$$r = \frac{20}{3}$$

The slant height of the cone is the radius of the sector:

$$h^2 + r^2 = 10^2$$
$$h = \sqrt{10^2 - \left(\frac{20}{3}\right)^2}$$
$$= \sqrt{100 - \frac{400}{9}}$$
$$= 10\sqrt{1 - \frac{4}{9}}$$
$$= 10\sqrt{\frac{5}{9}}$$
$$= \frac{10\sqrt{5}}{3}$$

Exercise 5B

1. $\theta = \frac{3}{1} = 3$ rads
2. $\theta = \frac{3}{2} = 1.5$ rads
3. $\theta = \frac{5}{1} = 5$
4. $\theta = \frac{5}{2} = 2.5$
5. $\theta = \frac{4}{1} = 4$
6. $\theta = \frac{8}{2} = 4$
7. $5° = 5 \times \frac{\pi}{180} = \frac{\pi}{36}$
8. $18° = 18 \times \frac{\pi}{180} = \frac{\pi}{10}$
9. $30° = 30 \times \frac{\pi}{180} = \frac{\pi}{6}$
10. $80° = 80 \times \frac{\pi}{180} = \frac{4\pi}{9}$
11. $144° = 144 \times \frac{\pi}{180} = \frac{4\pi}{5}$
12. $40° = 40 \times \frac{\pi}{180} = \frac{2\pi}{9}$
13. $145° = 145 \times \frac{\pi}{180} = \frac{29\pi}{36}$
14. $108° = 108 \times \frac{\pi}{180} = \frac{3\pi}{5}$
15. $165° = 165 \times \frac{\pi}{180} = \frac{11\pi}{12}$
16. $9° = 9 \times \frac{\pi}{180} = \frac{\pi}{20}$
17. $65° = 65 \times \frac{\pi}{180} = \frac{13\pi}{36}$
18. $110° = 110 \times \frac{\pi}{180} = \frac{11\pi}{18}$
19. $130° = 130 \times \frac{\pi}{180} = \frac{13\pi}{18}$
20. $126° = 126 \times \frac{\pi}{180} = \frac{7\pi}{10}$
21. $99° = 99 \times \frac{\pi}{180} = \frac{11\pi}{20}$
22. $155° = 155 \times \frac{\pi}{180} = \frac{31\pi}{36}$
23. $\frac{\pi}{6}$ rads $= \frac{\pi}{6} \times \frac{180}{\pi} = 30°$
24. $\frac{\pi}{12}$ rads $= \frac{\pi}{12} \times \frac{180}{\pi} = 15°$
25. $\frac{5\pi}{18}$ rads $= \frac{5\pi}{18} \times \frac{180}{\pi} = 50°$
26. $\frac{3\pi}{10}$ rads $= \frac{3\pi}{10} \times \frac{180}{\pi} = 54°$
27. $\frac{2\pi}{5}$ rads $= \frac{2\pi}{5} \times \frac{180}{\pi} = 72°$
28. $\frac{8\pi}{9}$ rads $= \frac{8\pi}{9} \times \frac{180}{\pi} = 160°$
29. π rads $= 180°$
30. $\frac{35\pi}{36}$ rads $= \frac{35\pi}{36} \times \frac{180}{\pi} = 175°$
31. $\frac{\pi}{2}$ rads $= \frac{\pi}{2} \times \frac{180}{\pi} = 90°$
32. $\frac{3\pi}{8}$ rads $= \frac{3\pi}{8} \times \frac{180}{\pi} = 67.5°$
33. $\frac{\pi}{3}$ rads $= \frac{\pi}{3} \times \frac{180}{\pi} = 60°$
34. $\frac{\pi}{5}$ rads $= \frac{\pi}{5} \times \frac{180}{\pi} = 36°$
35. $\frac{17\pi}{36}$ rads $= \frac{17\pi}{36} \times \frac{180}{\pi} = 85°$
36. $\frac{3\pi}{4}$ rads $= \frac{3\pi}{4} \times \frac{180}{\pi} = 135°$
37. $\frac{11\pi}{60}$ rads $= \frac{11\pi}{60} \times \frac{180}{\pi} = 33°$
38. $\frac{7\pi}{18}$ rads $= \frac{7\pi}{18} \times \frac{180}{\pi} = 70°$
39. $32° = 32 \times \frac{\pi}{180} \approx 0.56$
40. $63° = 63 \times \frac{\pi}{180} \approx 1.10$
41. $115° = 115 \times \frac{\pi}{180} \approx 2.01$
42. $170° = 170 \times \frac{\pi}{180} \approx 2.97$
43. $16° = 16 \times \frac{\pi}{180} \approx 0.28$
44. $84° = 84 \times \frac{\pi}{180} \approx 1.47$
45. $104° = 104 \times \frac{\pi}{180} \approx 1.82$
46. $26° = 26 \times \frac{\pi}{180} \approx 0.45$

47. $76° = 76 \times \frac{\pi}{180} \approx 1.33$
48. $51° = 51 \times \frac{\pi}{180} \approx 0.89$
49. $152° = 152 \times \frac{\pi}{180} \approx 2.65$
50. $158° = 158 \times \frac{\pi}{180} \approx 2.76$
51. $1.5^R = 1.5 \times \frac{180}{\pi} \approx 86°$
52. $2.3^R = 2.3 \times \frac{180}{\pi} \approx 132°$
53. $1.4^R = 1.4 \times \frac{180}{\pi} \approx 80°$
54. $0.6^R = 0.6 \times \frac{180}{\pi} \approx 34°$
55. $0.2^R = 0.2 \times \frac{180}{\pi} \approx 11°$
56. $0.32^R = 0.32 \times \frac{180}{\pi} \approx 18°$
57. $1.21^R = 1.21 \times \frac{180}{\pi} \approx 69°$
58. $3.1^R = 3.1 \times \frac{180}{\pi} \approx 178°$

For exact value problems, you should work towards knowing these in radians as well as degrees, so you don't have to first convert to degrees. This will come with time and effort. You should deliberately memorise the following table:

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	1	0
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	1	0	undefined

59. $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$
60. $\frac{5\pi}{6}$ makes an angle of $\frac{\pi}{6}$ with the x-axis and is in quadrant 2, so $\sin\frac{5\pi}{6} = \frac{1}{2}$
61. $\frac{3\pi}{4}$ makes an angle of $\frac{\pi}{4}$ with the x-axis and is in quadrant 2, so $\cos\frac{3\pi}{4} = -\frac{1}{\sqrt{2}}$
62. $\sin\frac{\pi}{2} = 1$
63. $\frac{2\pi}{3}$ makes an angle of $\frac{\pi}{3}$ with the x-axis and is in quadrant 2, so $\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$
64. $\frac{3\pi}{4}$ makes an angle of $\frac{\pi}{4}$ with the x-axis and is in quadrant 2, so $\sin\frac{3\pi}{4} = \frac{1}{\sqrt{2}}$
65. $\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$
66. $\frac{2\pi}{3}$ makes an angle of $\frac{\pi}{3}$ with the x-axis and is in quadrant 2, so $\tan\frac{2\pi}{3} = -\sqrt{3}$
67. $\cos\frac{\pi}{2} = 0$
68. $\tan\frac{\pi}{2}$ is undefined
69. $\frac{2\pi}{3}$ makes an angle of $\frac{\pi}{3}$ with the x-axis and is in quadrant 2, so $\cos\frac{2\pi}{3} = -\frac{1}{2}$
70. $\frac{5\pi}{6}$ makes an angle of $\frac{\pi}{6}$ with the x-axis and is in quadrant 2, so $\tan\frac{5\pi}{6} = -\frac{1}{\sqrt{3}}$
71. $\frac{5\pi}{6}$ makes an angle of $\frac{\pi}{6}$ with the x-axis and is in quadrant 2, so $\cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$
72. π makes an angle of 0 with the x-axis and is in quadrant 2, so $\tan\pi = 0$
73. $\cos\frac{\pi}{3} = \frac{1}{2}$
74. π makes an angle of 0 with the x-axis and is in quadrant 2, so $\sin\pi = 0$

Questions 75–90 are single-step calculator exercises, so there's no point in reproducing the solutions here. Refer to the answers in Sadler.

91. (a) 3 revolutions/second$= 3 \times 2\pi = 6\pi$ radians/second.
    (b) 15 revolutions/minute$= \frac{15}{60} \times 2\pi = \frac{\pi}{2}$ radians/second.
    (c) 90 degrees/second$= \frac{\pi}{4}$ radians/second.
92. (a) 2π radians/minute $= 1$ revolution/minute
    (b) $\frac{3\pi}{4}$ radians/second $= \frac{3\pi}{4} \times 60 = 45\pi$ radians/minute $= \frac{45\pi}{2\pi} = 22.5$ revolutions/minute
    (c) $\frac{\pi}{3}$ radians/second $= \frac{\pi}{3} \times 60 = 20\pi$ radians/minute $= \frac{20\pi}{2\pi} = 10$ revolutions/minute
93. $$\begin{aligned}\sin 1 &= \frac{6}{x}\\ x &= \frac{6}{\sin 1}\\ &\approx 7.1\end{aligned}$$
94. $$\begin{aligned}\tan 1.2 &= \frac{8}{x}\\ x &= \frac{8}{\tan 1.2}\\ &\approx 3.1\end{aligned}$$
95. Let h be the perpendicular height in cm.
$$\begin{aligned}\sin 0.6 &= \frac{h}{20}\\ h &= 20\sin 0.6\\ &\approx 11.3\\ x &= \sqrt{h^2+6^2}\\ &\approx 12.8\end{aligned}$$
96. $$\begin{aligned}\frac{x}{\sin 1.1} &= \frac{14}{\sin 1.8}\\ x &= \frac{14\sin 1.1}{\sin 1.8}\\ &= 12.8\end{aligned}$$
97. $$\begin{aligned}\theta &= \pi - 0.64\\ &\approx 2.50\\ x &= \sqrt{7^2+10^2-2\times 7\times 10\cos 2.50}\\ &\approx 16.2\end{aligned}$$

98. $7.2^2 = 5.0^2 + 6.1^2 - 2 \times 5.0 \times 6.1 \cos x$

$$x = \cos^{-1}\left(\frac{5.0^2 + 6.1^2 - 7.2^2}{2 \times 5.0 \times 6.1}\right)$$
$$\approx 1.4$$

99. (a) $\frac{1}{4} \times 2\pi = \frac{\pi}{2}$

(b) $\frac{2}{3} \times 2\pi = \frac{4\pi}{3}$

(c) $\frac{5}{6} \times 2\pi = \frac{5\pi}{3}$

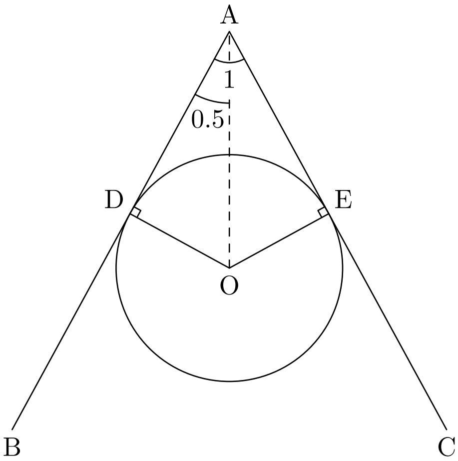
(d) $\frac{55}{60} \times 2\pi = \frac{11\pi}{6}$

100. (a) 50 grads$= 0.50 \times \frac{\pi}{2} = \frac{\pi}{4}$

(b) 75 grads$= 0.75 \times \frac{\pi}{2} = \frac{3\pi}{8}$

(c) 10 grads$= 0.10 \times \frac{\pi}{2} = \frac{\pi}{20}$

(d) 130 grads$= 0.10 \times \frac{\pi}{2} = \frac{13\pi}{20}$



101. (a) Let d be the diameter of the pipe.

$$\frac{\text{DO}}{\text{DA}} = \tan 0.5$$
$$\text{DA} = \frac{\text{DO}}{\tan 0.5} = \frac{d}{2\tan 0.5} = \frac{d}{1.09} = 0.915d$$

The scale along AB must be set up so that 1cm units are 0.915cm apart, starting with 0 at A.

(b) If BAC $= \frac{\pi}{2}$ then $\triangle$ODA is isosceles so OD=AD or AD $= 0.5d$. This would be simpler to construct as 1cm units along AB would be exactly 0.5cm apart.

Exercise 5C

1. $l = r\theta = 5 \times 0.8 = 4$cm

2. $l = r\theta = 10 \times 2.5 = 25$cm

3. $l = r\theta = 7.8 \times (2\pi - 4.5) \approx 13.9$cm

4. $a = \frac{1}{2}r^2\theta$
$= \frac{1}{2} \times 4^2 \times 1$
$= 8\text{cm}^2$

5. $a = \frac{1}{2}r^2\theta$
$= \frac{1}{2} \times 6^2 \times 2.5$
$= 45\text{cm}^2$

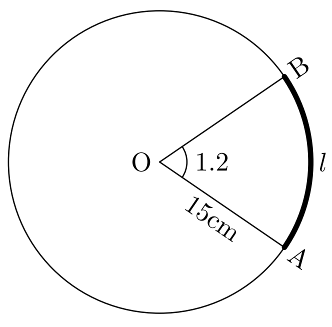
6. $a = \frac{1}{2}r^2\theta$
$= \frac{1}{2} \times 10^2 \times (2\pi - 4)$
$= 114.2\text{cm}^2$

7. $a = \frac{1}{2}r^2(\theta - \sin\theta)$
$= \frac{1}{2} \times 59^2(1 - \sin 1)$
$= 275.9\text{cm}^2$

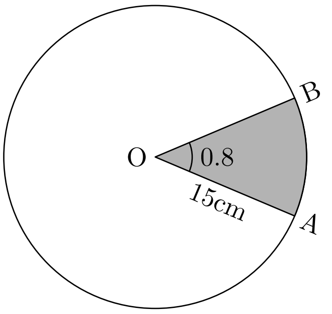
8. $a = \frac{1}{2}r^2(\theta - \sin\theta)$
$= \frac{1}{2} \times 5^2\left((2\pi - 3.5) - \sin(2\pi - 3.5)\right)$
$= 30.4\text{cm}^2$

9. $a = \frac{1}{2}r^2(\theta - \sin\theta)$
$= \frac{1}{2} \times 7.5^2(2.2 - \sin 2.2)$
$= 39.1\text{cm}^2$

10.

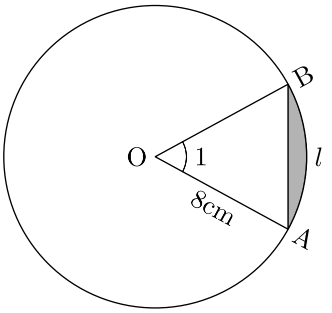


$l = r\theta = 15 \times 1.2 = 18$cm

11. 

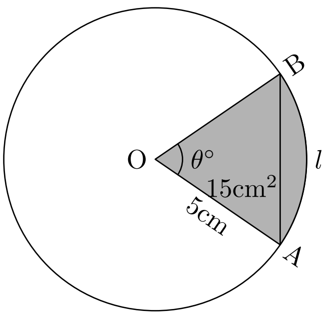
(a) $a = \frac{1}{2}r^2\theta = \frac{1}{2} \times 15^2 \times 0.8 = 90\text{cm}^2$

(b) $a = \pi \times 15^2 - 90 \approx 616.9\text{cm}^2$

12. 

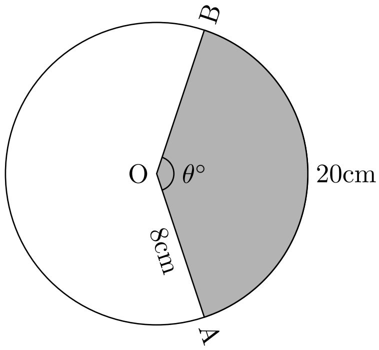
(a) $l = r\theta = 8 \times 1 = 8\text{cm}$

(b) $$\begin{aligned} a &= \frac{1}{2}r^2(\theta - \sin\theta) \\ &= \frac{1}{2} \times 8^2(1 - sin1) \\ &\approx 5.1\text{cm}^2 \end{aligned}$$

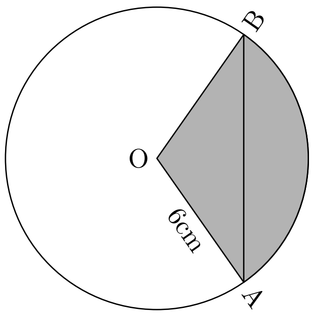
13. 

(a) $$\begin{aligned} \frac{1}{2}r^2\theta &= 15 \\ \frac{25}{2}\theta &= 15 \\ \theta &= \frac{6}{5} \\ l &= r\theta \\ &= 5 \times \frac{6}{5} \\ &= 6\text{cm} \end{aligned}$$

(b) $$\begin{aligned} a &= \frac{1}{2}r^2(\theta - \sin\theta) \\ &= \frac{1}{2} \times 5^2(\frac{6}{5} - sin\frac{6}{5}) \\ &\approx 3.35\text{cm}^2 \end{aligned}$$

14. 

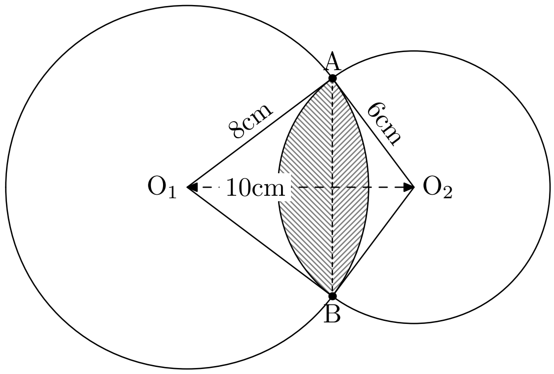
$$\begin{aligned} \theta &= l/r \\ &= 20/8 \\ &= 2.5 \\ a &= \frac{1}{2}r^2\theta \\ &= \frac{1}{2} \times 8^2 \times 2.5 \\ &= 80\text{cm}^2 \end{aligned}$$

15. 

$$\begin{aligned} \frac{1}{2}r^2\theta &= 9 \\ \frac{1}{2} \times 6^2\theta &= 9 \\ 18\theta &= 9 \\ \theta &= \frac{1}{2} \\ a &= \frac{1}{2}r^2(\theta - \sin\theta) \\ &= \frac{1}{2} \times 6^2\left(\frac{1}{2} - \sin\frac{1}{2}\right) \quad \approx 0.37\text{cm}^2 \end{aligned}$$

16. $$\begin{aligned} a &= \text{segment BOC} - \text{segment AOD} \\ &= \frac{1}{2}R^2\theta - \frac{1}{2}r^2\theta \\ &= \frac{\theta}{2}(R^2 - r^2) \\ &= \frac{1.5}{2}(12^2 - 6^2) \\ &= 81\text{cm}^2 \end{aligned}$$

17. $$\begin{aligned} a &= \frac{\theta}{2}(R^2 - r^2) \\ &= \frac{1.5}{2}(9^2 - 5^2) \\ &= 42\text{cm}^2 \end{aligned}$$

18. 

Consider $\triangle O_1AO_2$.

$$\begin{aligned}\angle AO_1O_2 &= \cos^{-1}\frac{8^2+10^2-6^2}{2\times 8\times 10}\\ &\approx 0.644\\ \angle AO_1B &= 2\angle AO_1O_2\\ &\approx 1.287\\ \text{segment } AO_1B &= \frac{1}{2}\times 8^2(1.287-\sin 1.287)\\ &\approx 10.46\end{aligned}$$

Similarly

$$\begin{aligned}\angle AO_2O_1 &= \cos^{-1}\frac{6^2+10^2-8^2}{2\times 6\times 10}\\ &\approx 0.927\\ \angle AO_2B &= 2\angle AO_2O_1\\ &\approx 1.855\\ \text{segment } AO_2B &= \frac{1}{2}\times 6^2(1.855-\sin 1.855)\\ &\approx 16.10\\ \text{total area} &= 10.46+16.10\\ &= 26.57\text{cm}^2\end{aligned}$$

19. $$\begin{aligned}\text{area}_{\text{segment BOC}} &= \frac{1}{2}\times 8^2\times 0.8\\ &= 25.6\\ \text{area}_{\triangle\text{AOD}} &= \frac{1}{2}\times 5^2\sin 0.8\\ &\approx 8.97\\ \text{area}_{\text{ABCD}} &= 25.6-8.97\\ &= 16.60\text{cm}^2\end{aligned}$$

20. The AC and BC are perpendicular to AO and BO respectively, since a tangent is perpendicular to a radius to the same point. This makes calculating the area of the halves of the quadrilateral simple.

$$\begin{aligned}\text{area}_{\triangle\text{AOC}} &= \frac{1}{2}\times 6\times 8\\ &= 24\\ \text{area}_{\text{AOBC}} &= 48\\ \angle AOC &= \tan^{-1}\frac{8}{6}\\ &\approx 0.927\end{aligned}$$

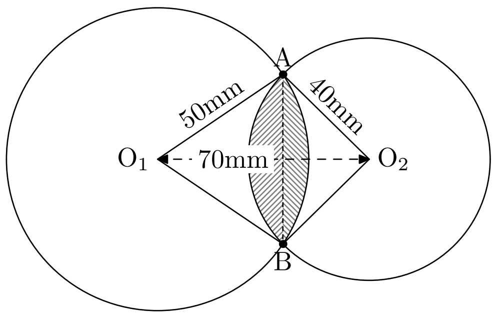
$$\begin{aligned}\angle AOB &= 2\angle AOC\\ &\approx 1.855\\ \text{area}_{\text{sector AOB}} &= \frac{1}{2}\times 6^2\times 1.855\\ &= 33.38\\ \text{area} &= 48-33.38\\ &= 14.62\text{cm}^2\end{aligned}$$

21. Area ABCD is equal to the area of the segment created by chord AD minus the area of the segment created by chord BC.

$$\begin{aligned}a &= \frac{1}{2}\times 5^2(2-\sin 2)-\frac{1}{2}\times 5^2(1-\sin 1)\\ &= \frac{1}{2}\times 5^2\left(2-\sin 2-(1-\sin 1)\right)\\ &= \frac{25}{2}(1-\sin 2+\sin 1)\\ &= 11.65\text{cm}^2\end{aligned}$$

22. (a) $l = r\theta = 75\times 0.8 = 60$cm each way, or 120cm total.

(b) BC$=2\times 75\sin 0.4\approx 58.4$cm
Arc BC exceeds chord BC by 1.6cm.

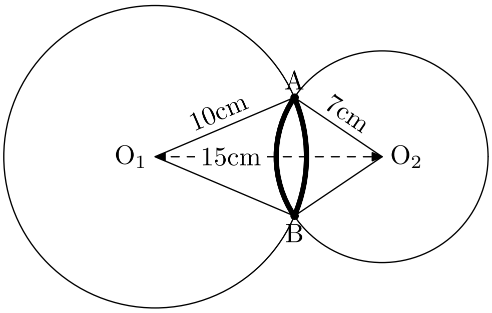
23. 

Consider $\triangle O_1AO_2$.

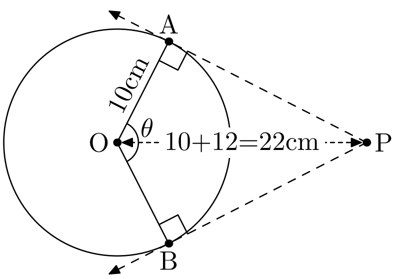
$$\begin{aligned}\angle AO_1O_2 &= \cos^{-1}\frac{50^2+70^2-40^2}{2\times 50\times 70}\\ &\approx 0.594\\ \angle AO_1B &= 2\angle AO_1O_2\\ &\approx 1.188\\ \text{segment } AO_1B &= \frac{1}{2}\times 50^2(1.188-\sin 1.188)\\ &\approx 325.9\end{aligned}$$

Similarly

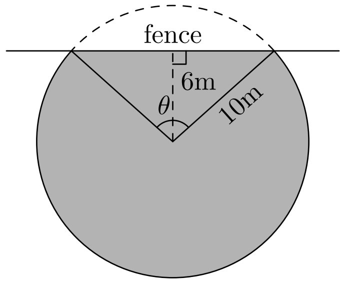
$$\begin{aligned}\angle AO_2O_1 &= \cos^{-1}\frac{40^2+70^2-50^2}{2\times 40\times 70}\\ &\approx 0.775\\ \angle AO_2B &= 2\angle AO_2O_1\\ &\approx 1.550\\ \text{segment } AO_2B &= \frac{1}{2}\times 40^2(1.550-\sin 1.550)\\ &\approx 440.5\\ \text{total area} &= 325.9+440.5\\ &\approx 770\text{mm}^2\end{aligned}$$

24. 

$$\begin{aligned}
\angle \mathrm{AO_1O_2} &= \cos^{-1}\frac{10^2+15^2-7^2}{2\times 10\times 15}\\
&\approx 0.403\\
\angle \mathrm{AO_1B} &= 2\angle \mathrm{AO_1O_2}\\
&\approx 0.805\\
\text{arc }\mathrm{AO_1B} &= 10\times 0.805\\
&\approx 8.05\\
\angle \mathrm{AO_2O_1} &= \cos^{-1}\frac{7^2+15^2-10^2}{2\times 7\times 15}\\
&\approx 0.594\\
\angle \mathrm{AO_2B} &= 2\angle \mathrm{AO_2O_1}\\
&\approx 1.188\\
\text{arc }\mathrm{AO_2B} &= 7\times 1.188\\
&\approx 8.32\\
\text{perimeter} &= 8.05+8.32\\
&\approx 16.4\text{cm}
\end{aligned}$$

25. 

$$\begin{aligned}
\cos\frac{\theta}{2} &= \frac{10}{10+12}\\
\theta &= 2\cos^{-1}\frac{10}{22}\\
&\approx 2.20\\
\text{percentage} &= \frac{2.20}{2\pi}\times 100\%\\
&\approx 35\%
\end{aligned}$$

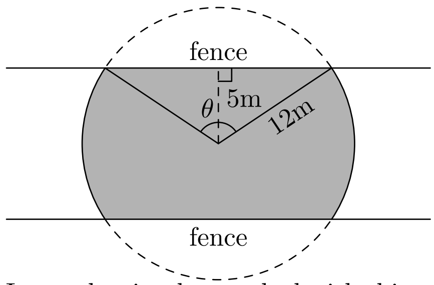
26. 

To

$$\begin{aligned}
\cos\frac{\theta}{2} &= \frac{6}{10}\\
\theta &= 2\cos^{-1}\frac{6}{10}\\
&\approx 1.85
\end{aligned}$$

find the major segment, use $2\pi-\theta$:

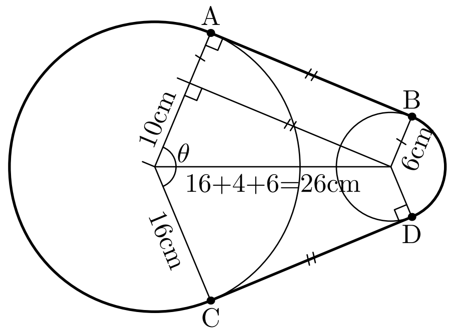
$$\begin{aligned}
a &= \frac{1}{2}\times 10^2\left(2\pi-\theta-\sin(2\pi-\theta)\right)\\
&\approx 269\text{m}^2
\end{aligned}$$

27. 

It may be simplest to deal with this as the area of the circle minus the area of two identical minor segments.

$$\begin{aligned}
a_\circ &= \pi\times 12^2\\
&\approx 452.39\text{m}^2\\
\cos\frac{\theta}{2} &= \frac{5}{12}\\
\theta &= 2\cos^{-1}\frac{5}{12}\\
&\approx 2.28\\
a_{\text{seg}} &= \frac{1}{2}r^2\left(\theta-\sin\theta\right)\\
&= \frac{12^2}{2}(2.28-\sin 2.28)\\
&\approx 109.76\text{m}^2\\
a &= a_\circ - 2a_{\text{seg}}\\
&\approx 452.39-2\times 109.76\\
&\approx 233\text{m}^2
\end{aligned}$$

28.



This is similar to the Situation at the beginning of the chapter.

Straight segments AB and CD

$$\begin{aligned} AB &= \sqrt{26^2 - 10^2} \\ &= 24\text{cm} \end{aligned}$$

Angle θ

$$\begin{aligned} \cos\frac{\theta}{2} &= \frac{10}{26} \\ \theta &= 2\cos^{-1}\frac{10}{26} \\ &\approx 2.35 \end{aligned}$$

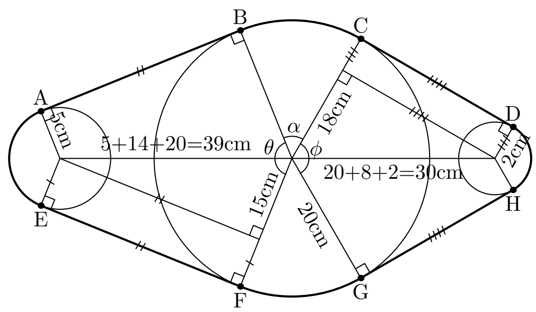
Major arc AC

$$\begin{aligned} AC &= r(2\pi - \theta) \\ &= 16(2\pi - 2.35) \\ &\approx 62.90 \end{aligned}$$

Minor arc BD

$$\begin{aligned} BD &= r\theta \\ &= 6 \times 2.35 \\ &\approx 14.11 \\ \text{Total length} &= 2 \times 24 + 62.90 + 14.11 \\ &\approx 125\text{cm} \end{aligned}$$

29.



Straight segments AB and EF

$$\begin{aligned} AB &= \sqrt{39^2 - 15^2} \\ &= 36\text{cm} \end{aligned}$$

Straight segments CD and GH

$$\begin{aligned} CD &= \sqrt{30^2 - 18^2} \\ &= 24\text{cm} \end{aligned}$$

Angle θ

$$\begin{aligned} \cos\frac{\theta}{2} &= \frac{15}{39} \\ \theta &= 2\cos^{-1}\frac{15}{39} \\ &\approx 2.35 \end{aligned}$$

Minor arc AE

$$\begin{aligned} AE &= r\theta \\ &= 5 \times 2.35 \\ &\approx 11.76 \end{aligned}$$

Angle ϕ

$$\begin{aligned} \cos\frac{\phi}{2} &= \frac{18}{30} \\ \phi &= 2\cos^{-1}\frac{18}{30} \\ &\approx 1.85 \end{aligned}$$

Minor arc DH

$$\begin{aligned} DH &= r\phi \\ &= 2 \times 1.85 \\ &\approx 3.71 \end{aligned}$$

Minor arcs BC and FG

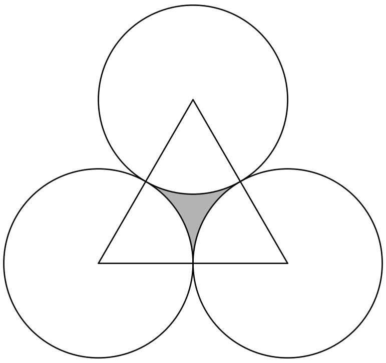
$$\begin{aligned} \alpha &= \frac{1}{2}(2\pi - \theta - \phi) \\ &\approx 1.04 \\ BG &= r\alpha \\ &= 20 \times 1.04 \\ &\approx 20.77 \\ \text{Total length} &= 2(AB + BC + CD) + AE + DH \\ &= 2(36 + 20.77 + 24) + 11.76 + 3.71 \\ &\approx 177\text{cm} \end{aligned}$$

30.

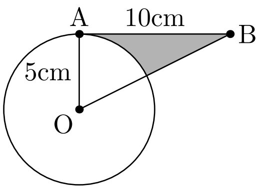
$$\begin{aligned} \text{perimeter:} \quad 2r + r\theta &= 14 \\ r\theta &= 14 - 2r \\ \theta &= \frac{14}{r} - 2 \\ \text{area:} \quad \frac{1}{2}r^2\theta &= 10 \\ \text{subst. for } \theta\text{:} \quad \frac{1}{2}r^2\left(\frac{14}{r} - 2\right) &= 10 \\ 7r - r^2 &= 10 \\ r^2 - 7r + 10 &= 0 \\ (r-5)(r-2) &= 0 \\ r &= 5 \\ \text{or } r &= 2 \\ \text{for } r = 5,\ \theta &= \frac{14}{5} - 2 \\ &= 0.8 \text{ (an acute angle)} \\ \text{for } r = 2,\ \theta &= \frac{14}{2} - 2 \\ &= 5 \text{ (a reflex angle)} \end{aligned}$$

(a) Radius = 5 cm.

(b) Radius = 2 cm.

31. 

$$\begin{aligned}
a_\triangle &= \frac{1}{2} \times 10 \times 10 \sin \frac{\pi}{3} \\
&= \frac{50\sqrt{3}}{2} \\
&= 25\sqrt{3} \\
a_{\text{sector}} &= \frac{1}{2} \times 5^2 \times \frac{\pi}{3} \\
&= \frac{25\pi}{6} \\
a_{\text{shaded}} &= a_\triangle - 3a_{\text{sector}} \\
&= 25\sqrt{3} - 3 \times \frac{25\pi}{6} \\
&= 25\sqrt{3} - \frac{25\pi}{2} \\
&= 25\left(\sqrt{3} - \frac{\pi}{2}\right) \text{cm}^2
\end{aligned}$$

32. 

$$\begin{aligned}
a_\triangle &= \frac{1}{2} \times 10 \times 5 \\
&= 25 \\
\theta &= \tan^{-1} \frac{10}{5} \\
&\approx 1.11 \\
a_{\text{sector}} &= \frac{1}{2} \times 5^2 \times 1.11 \\
&\approx 13.84 \\
a_{\text{shaded}} &= a_\triangle - a_{\text{sector}} \\
&= 25 - 13.84 \\
&\approx 11.2\text{cm}^2
\end{aligned}$$

33. r is the slant height of the cone, and θ is such that the arc length of the sector equals the circumference of the base of the cone.

$$\begin{aligned}
r &= \sqrt{28^2 + 8^2} \\
&= 4\sqrt{53} \\
&\approx 29.1\text{cm} \\
r\theta &= 2\pi \times 8 \\
\theta &= \frac{16\pi}{4\sqrt{53}} \\
&\approx 1.73^{\text{RADIANS}}
\end{aligned}$$

34. First, find the area of the major segment, then find the capacity:

$$\begin{aligned}
\cos \frac{\theta}{2} &= \frac{10}{60} \\
\theta &= 2\cos^{-1} \frac{1}{6} \\
&\approx 2.81 \\
a &= \frac{1}{2} \times 60^2 \left(2\pi - \theta - \sin(2\pi - \theta)\right) \\
&\approx 6849\text{cm}^2 \\
V &= al \\
&= 6849 \times 120 \\
&\approx 821915\text{cm}^3 \\
&\approx 822\text{L}
\end{aligned}$$

35. (a)

$$\begin{aligned}
a_{\text{I}} &= \frac{1}{2}bh \\
&= \frac{1}{2} \times 15 \times 40 \\
&= 300\text{cm}^2 \\
a_{\text{II}} &= a_{\text{I}} = 300\text{cm}^2 \\
a_{\text{III}} &= a_{\text{I}} + a_{\text{II}} = 600\text{cm}^2
\end{aligned}$$

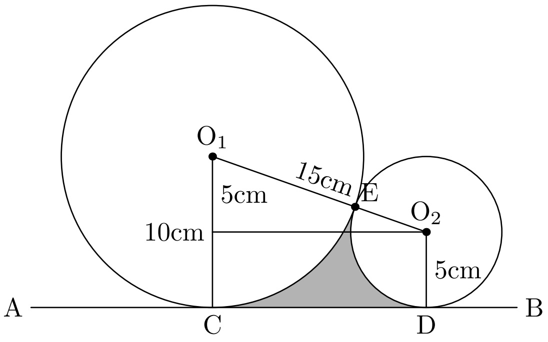
for segment IV:

$$\begin{aligned}
r &= \sqrt{40^2 + 15^2} \\
&= 5\sqrt{73} \\
&\approx 42.72 \\
\theta &= 2\tan^{-1} \frac{15}{40} \\
&\approx 0.718 \\
a_{\text{IV}} &= \frac{1}{2}r^2\theta - a_{III} \\
&= \frac{1}{2} \times 1825 \times 0.718 - 600 \\
&\approx 55\text{cm}^2
\end{aligned}$$

(b)

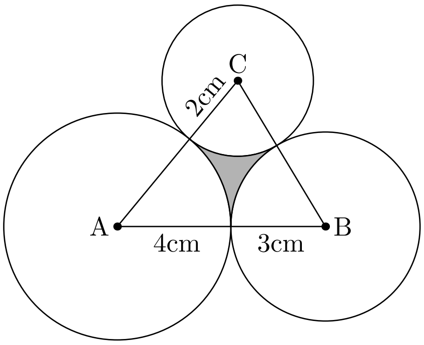
$$\begin{aligned}
l = 40 \times 2 &\quad +30 \times 2 \\
+ 5\sqrt{73} \times 2 &\quad +5\sqrt{73} \times 0.718 \\
\approx 256\text{cm}
\end{aligned}$$

36.



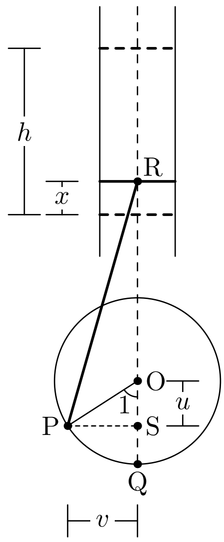
$$\begin{aligned}
CD &= \sqrt{15^2 - 5^2} \\
&= 10\sqrt{2} \\
a_{\text{O}_1\text{O}_2\text{DC}} &= \frac{1}{2}\text{CD}(r_1 + r_2) \\
&= \frac{1}{2} \times 10\sqrt{2}(10+5) \\
&= 75\sqrt{2} \\
&\approx 106.07 \\
\cos\angle\text{CO}_1\text{E} &= \frac{5}{15} \\
\angle\text{CO}_1\text{E} &\approx 1.23 \\
a_{\text{CO}_1\text{E}} &= \frac{1}{2} \times 10^2 \times 1.23 \\
&\approx 61.55 \\
\sin\angle\text{DO}_2\text{E} - \frac{\pi}{4} &= \frac{5}{15} \\
\angle\text{DO}_2\text{E} &\approx 0.34 + \frac{\pi}{4} \\
&\approx 1.91 \\
a_{\text{DO}_2\text{E}} &= \frac{1}{2} \times 5^2 \times 1.91 \\
&\approx 23.88 \\
a_{\text{shaded}} &= a_{\text{O}_1\text{O}_2\text{DC}} - a_{\text{CO}_1\text{E}} - a_{\text{dO}_2\text{E}} \\
&\approx 106.07 - 61.55 - 23.88 \\
&\approx 20.64\text{cm}^2
\end{aligned}$$

37.



$$\begin{aligned}
\angle\text{A} &= \cos^{-1}\frac{6^2 + 7^2 - 5^2}{2 \times 6 \times 7} \\
&\approx 0.775 \\
\angle\text{B} &= \cos^{-1}\frac{5^2 + 7^2 - 6^2}{2 \times 5 \times 7} \\
&\approx 0.997 \\
\angle\text{C} &= \cos^{-1}\frac{6^2 + 5^2 - 7^2}{2 \times 6 \times 5} \\
&\approx 1.369 \\
a_\triangle &= \frac{1}{2}ab\sin\text{C} \\
&= \frac{1}{2} \times 5 \times 6 \sin 1.369 \\
&\approx 14.70 \\
a_{\text{sector A}} &= \frac{1}{2} \times 4^2 \times 0.775 \\
&\approx 6.20 \\
a_{\text{sector B}} &= \frac{1}{2} \times 3^2 \times 0.997 \\
&\approx 4.49 \\
a_{\text{sector C}} &= \frac{1}{2} \times 2^2 \times 1.369 \\
&\approx 2.74 \\
a_{\text{shaded}} &= a_\triangle - a_{\text{sector A}} - a_{\text{sector B}} a_{\text{sector C}} \\
&= 14.70 - 6.20 - 4.49 - 2.74 \\
&\approx 1.27 \\
\frac{a_{\text{shaded}}}{a_\triangle} &= \frac{1.27}{14.70} \\
&= 8.6\%
\end{aligned}$$

38.



The lowest position of the piston is r above the top of the wheel. Therefore it is $3r$ above the bottom of the wheel. Thus the fixed length of the drive rod PR= $3r$.

It should be clear that $h = 2r$ since at the low position the arm goes straight down from the piston to the bottom of the wheel, and at the high position the arm goes straight down from the piston to the top of the wheel.

If arc length PQ is equal to r, the angle it subtends is $\angle$POQ=1^{RADIAN}.

The height of point P relative to point O is $u = r\cos 1$.

Similarly the horizontal position of point P is $v = -r\sin 1$.

The length from point S to R can be determined by Pythagoras' Theorem:

$$\begin{aligned}\text{SR} &= \sqrt{(3r)^2 - (r\sin 1)^2}\\ &= \sqrt{9r^2 - r^2\sin^2 1}\\ &= r\sqrt{9-\sin^2 1}\\ &= 2.88r\end{aligned}$$

$$\begin{aligned}x &= \text{SR} - 2r - u\\ &= 2.88r - 2r - r\cos 1\\ &= r(2.88 - 2 - \cos 1)\\ &= 0.33r\\ \frac{x}{h} &= \frac{0.33r}{2r}\\ &\approx 17\%\end{aligned}$$

Miscellaneous Exercise 5

1. No worked solution necessary.
2. No worked solution necessary.
3. (a) $x < 2.5$: $2x - 5 < 0$; $4 - x > 0$

$$\begin{aligned}-(2x-5) &= 4-x\\ -2x+5 &= 4-x\\ x &= 1\end{aligned}$$

(which is in the part of the domain we are considering, so we do not reject it.)

$2.5 \le x < 4$: $2x - 5 \ge 0$; $4 - x > 0$

$$\begin{aligned}2x-5 &= 4-x\\ 3x &= 9\\ x &= 3\end{aligned}$$

$x \ge 4$: $2x - 5 \ge 0$; $4 - x \le 0$

$$\begin{aligned}2x-5 &= -(4-x)\\ x &= 1\end{aligned}$$

We don't really need to do the third part (which yields a solution outside the part of the domain we're considering), as there can only be two solutions. (You can see that if you graph the left and right hand sides of the equation and see where they intersect.)

Solution: $x = 1$ or $x = 3$.

(b) $x < 2.5$: $2x - 5 < 0$; $4 - x > 0$

$$\begin{aligned}-(2x-5) &< 4-x\\ -2x+5 &< 4-x\\ x &> 1\end{aligned}$$

(which is in the part of the domain we are considering, so we have $1 < x < 2.5$.)

$2.5 \le x < 4$: $2x - 5 \ge 0$; $4 - x > 0$

$$\begin{aligned}2x-5 &< 4-x\\ 3x &< 9\\ x &< 3\end{aligned}$$

(which is in the part of the domain we are considering, so we have $2.5 \le x < 4$.)

Combining the two parts of solution we get:

Solution: $1 < x < 3$.

(c) This inequality must be true where the previous inequality is false, i.e.:

Solution: $x \le 1$ or $x \ge 3$.

There is another way we could look at this. We know the left and right hand sides are equal at 1 and 3, so the inequality must be true either between 1 and 3 or less than 1/greater than 3. We can decide which by testing a suitable value: substitute (for example) 2 for x and decide whether the inequality holds true. Here $|2 \times 2 - 5| = 1$ and $|4 - 2| = 2$ so LHS<RHS and the second inequality holds for the interval $1 < x < 3$.

4. $l = r\theta = \frac{40}{2} \times 3 = 60\text{cm/s}$

5. (a) $$\begin{aligned}\text{BC} &= \sqrt{\text{AC}^2 - \text{AB}^2}\\ &= \sqrt{10^2 - 6^2}\\ &= 8\text{cm}\end{aligned}$$

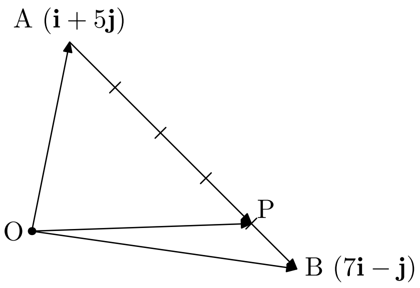
(b) $$\begin{aligned}\text{DB} &= \sqrt{\text{DA}^2 + \text{AB}^2}\\ &= \sqrt{15^2 + 6^2}\\ &= \sqrt{261}\\ &= 3\sqrt{29}\text{cm}\end{aligned}$$

(c) $$\begin{aligned}\cos\angle\text{CAB} &= \frac{AB}{AC}\\ \angle\text{CAB} &= \cos^{-1}\frac{AB}{AC}\\ &= \cos^{-1}\frac{6}{10}\\ &\approx 53^\circ\end{aligned}$$

(d) We want ∠FAE.

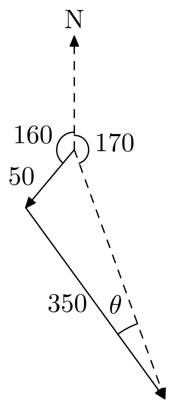
$$\begin{aligned}\tan\angle\text{FAE} &= \frac{FE}{EA}\\ \angle\text{FAE} &= \tan^{-1}\frac{FE}{EA}\\ &= \tan^{-1}\frac{8}{3\sqrt{29}}\\ &\approx 26^\circ\end{aligned}$$

6.



$$\begin{aligned}\overrightarrow{\text{AP}} &= \frac{4}{5}\overrightarrow{\text{AB}}\\ \overrightarrow{\text{OP}} - \overrightarrow{\text{OA}} &= \frac{4}{5}(\overrightarrow{\text{OB}} - \overrightarrow{\text{OA}})\\ \overrightarrow{\text{OP}} &= \frac{4}{5}(\overrightarrow{\text{OB}} - \overrightarrow{\text{OA}}) + \overrightarrow{\text{OA}}\\ &= \frac{4}{5}\overrightarrow{\text{OB}} + \frac{1}{5}\overrightarrow{\text{OA}}\\ &= 0.8(7\mathbf{i} - \mathbf{j}) + 0.2(\mathbf{i} + 5\mathbf{j})\\ &= (5.6\mathbf{i} - 0.8\mathbf{j}) + (0.2\mathbf{i} + \mathbf{j})\\ &= 5.8\mathbf{i} + 0.2\mathbf{j}\end{aligned}$$

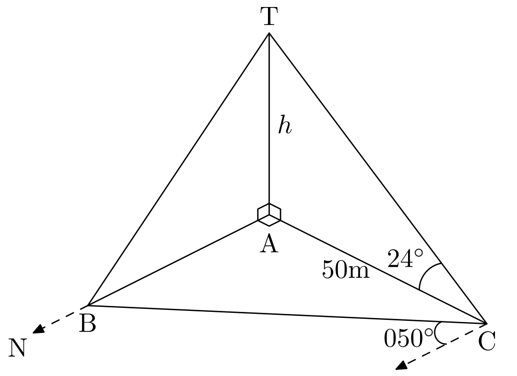
7.



$$\begin{aligned}\frac{\sin\theta}{50} &= \frac{\sin 30^\circ}{350}\\ \theta &= \sin^{-1}\frac{50\sin 30}{350}\\ &\approx 4.1^\circ\end{aligned}$$

The plane must fly on a bearing of $170 - 4 = 166^\circ$.

8.



(a)
$$\begin{aligned}\tan 24^\circ &= \frac{h}{50}\\ h &= 50\tan 24^\circ\\ &\approx 22\text{m}\end{aligned}$$

(b)
$$\begin{aligned}\angle\text{ACB} &= 90^\circ - 50^\circ\\ &= 40^\circ\\ \cos 40^\circ &= \frac{50}{\text{BC}}\\ \text{BC} &= \frac{50}{\cos 40^\circ}\\ &\approx 65\text{m}\end{aligned}$$

(c)
$$\begin{aligned}\tan 40^\circ &= \frac{\text{AB}}{50}\\ \text{AB} &= 50\tan 40^\circ\\ &\approx 42\text{m}\end{aligned}$$

(d)
$$\begin{aligned}\tan\angle\text{TBA} &= \frac{h}{AB}\\ \angle\text{TBA} &= \tan^{-1}\frac{22.26}{41.95}\\ &\approx 28^\circ\end{aligned}$$

9. $\angle\text{BOA} = 45^\circ$

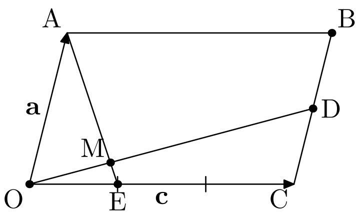
$$\begin{aligned}\text{BA} &= \sqrt{10^2 + 10^2 - 2\times 10\times 10\sin 45^\circ}\\ &= 10\left(\sqrt{2-\sqrt{2}}\right)\\ &\approx 7.65\text{m}\end{aligned}$$

By the similarity of the triangles BOA and JOI:

$$\begin{aligned}\text{JI} &= \frac{3}{5}\text{BA}\\ &\approx 4.59\text{m}\end{aligned}$$

10.
$$\begin{aligned}\mathbf{a} &= k\mathbf{c}\\ k &= \frac{|\mathbf{b}|}{|\mathbf{c}|}\\ &= \frac{\sqrt{7^2+24^2}}{\sqrt{3^2+4^2}}\\ &= \frac{25}{5}\\ &= 5\\ \mathbf{a} &= 5(3\mathbf{i}-4\mathbf{j})\\ &= 15\mathbf{i}-20\mathbf{j}\\ \mathbf{a}+\mathbf{b} &= 15\mathbf{i}-20\mathbf{j}-7\mathbf{i}+24\mathbf{j}\\ &= 8\mathbf{i}+4\mathbf{j}\\ |\mathbf{a}+\mathbf{b}| &= \sqrt{8^2+4^2}\\ &= \sqrt{80}\\ &= 4\sqrt{5}\end{aligned}$$

11.



$$\begin{aligned}\overrightarrow{\mathrm{AE}} &= -\mathbf{a}+\frac{1}{3}\mathbf{c}\\ \overrightarrow{\mathrm{AM}} &= h\overrightarrow{\mathrm{AE}}\\ &= -h\mathbf{a}+\frac{h}{3}\mathbf{c}\\ \overrightarrow{\mathrm{OM}} &= \mathbf{a}+\overrightarrow{\mathrm{AM}}\\ &= \mathbf{a}-h\mathbf{a}+\frac{h}{3}\mathbf{c}\\ &= (1-h)\mathbf{a}+\frac{h}{3}\mathbf{c}\\ \overrightarrow{\mathrm{OD}} &= \mathbf{c}+\frac{1}{2}\mathbf{a}\\ \overrightarrow{\mathrm{OM}} &= k\overrightarrow{\mathrm{OD}}\\ &= k(\mathbf{c}+\frac{1}{2}\mathbf{a})\\ &= \frac{k}{2}\mathbf{a}+k\mathbf{c}\end{aligned}$$

$$\therefore\quad (1-h)\mathbf{a}+\frac{h}{3}\mathbf{c} = \frac{k}{2}\mathbf{a}+k\mathbf{c}$$

$$(1-h)\mathbf{a}-\frac{k}{2}\mathbf{a} = k\mathbf{c}-\frac{h}{3}\mathbf{c}$$

$$\left(1-h-\frac{k}{2}\right)\mathbf{a} = \left(k-\frac{h}{3}\right)\mathbf{c}$$

$$\begin{aligned}\therefore\quad 1-h-\frac{k}{2} &= 0\\ h+\frac{k}{2} &= 1\\ 2h+k &= 2 && ①\\ \text{and } k-\frac{h}{3} &= 0\\ -h+3k &= 0 && ②\\ 7k &= 2 && (①+2\times②)\\ k &= \frac{2}{7}\\ -h+3\left(\frac{2}{7}\right) &= 0 && (\text{subst into } ②)\\ h &= \frac{6}{7}\end{aligned}$$

12. (a)
$$\begin{aligned}\overrightarrow{\mathrm{OA}} &= (19\mathbf{i}-6\mathbf{j})+t(-2\mathbf{i}+5\mathbf{j})\\ &= (19-2t)\mathbf{i}+(-6+5t)\mathbf{j}\end{aligned}$$

$$\begin{aligned}|\overrightarrow{\mathrm{OA}}| &= 25\\ (19-2t)^2+(-6+5t)^2 &= 25^2\\ 361-76t+4t^2+36-60t+25t^2 &= 625\\ 29t^2-136t+397 &= 625\\ 29t^2-136t-228 &= 0\\ (t-6)(29t+38) &= 0\\ t &= 6\text{s}\end{aligned}$$

(b)
$$\begin{aligned}\overrightarrow{\mathrm{AB}} &= \overrightarrow{\mathrm{OB}}-\overrightarrow{\mathrm{OA}}\\ &= (5\mathbf{i}+\mathbf{j})-((19-2t)\mathbf{i}+(-6+5t)\mathbf{j})\\ &= (5-19+2t)\mathbf{i}+(1+6-5t)\mathbf{j}\\ &= (-14+2t)\mathbf{i}+(7-5t)\mathbf{j}\end{aligned}$$

$$|\overrightarrow{\mathrm{OA}}| = |\overrightarrow{\mathrm{AB}}|$$

$$\begin{aligned}\therefore\quad 29t^2-136t+397 &= (-14+2t)^2+(7-5t)^2\\ 29t^2-136t+397 &= 196-56t+4t^2\\ &\quad +49-70t+25t\\ 29t^2-136t+397 &= 29t^2-126t+245\\ 10t &= 152\\ t &= 15.2\text{s}\end{aligned}$$

Chapter 6

Exercise 6A

1. $2^{-1} = \frac{1}{2^1} = \frac{1}{2}$
2. $4^8 \div 4^6 = 4^{8-6} = 4^2 = 16$
3. $\left(\frac{3}{2}\right)^2 = \frac{3^2}{2^2} = \frac{9}{4} = 2\frac{1}{4}$
4. $18^0 = 1$
5. $\left(4^{0.5}\right)^6 = 4^{0.5\times 6} = 4^3 = 64$
6. $5^6 \times 5^{-8} = 5^{6+-8} = 5^{-2} = \frac{1}{25}$
7. $$\begin{aligned}\frac{3^7 \times 27^2}{3^{14}} &= \frac{3^7 \times \left(3^3\right)^2}{3^{14}} \\ &= \frac{3^7 \times 3^6}{3^{14}} \\ &= \frac{3^{13}}{3^{14}} \\ &= \frac{1}{3}\end{aligned}$$
8. $$\begin{aligned}\frac{5^8 \div 5^4}{125} &= \frac{5^{8-4}}{5^3} \\ &= \frac{5^4}{5^3} \\ &= 5\end{aligned}$$
9. $$\begin{aligned}\frac{7^{10} \div 7^2}{49^2 \times 7^5} &= \frac{7^{10-2}}{\left(7^2\right)^2 \times 7^5} \\ &= \frac{7^8}{7^4 \times 7^5} \\ &= \frac{7^8}{7^9} \\ &= \frac{1}{7}\end{aligned}$$
10. $10\ 000 = 10^4$
11. $0.1 = 10^{-1}$
12. $10^9 \div 10^3 = 10^{9-3} = 10^6$
13. $10^5 \times 1000 = 10^5 \times 10^3 = 10^{5+3} = 10^8$
14. $10^5 \div 1000 = 10^5 \div 10^3 = 10^{5-3} = 10^2$
15. $$\begin{aligned}10^7 \times 10^5 \div 1\ 000\ 000 &= 10^{7+5} \div 10^6 \\ &= 10^{12-6} \\ &= 10^6\end{aligned}$$
16. $\sqrt{10} = 10^{\frac{1}{2}}$
17. $\sqrt[3]{10} = 10^{\frac{1}{3}}$
18. $\sqrt{1000} = \left(10^3\right)^{\frac{1}{2}} = 10^{\frac{3}{2}}$
19. $a^4 \div a^9 = a^{4-9} = a^{-5} \quad \Longrightarrow \quad n = -5$
20. $9 - n = 4 \quad \Longrightarrow \quad n = 5$
21. $n - 9 = 4 \quad \Longrightarrow \quad n = 13$
22. $2 \times 3 = n \quad \Longrightarrow \quad n = 6$
23. $\frac{1}{2} \times n = 5 \quad \Longrightarrow \quad n = 10$
24. $\frac{1}{2} + \frac{1}{3} = n \quad \Longrightarrow \quad n = \frac{5}{6}$
25. $2n + 1 = 7 \quad \Longrightarrow \quad n = 3$
26. $$\begin{aligned}(9-n) - (3+2) &= 3 \\ 9 - n - 5 &= 3 \\ 4 - n &= 3 \quad \Longrightarrow \quad n = 1\end{aligned}$$
27. $5 \times \frac{1}{2} - \frac{1}{2} = n \quad \Longrightarrow \quad n = 2$
28. $a^{4+3} = a^7$
29. $12x^3y^4$
30. $\frac{15a^3b}{10ab^3} = \frac{3}{2}a^2b^{-2} = \frac{3a^2}{2b^2}$
31. $9a^2 \times 8a^6b^3 = 72a^8b^3$
32. $\frac{9a^2}{8a^6b^3} = \frac{9}{8a^4b^3}$
33. $6a^{-1} \times 8b = \frac{48b}{a}$
34. $\frac{6}{3}a^{2--3}b^{-4-1} = 2a^5b^{-5} = \frac{2a^5}{b^5}$
35. $\frac{k^7}{k^3} + \frac{k^3}{k^3} = k^4 + 1$
36. $\frac{p^5}{p^2} - \frac{p^8}{p^2} = p^3 - p^6$
37. $5^{(k+2)-(k-1)} = 5^3 = 125$
38. $\frac{5^n \times 5^2 - 50}{5^n - 2} = \frac{25(5^n - 2)}{5^n - 2} = 25$
39. $\frac{2^h \times 2^3 + 8}{3(2^h + 1)} = \frac{8(2^h + 1)}{3(2^h + 1)} = \frac{8}{3} = 2\frac{2}{3}$

Exercise 6B

1. First determine a by taking the ratio of the numbers in one year to that of the previous year.

$$\frac{16800}{18000} = 0.933$$
$$\frac{15200}{16500} = 0.921$$
$$\frac{14000}{15200} = 0.921$$
$$a \approx 0.92$$

The population each year is about 0.92 that of the previous year.

By 2025 the population is expected to be $18000 \times 0.92^{2025-2002} \approx 2600$

2. First determine a

$$\frac{450}{530} = 0.849$$
$$\frac{385}{450} = 0.856$$
$$\frac{325}{385} = 0.844$$
$$a \approx 0.85$$

Now use this to determine k, the initial population:

$$k \times 0.85^5 = 530$$
$$k = \frac{530}{0.85^5}$$
$$\approx 1200 \text{ frogs}$$

3. (a) From the graph, when $t = 3$, $P \approx 67$.
   (b) From the graph, when $t = 8$, $P \approx 29$.
   (c)
$$a^{8-3} = \frac{29}{67}$$
$$a^5 = 0.43$$
$$a = 0.43^{\frac{1}{5}}$$
$$\approx 0.85$$
$$ka^3 = 67$$
$$k = \frac{67}{0.85^3}$$
$$\approx 110$$
   (d) When $t = 0$, $P = k \approx 110$.

4. (a) At $t = 0$ the populations of A and B are 10 000 and 1 000 respectively.
   (b) After 3 months ($t = 3$) populations are:
$$\mathrm{P_A} = 10\,000(0.75)^3$$
$$\approx 4\,200$$
$$\mathrm{P_B} = 1\,000(1.09)^3$$
$$\approx 1\,300$$
   (c) Calculator shows $xc = 6.1589539$ $yc = 1700.2215$ so we conclude that the populations are equal at $t = 6.2$.

5. (a)

t	0	3	6	10
N	850	640	460	320

Determine an estimate for a:

$$\left(\frac{640}{850}\right)^{\frac{1}{3}} = 0.910$$
$$\left(\frac{460}{640}\right)^{\frac{1}{3}} = 0.896$$
$$\left(\frac{320}{460}\right)^{\frac{1}{4}} = 0.913$$
$$a \approx 0.91$$

An estimate for k can be read directly from the graph—the value of N when t=0: $k \approx 850$.

   (b) Solving $(0.91)^t = \frac{1}{4}$ gives $t \approx 14.7$, so we expect releases to cease after 15 weeks.

Exercise 6C

There is no need for worked solutions for questions 1–32 as these are all single step problems.

33. $64 = 8^2$ so $\log_8 64 = 2$
34. $128 = 2^7$ so $\log_2 128 = 7$
35. $10\,000 = 10^4$ so $\log 10\,000 = 4$
36. $243 = 3^5$ so $\log_3 243 = 5$
37. $\frac{1}{2} = 2^{-1}$ so $\log_2\left(\frac{1}{2}\right) = -1$
38. $\frac{1}{16} = 2^{-4}$ so $\log_2\left(\frac{1}{16}\right) = -4$
39. $216 = 6^3$ so $\log_6\left(\frac{1}{216}\right) = -3$

40. $0.125 = \frac{1}{8} = 2^{-3}$ so $\log_2(0.125) = -3$
41. $243 = 3^5 = \left(9^{\frac{1}{2}}\right)^5 = 9^{2.5}$ so $\log_9 243 = 2.5$
42. $0.001 = 10^{-3}$ so $\log(0.001) = -3$
43. $6 = 6^1$ so $\log_6 6 = 1$
44. $1 = 7^0$ so $\log_7 1 = 0$
45. $1 = a^0$ so $\log_a 1 = 0$ (provided $a \neq 0$)
46. $32 = 2^5 = \left(4^{\frac{1}{2}}\right)^5 = 4^{2.5}$ so $\log_4 32 = 2.5$
47. $a = a^1$ so $\log_a a = 1$ $(a \neq 0)$
48. $\log_a\left(a^3\right) = 3$

Questions 49–56 are straightforward calculator exercises so there is no need for worked solutions.

57. (a) Yes: c is negative for all $b < 1$.
    (b) No: there is no value of c such that 10^c is negative so b can never be negative.

Exercise 6D

1. $\log xz$
2. $\log x^2 + \log y = \log x^2 y$
3. $\log x^2 + \log y^3 = \log x^2 y^3$
4. $\log x^2 - \log y = \log \frac{x^2}{y}$
5. $\log ab - \log c = \log \frac{ab}{c}$
6. $\log a^3 + \log b^4 - \log c^2 = \log \frac{a^3 b^4}{c^2}$
7. $$\begin{aligned} 5\log c - 3\log c + \log a &= 2\log c + \log a \\ &= \log c^2 + \log a \\ &= \log ac^2 \end{aligned}$$
8. $\log 100 + \log x = \log 100x$
9. $$\begin{aligned} \log 1000 - (\log x + \log y) &= \log 1000 - \log xy \\ &= \log \frac{1000}{xy} \end{aligned}$$
10. $\log 1000 - \log x + \log y = \log \frac{1000y}{x}$
11. $\log_2\left(\frac{24}{3}\right) = \log_2 8 = 3$
12. $\log_2 \frac{20\times 8}{10} = \log_2 16 = 4$
13. $4 - 1 = 3$
14. $3 + 2 - 4 = 1$
15. $$\begin{aligned} \log_3 45 + \log_3 2^2 - \log_3 20 &= \log_3 \frac{45\times 4}{20} \\ &= \log_3 9 \\ &= 2 \end{aligned}$$
16. $$\begin{aligned} \log_3 4 - \log_3 36 - 2 &= \log_3 \frac{4}{36} - 2 \\ &= \log_3 \frac{1}{9} - 2 \\ &= -2 - 2 \\ &= -4 \end{aligned}$$
17. $$\begin{aligned} \log 5 - (\log 2 + 2\log 5) &= \log 5 - \log(2\times 5^2) \\ &= \log 5 - \log 50) \\ &= \log \frac{5}{50} \\ &= \log \frac{1}{10} \\ &= -1 \end{aligned}$$
18. $$\begin{aligned} \log_a b + \log_a (ab)^2 - \log_a b^3 &= \log_a \frac{a^2 b^3}{b^3} \\ &= \log_a a^2 \\ &= 2 \end{aligned}$$
19. $\frac{3\log_a b}{2\log_a b} = \frac{3}{2} = 1.5$
20. $$\begin{aligned} \frac{\log_a \frac{48}{3}}{\log_a 2} &= \frac{\log_a 16}{\log_a 2} \\ &= \frac{\log_a 2^4}{\log_a 2} \\ &= \frac{4\log_a 2}{\log_a 2} \\ &= 4 \end{aligned}$$
21. (a) $\log_a(2\times 3) = \log_a 2 + \log_a 3 = p + q$
    (b) $\log_a(2\times 3^2) = \log_a 2 + 2\log_a 3 = p + 2q$
    (c) $\log_a(2^2\times 3) = 2\log_a 2 + \log_a 3 = 2p + q$
    (d) $\log_a \frac{2}{3} = \log_a 2 - \log_a 3 = p - q$
    (e) $\log_a 3^2 + \log_a a^4 = 2\log_a 3 + 4 = 2q + 4$
    (f) $$\begin{aligned} \log_a \frac{2}{9} &= \log_a 2 - \log_a 3^2 \\ &= \log_a 2 - 2\log_a 3 \\ &= p - 2q \end{aligned}$$
22. (a) $\log_5 7^2 = 2\log_5 7 = 2a$
    (b) $\log_5(2^2\times 7) = \log_5 7 + 2\log_5 2 = a + 2b$
    (c) $\log_5 \frac{7}{2^2} = \log_5 7 - 2\log_5 2 = a - 2b$
    (d) $\log_5(5^2\times 2) = \log_5 5^2 + \log_5 2 = 2 + b$

(e) $\log_5(7^2 \times 2 \times 5) = 2\log_5 7 + \log_5 2 + \log_5 5$
$= 2a + b + 1$

(f) $\log_5(7 \times 2^2 \times 5^2) = \log_5 7 + 2\log_5 2 + 2$
$= a + 2b + 2$

23. $y = a^x$

24. $y = 2x$

25. $\log_a y = \log_a x^3$
$y = x^3$

26. $\log_a y^2 = \log_a x^3$
$y^2 = x^3$
$y = x^{\frac{3}{2}}$

27. $\log_a y = \log_a ax$
$y = ax$

28. $\log_a y = \log_a a^2 + \log_a x$
$= \log_a a^2 x$
$y = a^2 x$

29. $\log_a y = \log_a x^{-1}$
$y = x^{-1}$
$y = \frac{1}{x}$

30. $\log_a xy = \log_a a^2$
$xy = a^2$
$y = \frac{a^2}{x}$

31. (a) $75 - 35\log(0+1) = 75$
(b) $75 - 35\log(2+1) \approx 58$
(c) $75 - 35\log(4+1) \approx 51$
(d) $75 - 35\log(t+1) = 40$
$35\log(t+1) = 75 - 40$
$\log(t+1) = 1$
$t + 1 = 10^1$
$t = 10 - 1$
$= 9$ fortnights

32. (a) $R = \log\left(\frac{1000I_0}{I_0}\right) = \log 1000 = 3$
(b) $5.4 = \log\left(\frac{I}{I_0}\right)$
$\frac{I}{I_0} = 10^{5.4}$
$I = 10^{5.4} I_0$
$\approx 250\,000 I_0$
(c) An earthquake measuring 6 has an intensity of $10^6 I_0$ while one measuring 5 has an intensity of $10^5 I_0$, so the former has an intensity **ten times** that of the latter.
(d) $\frac{10^{7.7} I_0}{10^{5.9} I_0} = 10^{7.7-5.9}$
$= 10^{1.8}$
≈ 63 times as intense

33. (a) $-\log_{10} 0.0001 = 4.0$
(b) $-\log_{10} 0.000\,031\,6 \approx 4.5$
(c) $-\log_{10} 0.000\,000\,25 \approx 6.6$
(d) $-\log_{10} 0.000\,000\,016 \approx 7.8$
(e) $-\log_{10} 0.000\,000\,042 \approx 7.4$
(f) $10^{-5.25} \approx 0.000\,005\ 62$ mol/L

34. (a) $10\log_{10}\left(\frac{I}{I_0}\right) = 40$
$\log_{10}\left(\frac{I}{I_0}\right) = 4$
$\frac{I}{I_0} = 10^4$
$I = 10\,000 I_0$
(b) $10\log_{10}\left(\frac{I}{I_0}\right) = 70$
$\log_{10}\left(\frac{I}{I_0}\right) = 7$
$\frac{I}{I_0} = 10^7$
$I = 10\,000\,000 I_0$
(c) $10\log_{10}\left(\frac{I_1}{I_2}\right) = 90 - 20$
$\log_{10}\left(\frac{I_1}{I_2}\right) = \frac{70}{10}$
$\frac{I_1}{I_2} = 10^7$
The intensity of a 90dB noise level is 10 000 000 times that of a 20dB noise level.

Exercise 6E

1. $$\begin{aligned}\log 3^x &= \log 7\\ x\log 3 &= \log 7\\ x &= \frac{\log 7}{\log 3}\end{aligned}$$

2. $$\begin{aligned}\log 7^x &= \log 1000\\ x\log 7 &= 3\\ x &= \frac{3}{\log 7}\end{aligned}$$

3. $$\begin{aligned}\log 10^x &= \log 27\\ x\log 10 &= \log 27\\ x &= \log 27\\ &= 3\log 3\end{aligned}$$

4. $$\begin{aligned}\log 2^x &= \log 11\\ x\log 2 &= \log 11\\ x &= \frac{\log 11}{\log 2}\end{aligned}$$

5. $$\begin{aligned}\log 3^x &= \log 17\\ x\log 3 &= \log 17\\ x &= \frac{\log 17}{\log 3}\end{aligned}$$

6. $$\begin{aligned}\log 7^x &= \log 80\\ x\log 7 &= \log 80\\ x &= \frac{\log 80}{\log 7}\\ &= \frac{\log\left(10\times 2^3\right)}{\log 7}\\ &= \frac{1+3\log 2}{\log 7}\end{aligned}$$

7. $$\begin{aligned}\log 5^x &= \log 21\\ x\log 5 &= \log 21\\ x &= \frac{\log 21}{\log 5}\end{aligned}$$

8. $$\begin{aligned}\log 10^x &= \log 15\\ x\log 10 &= \log 15\\ x &= \log 15\end{aligned}$$

9. $$\begin{aligned}\log 2^x &= \log 70\\ x\log 2 &= \log 70\\ x &= \frac{\log 70}{\log 2}\\ x &= \frac{\log(7\times 10)}{\log 2}\\ x &= \frac{1+\log 7}{\log 2}\end{aligned}$$

10. $$\begin{aligned}\log 6^{(x+2)} &= \log 17\\ (x+2)\log 6 &= \log 17\\ x+2 &= \frac{\log 17}{\log 6}\\ x &= \frac{\log 17}{\log 6}-2\\ x &= \frac{\log 17}{\log 6}-\frac{2\log 6}{\log 6}\\ &= \frac{\log 17-\log 36}{\log 6}\\ &= \frac{\log\frac{17}{36}}{\log 6}\end{aligned}$$

11. $$\begin{aligned}\log 3^{(x+1)} &= \log 51\\ (x+1)\log 3 &= \log 51\\ x+1 &= \frac{\log 51}{\log 3}\\ x &= \frac{\log(17\times 3)}{\log 3}-1\\ x &= \frac{\log 17+\log 3}{\log 3}-1\\ x &= \frac{\log 17}{\log 3}+1-1\\ x &= \frac{\log 17}{\log 3}\end{aligned}$$

12. $$\begin{aligned}\log 8^{(x-1)} &= \log 7\\ (x-1)\log 8 &= \log 7\\ x-1 &= \frac{\log 7}{\log 8}\\ x &= \frac{\log 7}{\log 8}+1\\ x &= \frac{\log 7}{\log 8}+\frac{\log 8}{\log 8}\\ &= \frac{\log 7+\log 8}{\log 2^3}\\ &= \frac{\log 56}{3\log 2}\end{aligned}$$

13. $$\begin{aligned}\log 5^{(x-1)} &= \log 3^{2x}\\ (x-1)\log 5 &= 2x\log 3\\ x\log 5-\log 5 &= 2x\log 3\\ x\log 5-2x\log 3 &= \log 5\\ x(\log 5-2\log 3) &= \log 5\\ x &= \frac{\log 5}{\log 5-2\log 3}\\ x &= \frac{\log 5}{\log\frac{5}{9}}\end{aligned}$$

14.
$$\begin{aligned}
\log 2^{(x+1)} &= \log 3^x \\
(x+1)\log 2 &= x\log 3 \\
x\log 2 + \log 2 &= x\log 3 \\
x\log 2 - x\log 3 &= -\log 2 \\
x(\log 2 - \log 3) &= -\log 2 \\
x &= -\frac{\log 2}{\log 2 - \log 3} \\
x &= -\frac{\log 2}{\log \frac{2}{3}} \qquad \text{(see below)} \\
&= \frac{\log \frac{1}{2}}{\log \frac{2}{3}}
\end{aligned}$$

$$\text{alternatively:} \quad x = \frac{\log 2}{\log 3 - \log 2} = \frac{\log \frac{1}{2}}{\log \frac{3}{2}}$$

Although they look different, these two answers are equivalent.

15.
$$\begin{aligned}
\log 4^{3x} &= \log 5^{(x+2)} \\
3x\log 4 &= (x+2)\log 5 \\
3x\log 2^2 &= x\log 5 + 2\log 5 \\
6x\log 2 &= x\log 5 + 2\log 5 \\
x(6\log 2 - \log 5) &= 2\log 5 \\
x &= \frac{2\log 5}{6\log 2 - \log 5} \\
x &= \frac{2\log 5}{\log \frac{64}{5}}
\end{aligned}$$

16.
$$\begin{aligned}
\log 3^{(2x+1)} &= \log 2^{(3x-1)} \\
(2x+1)\log 3 &= (3x-1)\log 2 \\
2x\log 3 + \log 3 &= 3x\log 2 - \log 2 \\
2x\log 3 - 3x\log 2 &= -\log 3 - \log 2 \\
x(2\log 3 - 3\log 2) &= -\log 6 \\
x(3\log 2 - 2\log 3) &= \log 6 \\
x &= \frac{\log 6}{3\log 2 - 2\log 3} \\
x &= \frac{\log 6}{\log \frac{8}{9}}
\end{aligned}$$

17.
$$\begin{aligned}
5\left(2^x\right) &= 3 - 2^x \times 2^2 \\
5\left(2^x\right) &= 3 - 4\left(2^x\right) \\
9\left(2^x\right) &= 3 \\
\left(2^x\right) &= \frac{1}{3} \\
\log\left(2^x\right) &= \log \frac{1}{3} \\
x\log 2 &= -\log 3 \\
x &= -\frac{\log 3}{\log 2}
\end{aligned}$$

18.
$$\begin{aligned}
5^x + 4\left(5^x \times 5^1\right) &= 63 \\
5^x + 20\left(5^x\right) &= 63 \\
5^x(1+20) &= 63 \\
5^x &= \frac{63}{21} \\
&= 3 \\
\log 5^x &= \log 3 \\
x\log 5 &= \log 3 \\
x &= \frac{\log 3}{\log 5}
\end{aligned}$$

19.
$$\begin{aligned}
y^2 + 3y - 18 &= 0 \\
(y+6)(y-3) &= 0 \\
y &= 3 \qquad (\text{rejecting } y < 0) \\
2^x &= 3 \\
x\log 2 &= \log 3 \\
x &= \frac{\log 3}{\log 2}
\end{aligned}$$

20.
$$\begin{aligned}
\left(2^x\right)^2 - 2^3\left(2^x\right) + 15 &= 0 \\
y^2 - 8y + 15 &= 0 \\
(y-3)(y-5) &= 0 \\
y &= 3 \qquad \text{or } y = 5 \\
2^x &= 3 \qquad \text{or } 2^x = 5 \\
x\log 2 &= \log 3 \qquad \text{or } x\log 2 = \log 5 \\
x &= \frac{\log 3}{\log 2} \qquad \text{or } x = \frac{\log 5}{\log 2}
\end{aligned}$$

21.
$$\begin{aligned}
2^x &= 7 \\
x\log 2 &= \log 7 \\
x &= \frac{\log 7}{\log 2}
\end{aligned}$$

22. These are single-step problems, following the example of question 21.

23. n must be at least that given by solving

$$\begin{aligned}
0.92^n &= 0.2 \\
n\log 0.92 &= \log 0.2 \\
n &= \frac{\log 0.2}{\log 0.92} \\
&= 19.3
\end{aligned}$$

The metal must be passed through the rollers 20 times.

24. (a) $N = 200(2.7)^{0.1\times 3} \approx 270$

(b) $N = 200(2.7)^{0.1\times 5} \approx 330$

(c) $200(2.7)^{0.1t} = 1000$

$$\begin{aligned}
2.7^{0.1t} &= 50.1t\log 2.7 \quad = \log 5 \\
0.1t &= \frac{\log 5}{\log 2.7} \\
t &= 10\frac{\log 5}{\log 2.7} \\
&\approx 16.2
\end{aligned}$$

The population first exceeded 1000 on the 17th day.

25.
$$\begin{aligned} 2.8^{20a} &= 51 \\ 20a \log 2.8 &= \log 51 \\ 20a &= \frac{\log 51}{\log 2.8} \\ a &= \frac{\log 51}{20 \log 2.8} \\ &= 0.19 \end{aligned}$$

The percentage blood alcohol level is 0.19%.

26. (a) $N = 100\,000 + 150\,000(1.1)^{-0.8\times 4} \approx 211\,000$

(b) $N = 100\,000 + 150\,000(1.1)^{-0.8\times 8} \approx 182\,000$

$$\begin{aligned} 100\,000 + 150\,000(1.1)^{-0.8t} &= 135\,000 \\ 150\,000(1.1)^{-0.8t} &= 35\,000 \\ 1.1^{-0.8t} &= \frac{35\,000}{150\,000} \\ &= \frac{7}{30} \\ -0.8t \log 1.1 &= \log \frac{7}{30} \\ -0.8t &= \frac{\log \frac{7}{30}}{\log 1.1} \\ t &= -\frac{\log \frac{7}{30}}{0.8 \log 1.1} \\ &\approx 19.1 \end{aligned}$$

Sales fall to 135 000 approximately 19 weeks after the campaign ceases.

27. (a) $P = 10\,000(1.08)^3 = \$12\,597$

(b) $P = 10\,000(1.08)^7 = \$17\,138$

(c)
$$\begin{aligned} 10\,000(1.08)^t &= 50\,000 \\ (1.08)^t &= 5 \\ t \log 1.08 &= \log 5 \\ t &= \frac{\log 5}{\log 1.08} \\ &\approx 21 \text{ years} \end{aligned}$$

(d) i.
$$\begin{aligned} t &= \frac{\log 5}{\log 1.10} \\ &\approx 17 \text{ years} \end{aligned}$$

ii. After the first 8 years,

$$\begin{aligned} P &= 10\,000(1.14)^8 \\ &= 28\,526 \end{aligned}$$

thereafter: $P = 28\,526(1.10)^{t-8}$ so we need to solve

$$\begin{aligned} 28\,526(1.10)^{t-8} &= 50\,000 \\ 1.10^{t-8} &= \frac{50\,000}{28\,526} \\ (t-8)\log(1.1) &= \log \frac{50\,000}{28\,526} \\ t - 8 &= \frac{\log \frac{50\,000}{28\,526}}{\log 1.1} \\ t &= \frac{\log \frac{50\,000}{28\,526}}{\log 1.1} + 8 \\ &\approx 14 \text{ years} \end{aligned}$$

(e)
$$\begin{aligned} (1+r)^5 &= 2 \\ 1 + r &= 2^{\frac{1}{5}} \\ r &= 2^{\frac{1}{5}} - 1 \\ &\approx 0.149 \end{aligned}$$

The required interest rate is 14.9%.

Miscellaneous Exercise 6

1.
$$\begin{aligned} \log 2^x &= \log 11 \\ x \log 2 &= \log 11 \\ x &= \frac{\log 11}{\log 2} \end{aligned}$$

2. (a)
$$\begin{aligned} \log_a 25 &= \log_a 5^2 \\ &= 2 \log_a 5 \\ &= 2p \end{aligned}$$

(b)
$$\begin{aligned} \log_a 20 &= \log_a(5 \times 4) \\ &= \log_a 5 + \log_a 4 \\ &= p + q \end{aligned}$$

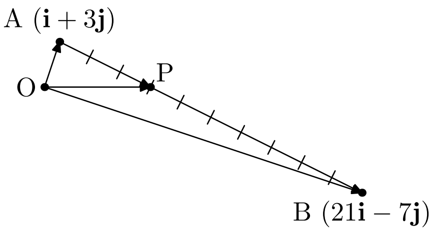
(c)
$$\begin{aligned} \log_a 80 &= \log_a(5 \times 4^2) \\ &= \log_a 5 + \log_a 4^2 \\ &= \log_a 5 + 2 \log_a 4 \\ &= p + 2q \end{aligned}$$

(d)
$$\begin{aligned} \log_a 0.8 &= \log_a \frac{4}{5} \\ &= \log_a 4 - \log_a 5 \\ &= q - p \end{aligned}$$

(e) $\log_a 20a^3 = \log_a(5 \times 4 \times a^3)$
$= \log_a 5 + \log_a 4 + \log_a a^3$
$= p + q + 3$

(f) $\log_5 4 = \dfrac{\log_a 4}{\log_a 5}$
$= \dfrac{q}{p}$

3.



$$\overrightarrow{AP} = \frac{3}{10}\overrightarrow{AB}$$
$$\overrightarrow{OP} - \overrightarrow{OA} = \frac{3}{10}(\overrightarrow{OB} - \overrightarrow{OA})$$
$$\overrightarrow{OP} = \frac{3}{10}\overrightarrow{OB} + \frac{7}{10}\overrightarrow{OA}$$
$$= (6.3\mathbf{i} - 2.1\mathbf{j}) + (0.7\mathbf{i} + 2.1\mathbf{i})$$
$$= 7\mathbf{i}$$

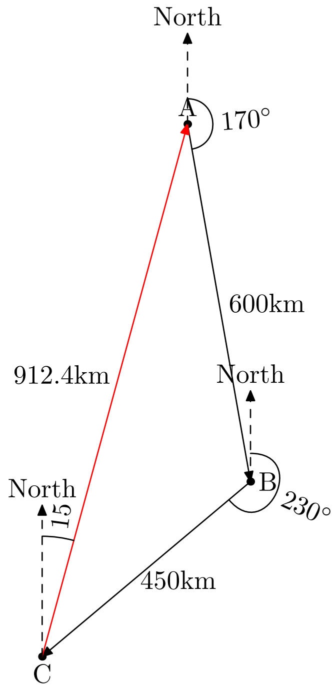
4. $Q = Q_0(1 - 0.12)^t = Q_0(0.88)^t$

For just 5% to remain, $\frac{Q}{Q_0} = 0.05$ so we must solve

$$0.88^t = 0.05$$
$$t \log 0.88 = \log 0.05$$
$$t = \frac{\log 0.05}{\log 0.88}$$
$$\approx 23.4 \text{ minutes}$$

If the 5% is a maximum, we should run the pump for about 24 minutes.

5.

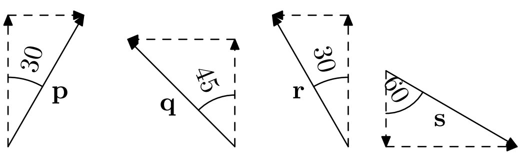


$$\angle ABC = 360 - 230 - 10$$
$$= 120°$$

$$d = \sqrt{600^2 + 450^2 - 2 \times 600 \times 450 \cos 120°}$$
$$\approx 912\text{km}$$

$$\frac{\sin \angle BAC}{450} = \frac{\sin 120°}{912.4}$$
$$\angle BAC = \sin^{-1}\frac{450 \sin 120°}{912.4}$$
$$\approx 25.4°$$
$$\text{bearing of C from A} = 170 + 25.4$$
$$= 195.4°$$
$$\text{bearing of A from C} = 195.4 + 25.4 - 180$$
$$\approx 15°$$

6.



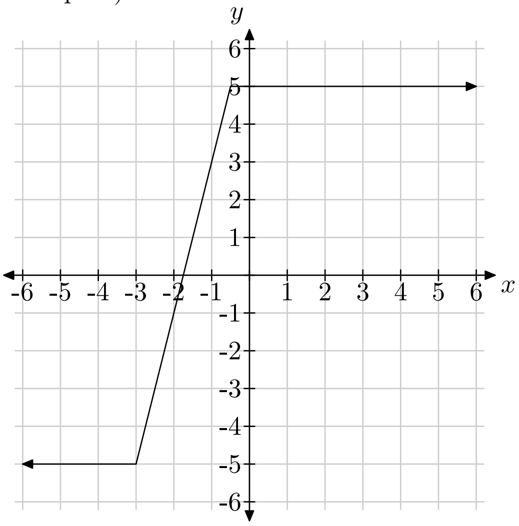
$$\mathbf{p} = 6 \sin 30°\mathbf{i} + 6 \cos 30°\mathbf{j}$$
$$= 3\mathbf{i} + 3\sqrt{3}\mathbf{j}$$

$$\mathbf{q} = -8\sqrt{2} \sin 45°\mathbf{i} + 8\sqrt{2} \cos 45°\mathbf{j}$$
$$= -8\mathbf{i} + 8\mathbf{j}$$

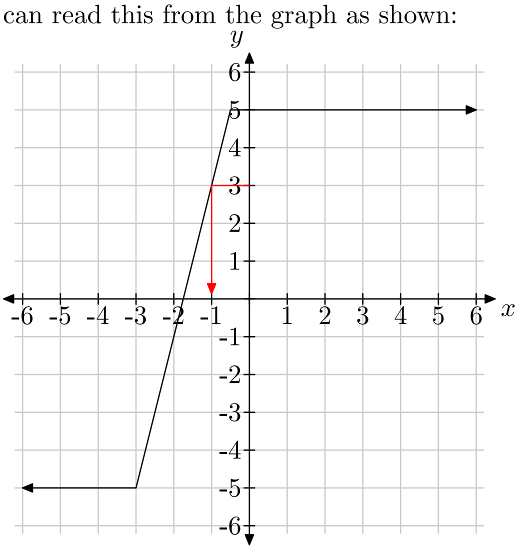
$$\mathbf{r} = -10 \sin 30°\mathbf{i} + 10 \cos 30°\mathbf{j}$$
$$= -5\mathbf{i} + 5\sqrt{3}\mathbf{j}$$

$$\mathbf{s} = 8 \sin 60°\mathbf{i} - 8 \cos 60°\mathbf{j}$$
$$= 4\sqrt{3}\mathbf{i} - 4\mathbf{j}$$

7. Start by drawing a graph of the left hand side: $y = 2|x+3| - |2x+1|$ (If you were to attempt this manually you would need to consider three parts of the domain, just as you would if you were to solve it algebraically. I'll assume that you can draw the graph, at least with the help of your Classpad.)

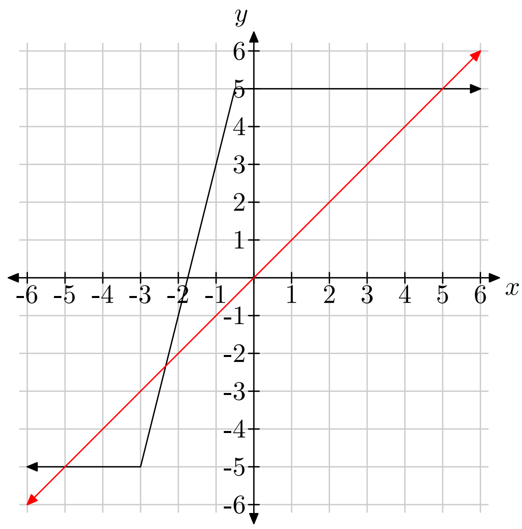


(a) There is just one value of x that results in the left hand side having a value of 3. You can read this from the graph as shown:



Solution: $x = -1$.

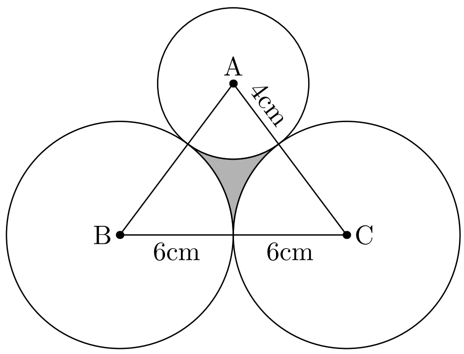
(b) To solve the second equation, superimpose on the graph the line $y = x$ and locate the points where RHS and LHS intersect.



Solution: $x = -5$, $x = -2\frac{1}{3}$, or $x = 5$

(c) It should be clear from the graph that the LHS is equal to 5 for any value of $x \geq -\frac{1}{2}$. Solution: $x \geq -\frac{1}{2}$

8.



$$\begin{aligned}
\angle A &= \cos^{-1} \frac{10^2 + 10^2 - 12^2}{2 \times 10 \times 10} \\
&\approx 1.287 \\
\angle B &= \cos^{-1} \frac{10^2 + 12^2 - 10^2}{2 \times 10 \times 12} \\
&\approx 0.927 \\
\angle C &= \angle B \\
&\approx 0.927 \\
a_{\triangle} &= \frac{1}{2} ab \sin C \\
&= \frac{1}{2} \times 12 \times 10 \sin 0.927 \\
&= 48 \\
a_{\text{sector A}} &= \frac{1}{2} \times 4^2 \times 1.287 \\
&\approx 10.30 \\
a_{\text{sector B}} &= \frac{1}{2} \times 6^2 \times 0.927 \\
&\approx 16.69 \\
a_{\text{sector C}} &= a_{\text{sector B}} \\
&\approx 16.69 \\
a_{\text{shaded}} &= a_{\triangle} - a_{\text{sector A}} - a_{\text{sector B}} - a_{\text{sector C}} \\
&= 48 - 10.30 - 2 \times 16.69 \\
&\approx 4.3\text{cm}^2
\end{aligned}$$

Chapter 7

Exercise 7A

1. (a) $\mathbf{r}_A = 2\mathbf{i} + 3\mathbf{j}$
   (b) $\mathbf{r}_B = 4\mathbf{i} + 5\mathbf{j}$
   (c) $\mathbf{r}_C = \mathbf{i} + 4\mathbf{j}$
   (d) ${}_A\mathbf{r}_B = \mathbf{r}_A - \mathbf{r}_B = -2\mathbf{i} - 2\mathbf{j}$
   (e) ${}_A\mathbf{r}_C = \mathbf{r}_A - \mathbf{r}_C = \mathbf{i} - \mathbf{j}$
   (f) ${}_A\mathbf{r}_D = \mathbf{r}_A - \mathbf{r}_D = -2\mathbf{i} + 2\mathbf{j}$
   (g) ${}_D\mathbf{r}_A = \mathbf{r}_D - \mathbf{r}_A = 2\mathbf{i} - 2\mathbf{j}$
   (h) ${}_B\mathbf{r}_C = \mathbf{r}_B - \mathbf{r}_C = 3\mathbf{i} + \mathbf{j}$
   (i) ${}_C\mathbf{r}_D = \mathbf{r}_C - \mathbf{r}_D = -3\mathbf{i} + 3\mathbf{j}$
   (j) ${}_D\mathbf{r}_C = \mathbf{r}_D - \mathbf{r}_C = 3\mathbf{i} - 3\mathbf{j}$

2. (a) $\mathbf{r}_A$ is given
   (b) $\mathbf{r}_B$ is given
   (c) ${}_C\mathbf{r}_A = \mathbf{r}_C - \mathbf{r}_A$ so
   $\mathbf{r}_C = {}_C\mathbf{r}_A + \mathbf{r}_A = 2\mathbf{i} + \mathbf{j}$
   (d) $\mathbf{r}_D = {}_D\mathbf{r}_A + \mathbf{r}_A = \mathbf{i} + 3\mathbf{j}$
   (e) $\mathbf{r}_E$ is given
   (f) $\mathbf{r}_F = {}_F\mathbf{r}_E + \mathbf{r}_E = -\mathbf{i} + 5\mathbf{j}$
   (g) $\mathbf{r}_G = {}_G\mathbf{r}_D + \mathbf{r}_D = -3\mathbf{i} + 2\mathbf{j}$
   (h) $\mathbf{r}_H = {}_H\mathbf{r}_G + \mathbf{r}_G = -2\mathbf{i} + \mathbf{j}$
   (i) $\mathbf{r}_I = {}_I\mathbf{r}_A + \mathbf{r}_A = 5\mathbf{i} + 5\mathbf{j}$
   (j) ${}_I\mathbf{r}_J = \mathbf{r}_I - \mathbf{r}_J$ so
   $\mathbf{r}_J = \mathbf{r}_I - {}_I\mathbf{r}_J = 5\mathbf{i} + 2\mathbf{j}$
   (k) $\mathbf{r}_K = \mathbf{r}_H - {}_H\mathbf{r}_K = 3\mathbf{i}$
   (l) $\mathbf{r}_L = \mathbf{r}_I - {}_I\mathbf{r}_L = 4\mathbf{j}$

3. $\mathbf{r}_A = 7\mathbf{i} + 11\mathbf{j}$
   ${}_B\mathbf{r}_A = -12\mathbf{i} - 8\mathbf{j}$
   $\mathbf{r}_B = \mathbf{r}_A + {}_B\mathbf{r}_A = (-5\mathbf{i} + 3\mathbf{j})$km.

4. $\overrightarrow{AB}$ represents the displacement *from* A *to* B, thus it is the displacement of B relative to A.
   Note the importance of understanding this result:
   $$\overrightarrow{AB} = {}_B\mathbf{r}_A$$
   Some of you will get this the wrong way around if you are not careful!

5. $\mathbf{r}_A = 11\mathbf{i} - 3\mathbf{j}$
   ${}_A\mathbf{r}_B = -8\mathbf{i} - 8\mathbf{j}$
   $\mathbf{r}_B = \mathbf{r}_A - {}_A\mathbf{r}_B = 19\mathbf{i} + 5\mathbf{j}$
   $|\mathbf{r}_B| = \sqrt{19^2 + 5^2} = \sqrt{386}$km.

6. $\mathbf{r}_A = 5\mathbf{i} + 2\mathbf{j}$
   ${}_A\mathbf{r}_B = 8\mathbf{i} + 3\mathbf{j}$
   $\mathbf{r}_B = \mathbf{r}_A - {}_A\mathbf{r}_B = -3\mathbf{i} - \mathbf{j}$
   $|\mathbf{r}_B| = \sqrt{3^2 + 1^2} = \sqrt{10}$km.

7. $\mathbf{r}_A = 2\mathbf{i} + 4\mathbf{j}$
   ${}_C\mathbf{r}_A = -4\mathbf{i} - \mathbf{j}$
   $\mathbf{r}_C = \mathbf{r}_A + {}_C\mathbf{r}_A = -2\mathbf{i} + 3\mathbf{j}$
   $\mathbf{r}_B = \mathbf{r}_C + {}_B\mathbf{r}_C = 5\mathbf{i} - \mathbf{j}$

8. LHS:
$$\begin{aligned} {}_D\mathbf{r}_E + {}_E\mathbf{r}_F &= (\mathbf{r}_D - \mathbf{r}_E) + (\mathbf{r}_E - \mathbf{r}_F) \\ &= \mathbf{r}_D - \mathbf{r}_E + \mathbf{r}_E - \mathbf{r}_F \\ &= \mathbf{r}_D - \mathbf{r}_F \\ &= {}_D\mathbf{r}_F \\ &= \text{ RHS. Q.E.D.} \end{aligned}$$

Exercise 7B

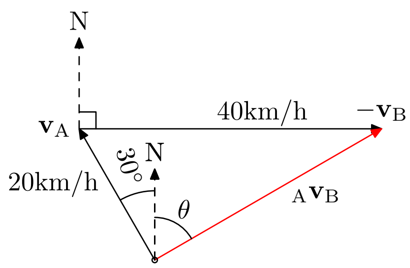
1. ${}_A\mathbf{v}_B = \mathbf{v}_A - \mathbf{v}_B = (2\mathbf{i} - 3\mathbf{j}) - (-4\mathbf{i} + 7\mathbf{j}) = 6\mathbf{i} - 10\mathbf{j}$

2. ${}_A\mathbf{v}_B = (4\mathbf{i} + 2\mathbf{j}) - (7\mathbf{i} - \mathbf{j}) = -3\mathbf{i} + 3\mathbf{j}$

3. ${}_A\mathbf{v}_B = (3\mathbf{i} - 2\mathbf{j}) - (4\mathbf{i} + 7\mathbf{j}) = -\mathbf{i} - 9\mathbf{j}$

4. ${}_A\mathbf{v}_B = (6\mathbf{i} + 2\mathbf{j}) - (3\mathbf{i} - \mathbf{j}) = 3\mathbf{i} + 3\mathbf{j}$

5. ${}_A\mathbf{v}_B = \mathbf{v}_A + (-\mathbf{v}_B)$



By the cosine rule,

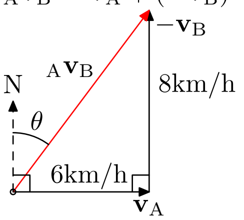
$$\begin{aligned} |{}_A\mathbf{v}_B|^2 &= 20^2 + 40^2 - 2 \times 20 \times 40 \cos 60° \\ |{}_A\mathbf{v}_B| &= 20\sqrt{3} \\ &\approx 36.64\text{km/h} \end{aligned}$$

By the sine rule,

$$\begin{aligned}\frac{\sin(\theta+30^\circ)}{40} &= \frac{\sin 60^\circ}{20\sqrt{3}}\\ \sin(\theta+30^\circ) &= \frac{40\times\frac{\sqrt{3}}{2}}{20\sqrt{3}}\\ &= 1\\ \therefore\quad \theta+30^\circ &= 90^\circ\\ \theta &= 60^\circ\end{aligned}$$

${}_{A}\mathbf{v}_{B} = 36.64$km/h on a bearing of 060°.

6. ${}_{A}\mathbf{v}_{B} = \mathbf{v}_A + (-\mathbf{v}_B)$



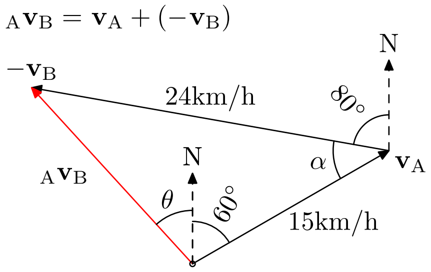
By Pythagoras,

$$\begin{aligned}|{}_{A}\mathbf{v}_{B}|^2 &= 6^2+8^2\\ |{}_{A}\mathbf{v}_{B}| &= 10\text{km/h}\end{aligned}$$

$$\begin{aligned}\tan(90^\circ-\theta) &= \frac{8}{6}\\ 90^\circ-\theta &\approx 53.13^\circ\\ \theta &\approx 36.87^\circ\end{aligned}$$

${}_{A}\mathbf{v}_{B} = 10$km/h on a bearing of 037°.

7. ${}_{A}\mathbf{v}_{B} = \mathbf{v}_A + (-\mathbf{v}_B)$



$$\begin{aligned}60+(80+\alpha) &= 180 \text{ (cointerior angles)}\\ \alpha &= 40^\circ\end{aligned}$$

By the cosine rule,

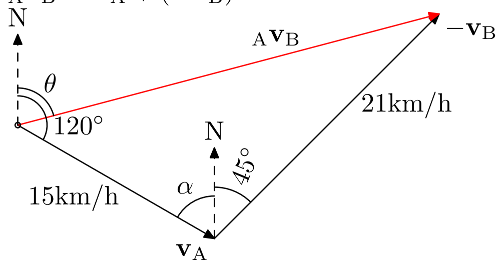
$$\begin{aligned}|{}_{A}\mathbf{v}_{B}|^2 &= 24^2+15^2-2\times24\times15\cos40^\circ\\ |{}_{A}\mathbf{v}_{B}| &\approx 15.79\text{km/h}\end{aligned}$$

By the cosine rule again (since the sine rule would be ambiguous here),

$$\begin{aligned}\theta+60 &= \cos^{-1}\frac{15^2+15.79^2-24^2}{2\times15\times15.79}\\ &= 102.38^\circ\\ \theta &= 42.38^\circ\end{aligned}$$

${}_{A}\mathbf{v}_{B} = 15.8$km/h on a bearing of 318°.

8. ${}_{A}\mathbf{v}_{B} = \mathbf{v}_A + (-\mathbf{v}_B)$



$$\begin{aligned}\alpha &= 180-120 \text{ (cointerior angles)}\\ &= 60^\circ\\ \alpha+45 &= 105^\circ\end{aligned}$$

By the cosine rule,

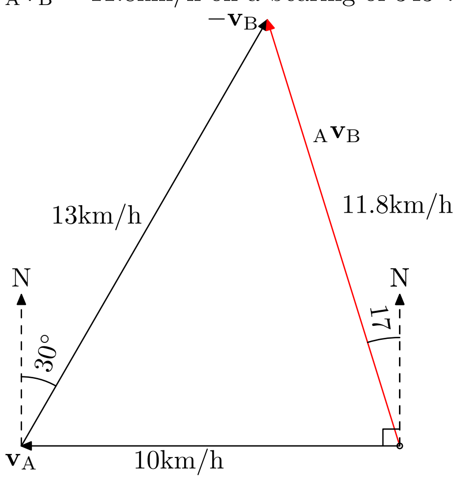
$$\begin{aligned}|{}_{A}\mathbf{v}_{B}|^2 &= 21^2+15^2-2\times21\times15\cos105^\circ\\ |{}_{A}\mathbf{v}_{B}| &= 28.79\text{km/h}\end{aligned}$$

By the sine rule (unambiguous here),

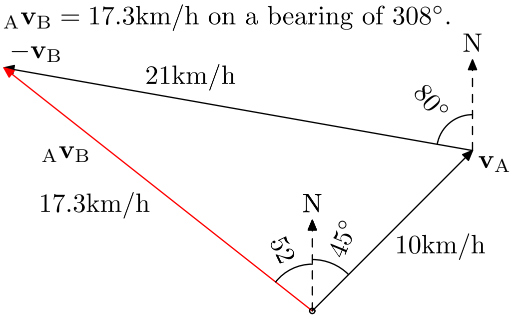
$$\begin{aligned}\frac{\sin(120^\circ-\theta)}{21} &= \frac{\sin 105^\circ}{28.79}\\ 120^\circ-\theta &= \sin^{-1}\frac{21\sin105^\circ}{28.79}\\ &= 44.79^\circ\\ \theta &= 120-44.79\\ &= 75.21^\circ\end{aligned}$$

${}_{A}\mathbf{v}_{B} = 28.8$km/h on a bearing of 075°.

9. ${}_{A}\mathbf{v}_{B} = 11.8$km/h on a bearing of 343°.



10. ${}_{A}\mathbf{v}_{B} = 17.3$km/h on a bearing of 308°.

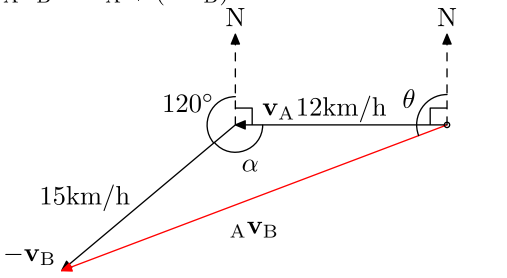


11. $\mathbf{v}_A = 7\mathbf{i}-10\mathbf{j}$
$\mathbf{v}_B = 2\mathbf{j}+20\mathbf{j}$

(a) ${}_{A}\mathbf{v}_{B} = \mathbf{v}_A - \mathbf{v}_B = 5\mathbf{i}-30\mathbf{j}$km/h

(b) ${}_{B}\mathbf{v}_{A} = \mathbf{v}_{B} - \mathbf{v}_{A} = -5\mathbf{i} + 30\mathbf{j}\text{km/h}$

12. (a) ${}_{A}\mathbf{v}_{B} = \mathbf{v}_{A} + (-\mathbf{v}_{B})$



$$\alpha = 360 - 90 - 120$$
$$= 150°$$

By the cosine rule,

$$|{}_{A}\mathbf{v}_{B}|^2 = 12^2 + 15^2 - 2 \times 12 \times 15 \cos 150°$$
$$|{}_{A}\mathbf{v}_{B}| = 26.09\text{km/h}$$

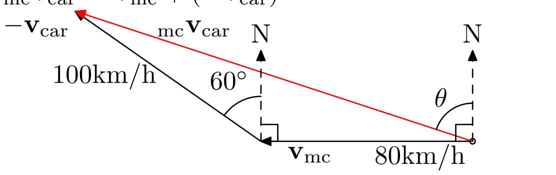
By the sine rule (unambiguous here),

$$\frac{\sin(\theta - 90)}{15} = \frac{\sin 150°}{26.09}$$
$$\theta - 90 = \sin^{-1}\frac{15 \sin 150°}{26.09}$$
$$= 16.71°$$
$$\theta = 106.71°$$

${}_{A}\mathbf{v}_{B} = 26.1\text{km/h}$ on a bearing of 253°.

(b) ${}_{B}\mathbf{v}_{A} = \mathbf{v}_{B} + (-\mathbf{v}_{A}) = -{}_{A}\mathbf{v}_{B}$
(Same diagram with all the arrows reversed.)
${}_{B}\mathbf{v}_{A} = 26.1\text{km/h}$ on a bearing of 073°.

13. ${}_{mc}\mathbf{v}_{car} = \mathbf{v}_{mc} + (-\mathbf{v}_{car})$



By the cosine rule,

$$|{}_{mc}\mathbf{v}_{car}|^2 = 100^2 + 80^2 - 2 \times 100 \times 80 \cos 150°$$
$$|{}_{mc}\mathbf{v}_{car}| = 173.94\text{km/h}$$

By the sine rule (unambiguous here),

$$\frac{\sin(90 - \theta)}{100} = \frac{\sin 150°}{173.94}$$
$$90 - \theta = \sin^{-1}\frac{100 \sin 150°}{173.94}$$
$$= 16.91°$$
$$\theta = 73.29°$$

${}_{mc}\mathbf{v}_{car} = 174\text{km/h}$ on a bearing of 287°.

14. ${}_{A}\mathbf{v}_{B} = \mathbf{v}_{A} - \mathbf{v}_{B}$
$\mathbf{v}_{B} = \mathbf{v}_{A} - {}_{A}\mathbf{v}_{B} = \mathbf{i} - 12\mathbf{j}$

15. ${}_{B}\mathbf{v}_{A} = \mathbf{v}_{B} - \mathbf{v}_{A}$
$\mathbf{v}_{B} = \mathbf{v}_{A} + {}_{B}\mathbf{v}_{A} = -2\mathbf{i} + 6\mathbf{j}$

16. LHS:

$$\begin{aligned} {}_{A}\mathbf{v}_{B} + {}_{B}\mathbf{v}_{C} &= (\mathbf{v}_{A} - \mathbf{v}_{B}) + (\mathbf{v}_{B} - \mathbf{v}_{C}) \\ &= \mathbf{v}_{A} - \mathbf{v}_{B} + \mathbf{v}_{B} - \mathbf{v}_{C} \\ &= \mathbf{v}_{A} - \mathbf{v}_{C} \\ &= {}_{A}\mathbf{r}_{C} \\ &= \text{ RHS. Q.E.D.} \end{aligned}$$

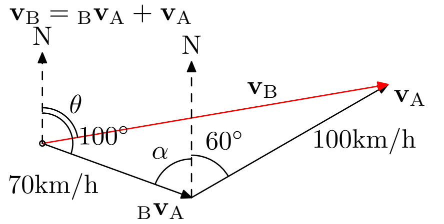
17. ${}_{A}\mathbf{v}_{B} = \mathbf{v}_{A} - \mathbf{v}_{B}$

$$\begin{aligned} \mathbf{v}_{B} &= \mathbf{v}_{A} - {}_{A}\mathbf{v}_{B} \\ &= 20 - 30 \\ &= -10\text{km/h due North} \\ &= 10\text{km/h due South} \end{aligned}$$

18. ${}_{A}\mathbf{v}_{B} = \mathbf{v}_{A} - \mathbf{v}_{B}$

$$\begin{aligned} \mathbf{v}_{B} &= \mathbf{v}_{A} - {}_{A}\mathbf{v}_{B} \\ &= 80 - 60 \\ &= 20\text{km/h due South} \end{aligned}$$

19. $\mathbf{v}_{B} = {}_{B}\mathbf{v}_{A} + \mathbf{v}_{A}$



$$\begin{aligned} \alpha &= 180 - 100 \text{ (cointerior angles)} \\ &= 80° \\ \alpha + 60 &= 140° \end{aligned}$$

By the cosine rule,

$$|\mathbf{v}_{B}|^2 = 70^2 + 100^2 - 2 \times 70 \times 100 \cos 140°$$
$$|{}_{A}\mathbf{v}_{B}| = 160.08\text{km/h}$$

By the sine rule (unambiguous here),

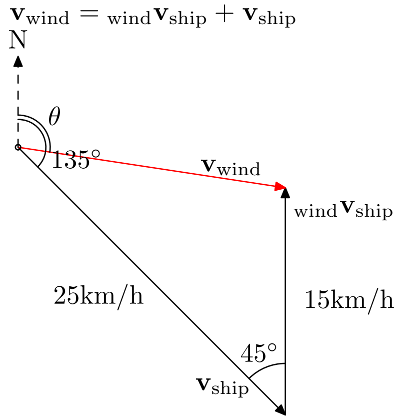
$$\frac{\sin(100° - \theta)}{100} = \frac{\sin 140°}{160.08}$$
$$100° - \theta = \sin^{-1}\frac{100 \sin 140°}{160.08}$$
$$= 23.68°$$
$$\theta = 100 - 23.68$$
$$= 76.32°$$

$\mathbf{v}_{B} = 160\text{km/h}$ on a bearing of 076°.

20. $\mathbf{v}_{wind} = {}_{wind}\mathbf{v}_{ship} + \mathbf{v}_{ship} = (13\mathbf{i} + \mathbf{j})\text{km/h}$

21. $\mathbf{v}_{wind} = {}_{wind}\mathbf{v}_{walker} + \mathbf{v}_{walker} = (4\mathbf{i} - 2\mathbf{j})\text{km/h}$

22. $\mathbf{v}_{\text{wind}} = {}_{\text{wind}}\mathbf{v}_{\text{ship}} + \mathbf{v}_{\text{ship}}$



By the cosine rule,

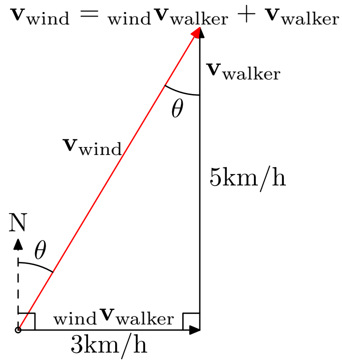
$$|\mathbf{v}_{\text{wind}}|^2 = 25^2 + 15^2 - 2 \times 25 \times 15 \cos 45^\circ$$
$$|\mathbf{v}_{\text{wind}}| = 17.88\text{km/h}$$

By the sine rule (unambiguous here),

$$\frac{\sin(135^\circ - \theta)}{15} = \frac{\sin 45^\circ}{17.88}$$
$$135^\circ - \theta = \sin^{-1}\frac{15 \sin 45^\circ}{17.88}$$
$$= 36.39^\circ$$
$$\theta = 135 - 36.39$$
$$= 98.61^\circ$$

$\mathbf{v}_{\text{wind}} = 17.9$km/h on a bearing of 099°. However, it is usual to give the direction a wind blows *from* so:
$\mathbf{v}_{\text{wind}} = 17.9$km/h blowing from 279°.

23. $\mathbf{v}_{\text{wind}} = {}_{\text{wind}}\mathbf{v}_{\text{walker}} + \mathbf{v}_{\text{walker}}$



By Pythagoras' Theorem,

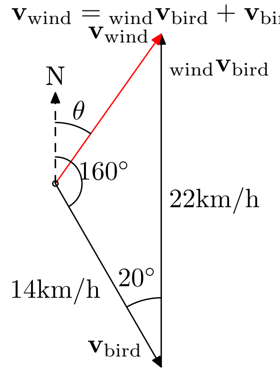
$$|\mathbf{v}_{\text{wind}}|^2 = 5^2 + 3^2$$
$$|\mathbf{v}_{\text{wind}}| = \sqrt{34}\text{km/h}$$
$$\approx 5.83\text{km/h}$$

$$\tan\theta = \frac{3}{5}$$
$$\theta = 30.96^\circ$$

$\mathbf{v}_{\text{wind}} = 5.8$km/h on a bearing of 031°. However, it is usual to give the direction a wind blows *from* so:
$\mathbf{v}_{\text{wind}} = 5.8$km/h blowing from 211°.

24.
- B appears stationary, so $\mathbf{v}_B = \mathbf{v}_A$ and B is travelling due north at 10km/h.
- C appears to be moving south at 3km/h. This means that C is actually moving north 3km/h slower than A. C is travelling due north at 7km/h.
- D appears to be moving north at 5km/h. It is actually travelling north 5km/h faster than A. D is travelling due north at 15km/h.

25. $\mathbf{v}_{\text{wind}} = {}_{\text{wind}}\mathbf{v}_{\text{bird}} + \mathbf{v}_{\text{bird}}$



By the cosine rule,

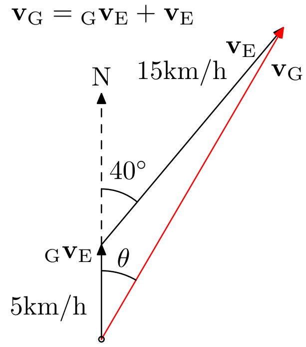
$$|\mathbf{v}_{\text{wind}}|^2 = 14^2 + 22^2 - 2 \times 14 \times 22 \cos 20^\circ$$
$$|\mathbf{v}_{\text{wind}}| = 10.06\text{km/h}$$

Without drawing a scale diagram, we don't know whether the angle $160^\circ - \theta$ is acute or obtuse, so the sine rule is ambiguous here. We'll use the cosine rule instead.

$$160^\circ - \theta = \cos^{-1}\frac{14^2 + 10.06^2 - 22^2}{2 \times 14 \times 10.06}$$
$$= 131.57^\circ$$
$$\theta = 28.43^\circ$$

$\mathbf{v}_{\text{wind}} = 10.1$km/h from 208°.

26.
- Ship F appears to be moving at the same speed as ship E but in the opposite direction ($220^\circ - 20^\circ = 180^\circ$). This implies that ship F is actually stationary.
- $\mathbf{v}_G = {}_G\mathbf{v}_E + \mathbf{v}_E$

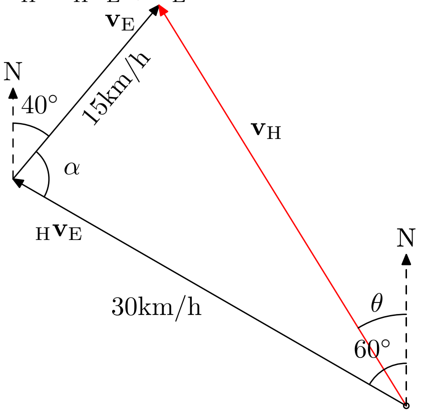


$$|\mathbf{v}_G|^2 = 5^2 + 15^2 - 2 \times 5 \times 15 \cos 140^\circ$$
$$|\mathbf{v}_G| = 19.10\text{km/h}$$

$$\frac{\sin(\theta)}{15} = \frac{\sin 140^\circ}{19.10}$$
$$\theta = \sin^{-1}\frac{15 \sin 140^\circ}{19.10}$$
$$= 30.31^\circ$$

$\mathbf{v}_G = 19.1$km/h on a bearing of 030°.

- $\mathbf{v}_\mathrm{H} = {}_\mathrm{H}\mathbf{v}_\mathrm{E} + \mathbf{v}_\mathrm{E}$



$$40 + \alpha = 180 - 60 \text{ (cointerior angles)}$$
$$\alpha = 80°$$

$$|\mathbf{v}_\mathrm{H}|^2 = 30^2 + 15^2 - 2 \times 30 \times 15 \cos 80°$$
$$|\mathbf{v}_\mathrm{H}| = 31.12\text{km/h}$$

$$\frac{\sin(60° - \theta)}{15} = \frac{\sin 80°}{31.12}$$
$$60 - \theta = \sin^{-1} \frac{15 \sin 80°}{31.12}$$
$$= 28.33°$$
$$\theta = 60 - 28.33$$
$$= 31.67°$$

$\mathbf{v}_\mathrm{H} = 31.1$km/h on a bearing of 328°.

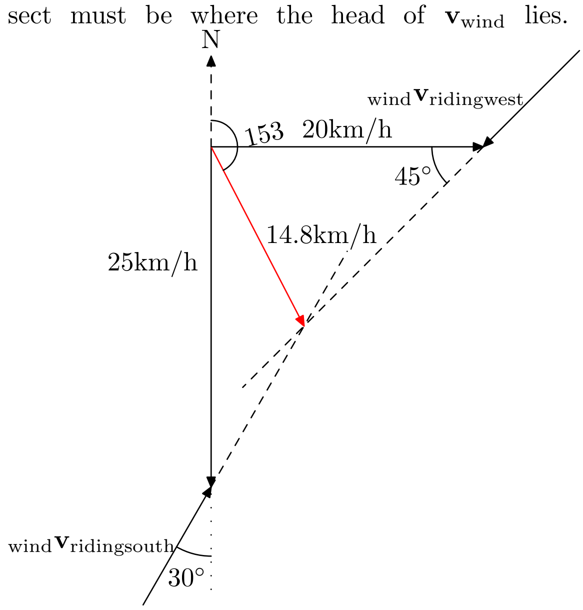
27. ${}_\mathrm{A}\mathbf{v}_\mathrm{B} = 7\mathbf{i} - 10\mathbf{j}$

$${}_\mathrm{A}\mathbf{v}_\mathrm{C} = 13\mathbf{i} - 2\mathbf{j}$$
$${}_\mathrm{B}\mathbf{v}_\mathrm{C} = \mathbf{v}_\mathrm{B} - \mathbf{v}_\mathrm{C}$$
$$= \mathbf{v}_\mathrm{B} - \mathbf{v}_\mathrm{C} + \mathbf{v}_\mathrm{A} - \mathbf{v}_\mathrm{A}$$
$$= \mathbf{v}_\mathrm{B} - \mathbf{v}_\mathrm{A} + \mathbf{v}_\mathrm{A} - \mathbf{v}_\mathrm{C}$$
$$= {}_\mathrm{B}\mathbf{v}_\mathrm{A} + {}_\mathrm{A}\mathbf{v}_\mathrm{C} \text{ (we might have started here)}$$
$$= -(7\mathbf{i} - 10\mathbf{j}) + (13\mathbf{i} - 2\mathbf{j})$$
$$= 6\mathbf{i} + 8\mathbf{j}$$

28. On the same diagram, draw the vectors representing the speed and direction of the cyclist travelling east and travelling south. From the head of each of these vectors draw the line representing the apparent direction of the wind. Since

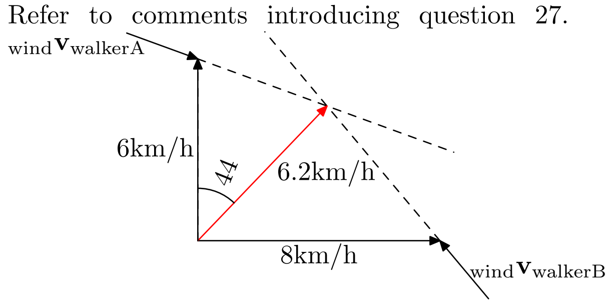
$$\mathbf{v}_\text{wind} = \mathbf{v}_\text{cyclist} + {}_\text{wind}\mathbf{v}_\text{cyclist}$$

with its foot at the origin, the head of $\mathbf{v}_\text{wind}$ must lie along this line. Thus, the point where these two extended lines intersect must be where the head of $\mathbf{v}_\text{wind}$ lies.



The wind velocity is 14.8km/h from 333°.

29. Refer to comments introducing question 27.



The wind velocity is 6.2km/h from 224°.

Miscellaneous Exercise 7

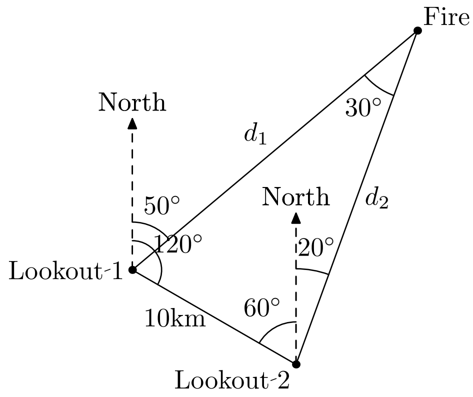
1. (a)
$$\begin{aligned} AC &= \sqrt{(\mathrm{AB})^2 + (\mathrm{BC})^2} \\ &= \sqrt{162^2 + 94^2} \\ &= \sqrt{35080} \\ &= 187.3\text{mm} \end{aligned}$$
   (b)
$$\begin{aligned} AE &= \sqrt{(\mathrm{AF})^2 + (\mathrm{EF})^2} \\ &= \sqrt{\left(\frac{\mathrm{AC}}{2}\right)^2 + (\mathrm{EF})^2} \\ &= \sqrt{\frac{35080}{4} + 118^2} \\ &= \sqrt{22694} \\ &= 150.6\text{mm} \end{aligned}$$
   (c) Let θ be the angle face ABE makes with the base ABCD.
   Let point G be the midpoint of AB.
$$\begin{aligned} \tan\theta &= \frac{\mathrm{EF}}{\mathrm{FG}} \\ \angle\mathrm{EBF} &= \tan^{-1}\frac{118}{94/2} \\ &= 68.3^\circ \end{aligned}$$

2. (a) Area$=\frac{1}{2}r^2\theta = .5 \times 5^2 \times 2 = 25\text{cm}^2$
   (b) Area$=\frac{1}{2}r^2(\theta - \sin\theta) = .5\times 8^2(1.8 - \sin 1.8) = 26.4\text{cm}^2$
   (c)
$$\begin{aligned} \sin\frac{\theta}{2} &= \frac{8}{10} \\ \theta &= 2\sin^{-1}0.8 \\ &= 1.85 \end{aligned}$$
   For the major segment we want the reflex angle:
$$\alpha = 2\pi - \theta = 4.43$$
$$\begin{aligned} Area &= \frac{1}{2}r^2(\alpha - \sin\alpha) \\ &= .5 \times 10^2(4.43 - \sin 4.43) \\ &= 269.4\text{cm}^2 \end{aligned}$$

3. (a) For $x < 3$, $|2x - 6| = -(2x - 6)$
$$\begin{aligned} -(2x-6) &\le x \\ -2x + 6 &\le x \\ 6 &\le 3x \\ 2 &\le x \\ x &\ge 2 \end{aligned}$$
   For $x \ge 3$, $|2x - 6| = 2x - 6$
$$\begin{aligned} 2x - 6 &\le x \\ x - 6 &\le 0 \\ x &\le 6 \end{aligned}$$
   Solution: $2 \le x \le 6$
   (b) For $x < 0$, $|x| = -x$
$$\begin{aligned} 3 - 2x &> -x \\ 3 &> x \\ x &< 3 \end{aligned}$$
   So the inequality is satisfied for all $x < 0$. For $x \ge 0$, $|x| = x$
$$\begin{aligned} 3 - 2x &> x \\ 3 &> 3x \\ x &< 1 \end{aligned}$$
   Solution: $x < 1$
   (c) For $x < -3$, $|x + 3| = -(x + 3)$
$$\begin{aligned} -(x+3) &< x + 1 \\ -x - 3 &< x + 1 \\ -4 &< 2x \\ -2 &< x \\ x &> -2 \end{aligned}$$
   No solution for $x < -3$. For $x \ge -3$, $|x + 3| = x + 3$
$$\begin{aligned} x + 3 &< x + 1 \\ 3 &< 1 \end{aligned}$$
   No solution for $x \ge -3$.
   No value of x satisfies the inequality.
   (d) For $x > -1$ this problem is essentially the same as the previous one which has no solution, so we only need to consider $x \le -1$ and the problem becomes
$$|x + 3| \le -(x + 1)$$
   For $x < -3$, $|x + 3| = -(x + 3)$
$$\begin{aligned} -(x+3) &\le -(x+1) \\ -x - 3 &\le -x - 1 \\ -3 &\le -1 \end{aligned}$$
   The inequality is satisfied for all $x < -3$. For $-3 \le x \le -1$, $|x + 3| = x + 3$
$$\begin{aligned} x + 3 &\le -(x+1) \\ x + 3 &\le -x - 1 \\ 2x &\le -4 \\ x &\le -2 \end{aligned}$$
   Solution: $x \le -2$

4. (a) $d = r\theta = \frac{2\pi}{5} \times 1.8 = 2.26\text{m}$
   (b) $d = r\theta = \frac{2\pi}{5} \times 1 = 1.26\text{m}$

5.



$$\frac{d_1}{\sin 80^\circ} = \frac{10}{\sin 30^\circ}$$
$$d_1 = \frac{10\sin 80^\circ}{\sin 30^\circ}$$
$$= 19.7\text{km}$$

$$\frac{d_2}{\sin 70^\circ} = \frac{10}{\sin 30^\circ}$$
$$d_2 = \frac{10\sin 70^\circ}{\sin 30^\circ}$$
$$= 18.8\text{km}$$

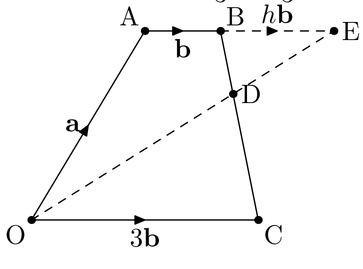
6.
$$\mathbf{a} = k\mathbf{b}$$
$$2\mathbf{i} + 2\mathbf{j} = k(x\mathbf{i} + 5\mathbf{j})$$
$$x = 5$$

$$|\mathbf{b}| = |\mathbf{c}|$$
$$|5\mathbf{i} + 5\mathbf{j}| = |7\mathbf{i} + y\mathbf{j}|$$
$$\sqrt{5^2 + 5^2} = \sqrt{7^2 + y^2}$$
$$50 = 49 + y^2$$
$$y = 1$$

7. (a) $\overrightarrow{BC} = -\mathbf{b} - \mathbf{a} + 3\mathbf{b} = 2\mathbf{b} - \mathbf{a}$

(b) $\overrightarrow{BD} = \frac{1}{3}\overrightarrow{BC} = \frac{2}{3}\mathbf{b} - \frac{1}{3}\mathbf{a}$

(c)
$$\overrightarrow{OD} = \mathbf{a} + \mathbf{b} + \overrightarrow{BD}$$
$$= \mathbf{a} + \mathbf{b} + \frac{2}{3}\mathbf{b} - \frac{1}{3}\mathbf{a}$$
$$= \frac{2}{3}\mathbf{a} + \frac{5}{3}\mathbf{b}$$



$$\overrightarrow{OE} = \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BD}$$
$$= \mathbf{a} + \mathbf{b} + h\mathbf{b}$$
$$= \mathbf{a} + (h+1)\mathbf{b}$$
$$\overrightarrow{OE} = \overrightarrow{OD} + \overrightarrow{OE}$$
$$= (k+1)\overrightarrow{OD}$$
$$= (k+1)(\frac{2}{3}\mathbf{a} + \frac{5}{3}\mathbf{b})$$
$$\therefore \quad \mathbf{a} + (h+1)\mathbf{b} = (k+1)(\frac{2}{3}\mathbf{a} + \frac{5}{3}\mathbf{b})$$
$$\mathbf{a} - \frac{2(k+1)}{3}\mathbf{a} = \frac{5(k+1)}{3}\mathbf{b} - (h+1)\mathbf{b}$$
$$\left(1 - \frac{2k}{3} - \frac{2}{3}\right)\mathbf{a} = \left(\frac{5k}{3} + \frac{5}{3} - h - 1\right)\mathbf{b}$$

LHS:
$$\frac{1}{3} - \frac{2k}{3} = 0$$
$$k = \frac{1}{2}$$

RHS:
$$\frac{5k}{3} + \frac{2}{3} - h = 0$$
$$\frac{5}{6} + \frac{2}{3} - h = 0$$
$$\frac{9}{6} - h = 0$$
$$h = \frac{3}{2}$$

8. (a) $\log_a x + \log_a y = \log_a xy$ so $p = xy$

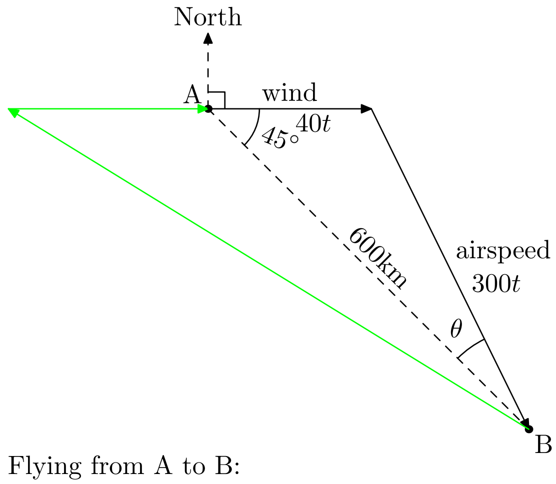
(b) $p = x^y$

(c)
$$3\log_a x - \log_a y = \log_a p$$
$$\log_a x^3 - \log_a y = \log_a p$$
$$\log_a \frac{x^3}{y} = \log_a p$$
$$p = \frac{x^3}{y}$$

(d)
$$2 + .5\log_{10} y = \log_{10} p$$
$$\log_{10} 10^2 + .5\log_{10} y = \log_{10} p$$
$$\log_{10} 100 + \log_{10} y^{0.5} = \log_{10} p$$
$$\log_{10} 100 + \log_{10} \sqrt{y} = \log_{10} p$$
$$\log_{10}(100\sqrt{y}) = \log_{10} p$$
$$p = 100\sqrt{y}$$

9. Let point P be the position of the object after 2 seconds.

$$\begin{aligned}
{}_{\mathrm{P}}\mathbf{r}_{\mathrm{B}} &= 2(3\mathbf{i} - 2\mathbf{j}) \\
&= 6\mathbf{i} - 4\mathbf{j} \\
{}_{\mathrm{P}}\mathbf{r}_{\mathrm{A}} &= {}_{\mathrm{P}}\mathbf{r}_{\mathrm{B}} + {}_{\mathrm{B}}\mathbf{r}_{\mathrm{A}} \\
&= 6\mathbf{i} - 4\mathbf{j} + 8\mathbf{i} + 3\mathbf{j} \\
&= 14\mathbf{i} - \mathbf{j} \\
\mathbf{r}_{\mathrm{P}} &= {}_{\mathrm{P}}\mathbf{r}_{\mathrm{A}} + \mathbf{r}_{\mathrm{A}} \\
&= 14\mathbf{i} - \mathbf{j} + -2\mathbf{i} + 7\mathbf{j} \\
&= 12\mathbf{i} + 6\mathbf{j} \\
|\mathbf{r}_{\mathrm{P}}| &= |12\mathbf{i} + 6\mathbf{j}| \\
&= 6|2\mathbf{i} + \mathbf{j}| \\
&= 6\sqrt{2^2 + 1^2} \\
&= 6\sqrt{5} \text{ m from the origin.}
\end{aligned}$$

10. 

Flying from A to B:

$$\begin{aligned}
(300t)^2 &= (40t)^2 + 600^2 - 2 \times 40t \times 600 \cos 45^\circ \\
90000t^2 &= 1600t^2 + 360000 - 48000t \times \frac{\sqrt{2}}{2} \\
225t^2 &= 4t^2 + 900 - 120t \times \frac{\sqrt{2}}{2} \\
0 &= 221t^2 + 60\sqrt{2}t - 900 \\
t &= 1.835\text{hours} \\
t &= 110\text{minutes}
\end{aligned}$$

Flying from B to A:

$$\begin{aligned}
(300t)^2 &= (40t)^2 + 600^2 - 2 \times 40t \times 600 \cos 135^\circ \\
90000t^2 &= 1600t^2 + 360000 + 48000t \times \frac{\sqrt{2}}{2} \\
225t^2 &= 4t^2 + 900 + 120t \times \frac{\sqrt{2}}{2} \\
0 &= 221t^2 - 60\sqrt{2}t - 900 \\
t &= 2.219\text{hours} \\
t &= 133\text{minutes}
\end{aligned}$$

Chapter 8

Exercise 8A

1. Cubic, $y = x^3$ (by observation)

2. Quadratic, $y = x^2 - 1$ (by observation)

3. Note the common product. Relationship is reciprocal $xy = -24$ so $y = -\frac{24}{x}$

4. There is a common ratio of $\frac{1}{2}$ so the relationship is exponential. $y = 128(0.5)^x$

5. There is a common first difference of -5, so the relationship is linear. $y = -5x + 13$

6. First differences are -5, -3, -1, 1, 3,5, 7, 9. Common second difference of 2, so the relationship is quadratic: $y = x^2 + 2x - 3$

7. Looks like it might be a common ratio. Checking gives a common ratio of 1.5; the relationship is exponential. $y = 1280(1.5)^x$

8. Common first difference of 2.5 $\implies$ linear. $y = 2.5x + 1.5$

9. First differences are -12,-10,-8,-6,-4,-2,0,2. Common second difference of 2 so it's a quadratic with $a = 1$. When $x = 0$, $y = 1$ so $c = 1$. $a + b = -4$ so $b = -5$ giving $y = x^2 - 5x + 1$

10. Looks like it could be a cubic. Construct a table of differences.

x	-4	-3	-2	-1	0	1	2	3	4
y	-69	-32	-13	-6	-5	-4	3	22	59
	37	19	7	1	1	7	19	37	
	-18	-12	-6	0	6	12	18		
	6	6	6	6	6	6			

The common third difference confirms that this is a cubic. From here you could use your calculator to find the cubic of best fit, or proceed as follows.

For the general cubic $y = ax^3 + bx^2 + cx + d$. Constructing a difference table for this gives:

x	0	1	2	3
y	d	$a+b+c+d$	$8a+4b+2c+d$	$27a+9b+3c+d$
	$a+b+c$	$7a+3b+c$	$19a+5b+c$	
	$6a+2b$	$12a+2b$		
	$6a$			

Compare this with the corresponding part of the difference table above and the following results appear:

$$6a = 6 \qquad a = 1$$
$$6a + 2b = 6 \qquad b = 0$$
$$a + b + c = 1 \qquad c = 0$$
$$d = -5$$

giving $y = x^3 - 5$.
You might have spotted this by observation without going through all this effort, but this is a generic approach that will work for any cubic.

11. This is the absolute value function $y = |x|$ and is none of linear, quadratic, cubic, exponential nor reciprocal.

12. This is is none of linear, quadratic, cubic, exponential nor reciprocal. (It is actually a log function, $y = \log_2 x$)

13. It's a linear pattern, increasing by 6 each time. The n^{th} term is $6n - 1$

14. First differences are 4, 6, 8, 10,12,14. It's a quadratic pattern where the n^{th} term is $n(n + 1)$ or $n^2 + n$.

15. It's an exponential pattern with each term 3 times the previous term. The n^{th} term is 2×3^n.

16. It's an exponential pattern with each term twice the previous term. The n^{th} term is $\frac{3}{2} \times 2^n$.

Exercise 8B

1. (a) One-to-one . . . function
   (b) One-to-many . . . not a function
   (c) Many-to-one . . . function
   (d) Many-to-many . . . not a function
   (e) Many-to-one . . . function
   (f) Many-to-many . . . not a function

2. (a) x maps to unique y . . . function
   (b) x maps to unique y . . . function
   (c) x maps to either zero, one or two values of y . . . not a function
   (d) x maps to either zero, one or two values of y . . . not a function
   (e) x maps to unique y . . . function
   (f) x maps to either one, two or three values of y . . . not a function

3. (a) Range=$\{1\times 2+3,\ 2\times 2+3,\ 3\times 2+3,\ 4\times 2+3\}$=$\{5, 7, 9, 11\}$
   (b) Range=$\{(1+3)\times 2, (2+3)\times 2, (3+3)\times 2, (4+3)\times 2\}$=$\{8, 10, 12, 14\}$
   (c) Range=$\{1\div 1, 2\div 2, 3\div 3, 4\div 4\}$=$\{1\}$
   (d) The function machine maps $x\to x^2$. The range is the set of non-negative real numbers.

4. (a) $f(4)=5\times 4-2=18$
   (b) $f(-1)=5\times -1-2=-7$
   (c) $f(3)=5\times 3-2=13$
   (d) $f(1.2)=5\times 1.2-2=4$
   (e) $f(3)+f(2)=(5\times 3-2)+(5\times 2-2)=21$
   (f) $f(5)=5\times 5-2=23$
   (g) $f(-5)=5\times -5-2=-27$
   (h) $f(a)=5\times a-2=5a-2$
   (i) $f(2a)=5\times 2a-2=10a-2$
   (j) $f(a^2)=5\times a^2-2=5a^2-2$
   (k) $3f(2)=3(5\times 2-2)=3\times 8=24$
   (l) $f(a+b)=5\times(a+b)-2=5a+5b-2$
   (m) $$\begin{aligned} f(p)&=33\\ 5p-2&=33\\ 5p&=35\\ p&=7\end{aligned}$$
   (n) $$\begin{aligned} f(q)&=-12\\ 5q-2&=-12\\ 5q&=-10\\ q&=-2\end{aligned}$$

5. (a) $f(4)=4(4)-7=9$
   (b) $f(0)=4(0)-7=-7$
   (c) $g(3)=3^2-12=-3$
   (d) $g(-3)=(-3)^2-12=-3$
   (e) $h(-5)=(-5)^2-3(-5)+3=43$
   (f) $h(5)=5^2-3(5)+3=13$
   (g) $h(-2)=(-2)^2-3(-2)+3=13$
   (h) $3f(a)=3(4a-7)=12a-21$
   (i) $f(3a)=4(3a)-7=12a-7$
   (j) $3g(a)=3(a^2-12)=3a^2-36$
   (k) $g(3a)=(3a)^2-12=9a^2-12$
   (l) $$\begin{aligned} g(p)&=24\\ p^2-12&=24\\ p^2&=36\\ p&=\pm 6\end{aligned}$$
   (m) $$\begin{aligned} g(q)&=h(q)\\ q^2-12&=q^2-3q+3\\ -12&=-3q+3\\ -15&=-3q\\ q&=5\end{aligned}$$
   (n) $$\begin{aligned} h(r)&=f(r)+28\\ r^2-3r+3&=(4r-7)+28\\ r^2-3r+3&=4r-7+28\\ r^2-3r+3&=4r+21\\ r^2-7r+3&=21\\ r^2-7r-18&=0\\ (r-9)(r+2)&=0\\ r&=9\\ \text{or } r&=-2\end{aligned}$$

6. Add 5 to domain to get range. $\{y\in\mathbb{R}: 5\le y\le 8\}$

7. Subtract 3 from domain to get range. $\{y\in\mathbb{R}: -3\le y\le 0\}$

8. Multiply domain by 3 to get range. $\{y\in\mathbb{R}: -6\le y\le 15\}$

9. Multiply domain by 4 to get range. $\{y\in\mathbb{R}: 20\le y\le 40\}$

10. Multiply domain by 2 and subtract 1 to get range. $\{y\in\mathbb{R}: -1\le y\le 9\}$

11. Here the minimum for the domain maps to the maximum for the range and vice versa, so the range is $\{y\in\mathbb{R}: (1-5)\le y\le (1-0)\}=\{y\in\mathbb{R}: -4\le y\le 1\}$

12. Here the minimum value of the function is zero (when $x=0$) and the maximum is 3^2. The range is $\{y\in\mathbb{R}: 0\le y\le 9\}$

13. Here the minimum value of the function is zero (when $x=-1$) and the maximum is $(3+1)^2$. The range is $\{y\in\mathbb{R}: 0\le y\le 16\}$

14. The minimum value of x^2+1 is 1 (when $x=0$. The maximum for the given domain is 3^2+1. The range is $\{y\in\mathbb{R}: 1\le y\le 10\}$

15. Here the minimum for the domain maps to the maximum for the range and vice versa, so the range is $\{y\in\mathbb{R}: \frac{1}{4}\le y\le 1\}$

16. This function has a minimum value of 1 but it has no maximum. $\{y\in\mathbb{R}: y\ge 1\}$

17. Absolute value has a minimum of 0. $\{y\in\mathbb{R}: 0\le y\le 3\}$

18. Absolute value has a minimum of 0. $\{y\in\mathbb{R}: y\ge 0\}$

19. $|x|$ has a minimum of 0 so $|x|+2$ has a minimum of 2. $\{y\in\mathbb{R}: y\ge 2\}$

20. x^2 has a minimum of 0 so x^2-1 has a minimum of -1. $\{y\in\mathbb{R}: y\ge -1\}$

21. x^2 has a minimum of 0 so x^2+4 has a minimum of 4. $\{y\in\mathbb{R}: y\ge 4\}$

22. This function has no minimum or maximum, but it cannot have a value of zero. $\{y \in \mathbb{R} : y \neq 0\}$

23. It's difficult to visualise this function without graphing it. Use your Classpad to graph it. It should be clear that the output of the function can be any real number except 1. $\{y \in \mathbb{R} : y \neq 1\}$

24. one-to-one

25. one-to-one (since every positive number has its own unique square)

26. many-to-one (since a positive and negative number can map to the same square)

27. many-to-one (as for the previous question)

28. one-to-one (any number can be the square root of at most one other number)

29. The natural domain of the function is $\{x \in \mathbb{R} : x \geq 0\}$ (since the function is not defined for negative x), and it is a one-to-one function (like the previous question).

30. Both x and y can take any real value.
$\{x \in \mathbb{R}\}, \quad \{y \in \mathbb{R}\}$

31. x can take any real value, but y can not be negative.
$\{x \in \mathbb{R}\}, \quad \{y \in \mathbb{R} : y \geq 0\}$

32. Neither x nor y can be negative.
$\{x \in \mathbb{R} : x \geq 0\}, \quad \{y \in \mathbb{R} : y \geq 0\}$

33. x must not be less than 3 (so that $x - 3$ is not negative) and y can not be negative.
$\{x \in \mathbb{R} : x \geq 3\}, \quad \{y \in \mathbb{R} : y \geq 0\}$

34. x must not be less than -3 (so that $x + 3$ is not negative) and y can not be negative.
$\{x \in \mathbb{R} : x \geq -3\}, \quad \{y \in \mathbb{R} : y \geq 0\}$

35. x must not be less than 3 (so that $x - 3$ is not negative) and y can not be less than 5.
$\{x \in \mathbb{R} : x \geq 3\}, \quad \{y \in \mathbb{R} : y \geq 5\}$

36. Neither x nor y can be zero.
$\{x \in \mathbb{R} : x \neq 0\}, \quad \{y \in \mathbb{R} : y \neq 0\}$

37. Neither $x - 1$ nor y can be zero.
$\{x \in \mathbb{R} : x \neq 1\}, \quad \{y \in \mathbb{R} : y \neq 0\}$

38. Neither $x - 3$ nor y can be zero.
$\{x \in \mathbb{R} : x \neq 3\}, \quad \{y \in \mathbb{R} : y \neq 0\}$

39. $x - 3$ must be non-negative and non-zero and y similarly can be neither zero nor a negative number.
$\{x \in \mathbb{R} : x > 3\}, \quad \{y \in \mathbb{R} : y > 0\}$

Exercise 8C

1. (a) $x \to \mathrm{f}(x) \to \mathrm{g\,f}(x)$

$$0 \to 1 \to -1$$
$$1 \to 2 \to 1$$
$$2 \to 3 \to 3$$
$$3 \to 4 \to 5$$
$$4 \to 5 \to 7$$

Range is {-1, 1, 3, 5, 7}.

(b) $x \to \mathrm{g}(x) \to \mathrm{f\,g}(x)$

$$0 \to -3 \to -2$$
$$1 \to -1 \to 0$$
$$2 \to 1 \to 2$$
$$3 \to 3 \to 4$$
$$4 \to 5 \to 6$$

Range is {-2, 0, 2, 4, 6}.

(c) $x \to \mathrm{g}(x) \to \mathrm{g\,g}(x)$

$$0 \to -3 \to -9$$
$$1 \to -1 \to -5$$
$$2 \to 1 \to -1$$
$$3 \to 3 \to 3$$
$$4 \to 5 \to 7$$

Range is {-9, -5, -1, 3, 7}.

2. (a) $x \to \mathrm{f}(x) \to \mathrm{g\,f}(x)$

$$1 \to 4 \to 9$$
$$2 \to 5 \to 16$$
$$3 \to 6 \to 25$$

Range is {9, 16, 25}.

(b) $x \to \mathrm{h}(x) \to \mathrm{g\,h}(x) \to \mathrm{f\,g\,h}(x)$

$$1 \to 1 \to 0 \to 3$$
$$2 \to 8 \to 49 \to 52$$
$$3 \to 27 \to 676 \to 679$$

Range is {3, 52, 679}.

(c) $x \to \mathrm{f}(x) \to \mathrm{g\,f}(x) \to \mathrm{h\,g\,f}(x)$

$$1 \to 4 \to 9 \to 729$$
$$2 \to 5 \to 16 \to 4096$$
$$3 \to 6 \to 25 \to 15\,625$$

Range is {729, 4096, 15 625}.

3. (a) Domain: $\{x \in \mathbb{R}\}$; Range: $\{y \in \mathbb{R}\}$

(b) Domain: $\{x \in \mathbb{R}\}$; Range: $\{y \in \mathbb{R}\}$

(c) $\mathrm{f}(x) + \mathrm{g}(x) = (x + 5) + (x - 5) = 2x$
Domain: $\{x \in \mathbb{R}\}$; Range: $\{y \in \mathbb{R}\}$

(d) $\mathrm{f}(x) - \mathrm{g}(x) = (x + 5) - (x - 5) = 10$
Domain: $\{x \in \mathbb{R}\}$; Range: $\{10\}$

(e) $\mathrm{f}(x) \cdot \mathrm{g}(x) = (x+5)(x-5)$
$= x^2 - 25$
Domain: $\{x \in \mathbb{R}\}$; Range: $\{y \in \mathbb{R} : y \geq -25\}$

(f) $\dfrac{\mathrm{f}(x)}{\mathrm{g}(x)} = \dfrac{x+5}{x-5}$
$= \dfrac{(x-5)+10}{x-5}$
$= 1 + \dfrac{10}{x-5}$
Domain: $\{x \in \mathbb{R} : x \neq 5\}$; Range: $\{y \in \mathbb{R} : y \neq 1\}$

4. (a) $\frac{2}{3x+2} = \frac{2}{\mathrm{f}(x)} = \mathrm{g}\,\mathrm{f}(x)$
(b) $\sqrt{3x+2} = \sqrt{\mathrm{f}(x)} = \mathrm{h}\,\mathrm{f}(x)$
(c) $\frac{6}{x} + 2 = 3 \times \frac{2}{x} + 2 = 3\mathrm{g}(x) + 2 = \mathrm{f}\,\mathrm{g}(x)$
(d) $3\sqrt{x} + 2 = 3\mathrm{h}(x) + 2 = \mathrm{f}\,\mathrm{h}(x)$
(e) $\frac{2}{\sqrt{x}} = \frac{2}{\mathrm{h}(x)} = \mathrm{g}\,\mathrm{h}(x)$
(f) $\sqrt{\frac{2}{x}} = \sqrt{\mathrm{g}(x)} = \mathrm{h}\,\mathrm{g}(x)$
(g) $9x+8 = (9x+6)+2 = 3(3x+2)+2 = \mathrm{f}\,\mathrm{f}(x)$
(h) $x^{0.25} = \sqrt{\sqrt{x}} = \mathrm{h}\,\mathrm{h}(x)$
(i) $27x + 26 = 3(9x+8) + 2 = \mathrm{f}\,\mathrm{f}\,\mathrm{f}(x)$

5. (a) $\mathrm{f} \circ \mathrm{f}(x) = 2(\mathrm{f}(x)) - 3$
$= 2(2x-3) - 3$
$= 4x - 6 - 3$
$= 4x - 9$

(b) $\mathrm{g} \circ \mathrm{g}(x) = 4(g(x)) + 1$
$= 4(4x+1) + 1$
$= 16x + 4 + 1$
$= 16x + 5$

(c) $\mathrm{h} \circ \mathrm{h}(x) = (\mathrm{h}(x))^2 + 1$
$= (x^2+1)^2 + 1$
$= x^4 + 2x^2 + 1 + 1$
$= x^4 + 2x^2 + 2$

(d) $\mathrm{f} \circ \mathrm{g}(x) = 2(g(x)) - 3$
$= 2(4x+1) - 3$
$= 8x + 2 - 3$
$= 8x - 1$

(e) $\mathrm{g} \circ \mathrm{f}(x) = 4(\mathrm{f}(x)) + 1$
$= 4(2x-3) + 1$
$= 8x - 12 + 1$
$= 8x - 11$

(f) $\mathrm{f} \circ \mathrm{h}(x) = 2(\mathrm{h}(x)) - 3$
$= 2(x^2+1) - 3$
$= 2x^2 + 2 - 3$
$= 2x^2 - 1$

(g) $\mathrm{h} \circ \mathrm{f}(x) = (\mathrm{f}(x))^2 + 1$
$= (2x-3)^2 + 1$
$= 4x^2 - 12x + 9 + 1$
$= 4x^2 - 12x + 10$

(h) $\mathrm{g} \circ \mathrm{h}(x) = 4(\mathrm{h}(x)) + 1$
$= 4(x^2+1) + 1$
$= 4x^2 + 4 + 1$
$= 4x^2 + 5$

(i) $\mathrm{h} \circ \mathrm{g}(x) = (\mathrm{g}(x))^2 + 1$
$= (4x+1)^2 + 1$
$= 16x^2 + 8x + 1 + 1$
$= 16x^2 + 8x + 2$

6. (a) $\mathrm{f} \circ \mathrm{f}(x) = 2(\mathrm{f}(x)) + 5$
$= 2(2x+5) + 5$
$= 4x + 10 + 5$
$= 4x + 15$

(b) $\mathrm{g} \circ \mathrm{g}(x) = 3(g(x)) + 1$
$= 3(3x+1) + 1$
$= 9x + 3 + 1$
$= 9x + 4$

(c) $\mathrm{h} \circ \mathrm{h}(x) = 1 + \dfrac{2}{\mathrm{h}(x)}$
$= 1 + \dfrac{2}{1 + \frac{2}{x}}$
$= 1 + \dfrac{2}{\frac{x+2}{x}}$
$= 1 + \dfrac{2x}{x+2}$

(d) $\mathrm{f} \circ \mathrm{g}(x) = 2(g(x)) + 5$
$= 2(3x+1) + 5$
$= 6x + 2 + 5$
$= 6x + 7$

(e) $\mathrm{g} \circ \mathrm{f}(x) = 3(\mathrm{f}(x)) + 1$
$= 3(2x+5) + 1$
$= 6x + 15 + 1$
$= 6x + 16$

(f) $\mathrm{f} \circ \mathrm{h}(x) = 2(\mathrm{h}(x)) + 5$
$= 2(1 + \dfrac{2}{x}) + 5$
$= 2 + \dfrac{4}{x} + 5$
$= \dfrac{4}{x} + 7$

(g) $\mathrm{h} \circ \mathrm{f}(x) = 1 + \dfrac{2}{\mathrm{f}(x)}$
$= 1 + \dfrac{2}{2x+5}$

(h) $\mathrm{g} \circ \mathrm{h}(x) = 3(\mathrm{h}(x)) + 1$
$= 3(1 + \dfrac{2}{x}) + 1$
$= 3 + \dfrac{6}{x} + 1$
$= \dfrac{6}{x} + 4$

(i) $$\text{h} \circ \text{g}(x) = 1 + \frac{2}{\text{g}(x)}$$
$$= 1 + \frac{2}{3x+1}$$

7. $\text{g}\left[\text{f}(x)\right] = \sqrt{x-4}$
$x - 4 \geq 0$
$x \geq 4$

8. $\text{g}\left[\text{f}(x)\right] = \sqrt{4-x}$
$4 - x \geq 0$
$x \leq 4$

9. $\text{g}\left[\text{f}(x)\right] = \sqrt{4-x^2}$
$4 - x^2 \geq 0$
$x^2 \leq 4$
$-2 \leq x \leq 2$

10. $\text{g}\left[\text{f}(x)\right] = \sqrt{4-|x|}$
$4 - |x| \geq 0$
$|x| \leq 4$
$-4 \leq x \leq 4$

11. $\text{g}\left[\text{f}(x)\right] = \sqrt{(x+3)-5}$
$(x+3) - 5 \geq 0$
$x - 2 \geq 0$
$x \geq 2$

12. $\text{g}\left[\text{f}(x)\right] = \sqrt{(x-6)+3}$
$(x-6) + 3 \geq 0$
$x - 3 \geq 0$
$x \geq 3$

13. (a) $\text{f}(3) = (3)^2 + 3 = 12$

(b) $\text{f}(-3) = (-3)^2 + 3 = 12$

(c) $\text{g}(2) = \frac{1}{2}$

(d) $$\text{fg}(1) = \text{f}(\frac{1}{1})$$
$$= f(1)$$
$$= (1)^2 + 3$$
$$= 4$$

(e) $$\text{gf}(1) = \text{g}\left((1)^2 + 3\right)$$
$$= g(4)$$
$$= \frac{1}{4}$$

(f) $\mathbb{R} \to \boxed{\text{f}(x) = x^2 + 3} \to \{y \in \mathbb{R} : y \geq 3\}$

(g) $\{x \in \mathbb{R} : x \neq 0\} \to \boxed{\text{g}(x) = \frac{1}{x}} \to \{y \in \mathbb{R} : y \neq 0\}$

(h) It may be useful to think of these compound functions as sequential mappings, something like

$$\{x\} \xrightarrow{\text{f}(x)} \{u\} \xrightarrow{\text{g}(u)} \{y\}$$

where $\{x\}$ is the domain of the compound function and $\{y\}$ is its range. The intermediate set $\{u\}$ is the intersection of the range of f and the domain of g. Thus to determine the natural domain we work right to left, then to determine the range we work left to right.

The whole of the range of $\text{f}(x)$ lies within the domain of $\text{g}(x)$ (since the only real number excluded from the domain of $\text{g}(x)$ is 0 and this is outside the range of $\text{f}(x)$) so there is no additional restriction to the domain and the domain of $\text{gf}(x)$ is the same the domain of $\text{f}(x)$: $x \in \mathbb{R}$.

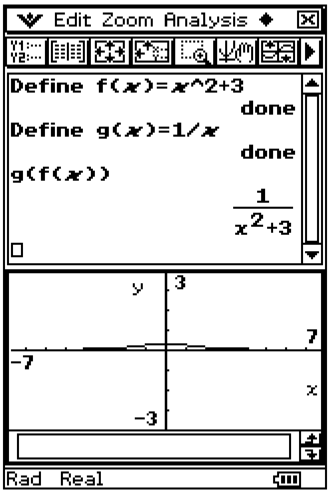
When the domain of $\text{g}(x)$ is restricted to the range of $\text{f}(x)$ (i.e. $x \in \mathbb{R} : x \geq 3$) the range is $y \in \mathbb{R} : 0 < y \leq \frac{1}{3}$.

$\mathbb{R} \to \boxed{\text{gf}(x)} \to \left\{y \in \mathbb{R} : 0 < y \leq \frac{1}{3}\right\}$

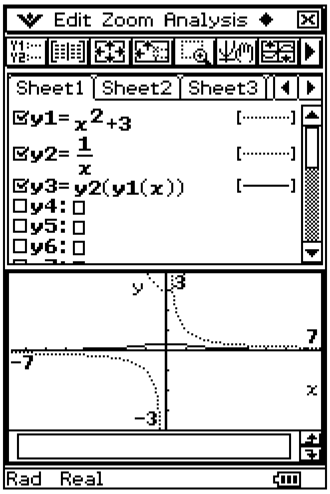
There are two main ways of graphing this on the ClassPad 330.

From the Main app
Interactive->Define
Func name: f
Variable/s: x
Expression: xˆ2+3
OK

Interactive->Define
Func name: g
Variable/s: x
Expression: 1/x
OK
g(f(x))
Tap the graph icon then highlight g(f(x)) and drag and drop it onto the graph.



From the Graph&Tab app
y1=x^2+3
y2=1/x
y3=y2(y1(x))
Tap the graph icon.



Note that to enter y2(y1(x)) you must use y in the abc tab, not the y button on the keyboard or under the VAR tab.

Once you have obtained the graph you can examine it in all the usual ways.

(i) The whole of the range of $g(x)$ lies within the domain of $f(x)$ (since $f(x)$ is defined for all real numbers) so there is no additional restriction to the domain and the domain of $fg(x)$ is the same the domain of $g(x)$: $\{x \in \mathbb{R} : x \neq 0\}$.

When the domain of $f(x)$ is restricted to the range of $g(x)$ (i.e. $x \in \mathbb{R} : x \neq 0$) the range is $y \in \mathbb{R} : y > 3$.

$$\{x \in \mathbb{R} : x \neq 0\} \rightarrow \boxed{fg(x)} \rightarrow \{y \in \mathbb{R} : y > 3\}$$

14. (a) $f(5) = 25 - (5)^2 = 0$

(b) $f(-5) = 25 - (-5)^2 = 0$

(c) $g(4) = \sqrt{4} = 2$

(d)
$$\begin{aligned} fg(4) &= f(\sqrt{4}) \\ &= f(2) \\ &= 25 - (2)^2 \\ &= 21 \end{aligned}$$

(e)
$$\begin{aligned} gf(4) &= g(25 - (4)^2) \\ &= g(9) \\ &= \sqrt{9} \\ &= 3 \end{aligned}$$

(f) $\mathbb{R} \rightarrow \boxed{f(x) = 25 - x^2} \rightarrow \{y \in \mathbb{R} : y \leq 25\}$

(g) $\{x \in \mathbb{R} : x \geq 0\} \rightarrow \boxed{g(x) = \sqrt{x}} \rightarrow \{y \in \mathbb{R} : y \geq 0\}$

(h) $gf(x)$:

$$\{x \in \mathbb{R} : -5 \leq x \leq 5\} \xrightarrow{f(x)=25-x^2} \{u \in \mathbb{R} : 0 \leq u \leq 25\} \xrightarrow{g(x)=\sqrt{x}} \{y \in \mathbb{R} : 0 \leq y \leq 5\}$$

Hence the domain is $\{x \in \mathbb{R} : -5 \leq x \leq 5\}$ and the range is $\{y \in \mathbb{R} : 0 \leq y \leq 5\}$.

(i) $fg(x)$:

$$\{x \in \mathbb{R} : x \geq 0\} \xrightarrow{g(x)=\sqrt{x}} \{u \in \mathbb{R} : u \geq 0\} \xrightarrow{f(x)=25-x^2} \{y \in \mathbb{R} : 0 \leq y \leq 25\}$$

Hence the domain is $\{x \in \mathbb{R} : x \geq 0\}$ and the range is $\{y \in \mathbb{R} : 0 \leq y \leq 25\}$.

15. (a) $g(x)$ is not defined for $x = 3$ so we must exclude $x = 1$ from the domain. $g \circ f(x)$:

$$\{x \in \mathbb{R} : x \neq 1\} \xrightarrow{f(x)=x+2} \{u \in \mathbb{R} : u \neq 3\} \xrightarrow{g(x)=\frac{1}{x-3}} \{y \in \mathbb{R} : y \neq 0\}$$

Hence the domain is $\{x \in \mathbb{R} : x \neq 1\}$ and the range is $\{y \in \mathbb{R} : y \neq 0\}$.

(b) $f \circ g(x)$:

$$\{x \in \mathbb{R} : x \neq 3\} \xrightarrow{g(x)=\frac{1}{x-3}} \{u \in \mathbb{R} : u \neq 0\} \xrightarrow{f(x)=x+2} \{y \in \mathbb{R} : y \neq 2\}$$

Hence the domain is $\{x \in \mathbb{R} : x \neq 3\}$ and the range is $\{y \in \mathbb{R} : y \neq 2\}$.

16. (a) $g \circ f(x)$:

$$\{x \in \mathbb{R} : x \geq 0\} \xrightarrow{f(x)=\sqrt{x}} \{u \in \mathbb{R} : u \geq 0\} \xrightarrow{g(x)=2x-1} \{y \in \mathbb{R} : y \geq -1\}$$

Hence the domain is $\{x \in \mathbb{R} : x \geq 0\}$ and the range is $\{y \in \mathbb{R} : y \geq -1\}$.

(b) $\mathrm{f} \circ \mathrm{g}(x)$:

The input to f must be non-negative so for g:

$$\begin{aligned} 2x - 1 &\geq 0 \\ 2x &\geq 1 \\ x &\geq 0.5 \end{aligned}$$

$$\{x \in \mathbb{R} : x \geq 0.5\} \xrightarrow{\mathrm{g}(x)=2x-1} \{u \in \mathbb{R} : u \geq 0\} \xrightarrow{\mathrm{f}(x)=\sqrt{x}} \{y \in \mathbb{R} : y \geq 0\}$$

Hence the domain is $\{x \in \mathbb{R} : x \geq 0.5\}$ and the range is $\{y \in \mathbb{R} : y \geq 0\}$.

17. (a) $\mathrm{g} \circ \mathrm{f}(x)$:

$$\{x \in \mathbb{R} : x \neq 0\} \xrightarrow{\mathrm{f}(x)=\frac{1}{x^2}} \{u \in \mathbb{R} : u > 0\} \xrightarrow{\mathrm{g}(x)=\sqrt{x}} \{y \in \mathbb{R} : y > 0\}$$

Hence the domain is $\{x \in \mathbb{R} : x \neq 0\}$ and the range is $\{y \in \mathbb{R} : y > 0\}$.

(b) $\mathrm{f} \circ \mathrm{g}(x)$:

$$\{x \in \mathbb{R} : x > 0\} \xrightarrow{\mathrm{g}(x)=\sqrt{x}} \{u \in \mathbb{R} : u > 0\} \xrightarrow{\mathrm{f}(x)=\frac{1}{x^2}} \{y \in \mathbb{R} : y > 0\}$$

Hence the domain is $\{x \in \mathbb{R} : x > 0\}$ and the range is $\{y \in \mathbb{R} : y > 0\}$.

18. The natural domain of $\mathrm{g}(x) = \sqrt{x}$ is $\{x \in \mathbb{R} : x \geq 0\}$ and the corresponding range is $\{y \in \mathbb{R} : y \geq 0\}$. All of this range lies within the natural domain of $\mathrm{f}(x) = x+3$ so $\mathrm{f}(x)$ is defined for every element in the range of $\mathrm{g}(x)$ and hence $\mathrm{f}[\mathrm{g}(x)]$ is a function for all x in the natural domain of $\mathrm{g}(x)$.

    The natural domain of $\mathrm{f}(x) = x + 3$ is $\mathbb{R}$ and the corresponding range is also $\mathbb{R}$. However $\mathrm{g}(x) = \sqrt{x}$ is only defined for $\{x \in \mathbb{R} : x \geq 0\}$ so $\mathrm{g}[\mathrm{f}(x)]$ is undefined for values of x that result in $\mathrm{f}(x) < 0$, i.e. $x < -3$, even though these values lie within the domain of $\mathrm{f}(x)$.

19. The natural domain of $\mathrm{g}(x) = \frac{1}{x-5}$ is $\{x \in \mathbb{R} : x \neq 5\}$ and the corresponding range is $\{y \in \mathbb{R} : y \neq 0\}$. All of this range lies within the natural domain of $\mathrm{f}(x) = x + 3$ so $\mathrm{f}(x)$ is defined for every element in the range of $\mathrm{g}(x)$ and hence $\mathrm{f}[\mathrm{g}(x)]$ is a function for all x in the natural domain of $\mathrm{g}(x)$.

    The natural domain of $\mathrm{f}(x) = x + 3$ is $\mathbb{R}$ and the corresponding range is also $\mathbb{R}$. However $\mathrm{g}(x) = \frac{1}{x-5}$ is only defined for $\{x \in \mathbb{R} : x \neq 5\}$ so $\mathrm{g}[\mathrm{f}(x)]$ is undefined for the value of x that results in $\mathrm{f}(x) = 5$, i.e. $x = 2$, even though this value lies within the domain of $\mathrm{f}(x)$.

20. (a) $\mathrm{g} \circ \mathrm{f}(x)$:

$$\{x\} \xrightarrow{\mathrm{f}(x)=x^2-9} \{u\} \xrightarrow{\mathrm{g}(x)=\frac{1}{x}} \{y\}$$

The natural domain of $\mathrm{g}(x)$ is $\{x \in \mathbb{R} : x \neq 0\}$ so we must exclude 0 from the output of $\mathrm{f}(x)$ and hence limit the domain such that

$$\begin{aligned} x^2 - 9 &\neq 0 \\ (x+3)(x-3) &\neq 0 \\ x &\neq -3 \\ \text{and } x &\neq 3 \end{aligned}$$

The domain of $\mathrm{g} \circ \mathrm{f}(x)$ is

$$\{x \in \mathbb{R} : x \neq -3, x \neq 3\}$$

.

This, then, gives us $\{x \in \mathbb{R} : x \geq -9, x \neq 0\}$ for the domain of $\mathrm{g}(x)$. To determine the range, consider this domain in parts:

$$\{x \in \mathbb{R} : -9 \leq x < 0\} \xrightarrow{\mathrm{g}(x)} \{y \in \mathbb{R} : y \leq -\frac{1}{9}\}$$

and

$$\{x \in \mathbb{R} : x > 0\} \xrightarrow{\mathrm{g}(x)} \{y \in \mathbb{R} : y > 0\}$$

The range of $\mathrm{g} \circ \mathrm{f}(x)$ is

$$\{y \in \mathbb{R} : y \leq -\frac{1}{9}\} \cup \{y \in \mathbb{R} : y > 0\}$$

(b) $\mathrm{f} \circ \mathrm{g}(x)$:

$$\{x\} \xrightarrow{\mathrm{g}(x)=\frac{1}{x}} \{u\} \xrightarrow{\mathrm{f}(x)=x^2-9} \{y\}$$

The natural domain of $\mathrm{f}(x)$ is $\mathbb{R}$ so we need place no additional restriction on the natural domain of $\mathrm{g}(x)$

The domain of $\mathrm{f} \circ \mathrm{g}(x)$ is $\{x \in \mathbb{R} : x \neq 0\}$.

This gives us $\{x \in \mathbb{R} : x \neq 0\}$ for the domain of $\mathrm{f}(x)$. The range then is the natural range of $\mathrm{f}(x)$ excluding $f(0) = -9$, i.e. $\{y \in \mathbb{R} : y \geq -9, y \neq -9\}$. This simplifies to give the range of $\mathrm{f} \circ \mathrm{g}(x)$ as $\{y \in \mathbb{R} : y > -9\}$

Exercise 8D

1. (a) Yes. All linear functions have an inverse on their natural domains except those in the form $f(x) = k$.
   (b) Yes (linear)
   (c) Yes (linear)
   (d) No. Quadratic functions do not have an inverse on their natural domains.
   (e) No (quadratic)
   (f) No (quadratic)
   (g) Yes. All reciprocal functions have an inverse on their natural domains.
   (h) Yes (reciprocal)
   (i) No. Does not pass the horizontal line test because $f(-x) = f(x)$ (so it's not a one-to-one function).
2. $f^{-1}(x) = x + 2$; domain $\mathbb{R}$, range $\mathbb{R}$
3. $f^{-1}(x) = \frac{x+5}{2}$; domain $\mathbb{R}$, range $\mathbb{R}$
4. $f^{-1}(x) = \frac{x-2}{5}$; domain $\mathbb{R}$, range $\mathbb{R}$
5. $$\begin{aligned} y &= \frac{1}{x-4} \\ x - 4 &= \frac{1}{y} \\ x &= \frac{1}{y} + 4 \\ f^{-1}(x) &= \frac{1}{x} + 4 \end{aligned}$$
   Domain $\{x \in \mathbb{R} : x \neq 0\}$, range $\{y \in \mathbb{R} : y \neq 4\}$
6. $f^{-1}(x) = \frac{1}{x} - 3$ (following the pattern of the previous question); domain $\{x \in \mathbb{R} : x \neq 0\}$, range $\{y \in \mathbb{R} : y \neq -3\}$
7. $$\begin{aligned} y &= \frac{1}{2x-5} \\ 2x - 5 &= \frac{1}{y} \\ 2x &= \frac{1}{y} + 5 \\ x &= \frac{\frac{1}{y} + 5}{2} \\ &= \frac{1}{2y} + \frac{5}{2} \\ f^{-1}(x) &= \frac{1}{2x} + \frac{5}{2} \end{aligned}$$
   Domain $\{x \in \mathbb{R} : x \neq 0\}$, range $\{y \in \mathbb{R} : y \neq \frac{5}{2}\}$
8. $$\begin{aligned} y &= 1 + \frac{1}{2+x} \\ y - 1 &= \frac{1}{2+x} \\ 2 + x &= \frac{1}{y-1} \\ x &= \frac{1}{y-1} - 2 \\ f^{-1}(x) &= \frac{1}{x-1} - 2 \end{aligned}$$
   Domain $\{x \in \mathbb{R} : x \neq 1\}$, range $\{y \in \mathbb{R} : y \neq -2\}$
9. $$\begin{aligned} y &= 3 - \frac{1}{x-1} \\ 3 - y &= \frac{1}{x-1} \\ x - 1 &= \frac{1}{3-y} \\ x &= \frac{1}{3-y} + 1 \\ f^{-1}(x) &= \frac{1}{3-x} + 1 \end{aligned}$$
   Domain $\{x \in \mathbb{R} : x \neq 3\}$, range $\{y \in \mathbb{R} : y \neq 1\}$
10. $$\begin{aligned} y &= 4 + \frac{2}{2x-1} \\ y - 4 &= \frac{2}{2x-1} \\ 2x - 1 &= \frac{2}{y-4} \\ 2x &= \frac{2}{y-4} + 1 \\ x &= \frac{1}{y-4} + \frac{1}{2} \\ f^{-1}(x) &= \frac{1}{x-4} + \frac{1}{2} \end{aligned}$$
    Domain $\{x \in \mathbb{R} : x \neq 4\}$, range $\{y \in \mathbb{R} : y \neq \frac{1}{2}\}$
11. $f^{-1}(x) = x^2$; domain $\{x \in \mathbb{R} : x \geq 0\}$ (because that's the range of $f(x)$), range $\{y \in \mathbb{R} : y \geq 0\}$
12. $$\begin{aligned} y &= \sqrt{x+1} \\ y^2 &= x + 1 \\ x &= y^2 - 1 \\ f^{-1}(x) &= x^2 - 1 \end{aligned}$$
    Domain $\{x \in \mathbb{R} : x \geq 0\}$ (range of $f(x)$), range $\{y \in \mathbb{R} : y \geq -1\}$
13. $$\begin{aligned} y &= \sqrt{2x-3} \\ y^2 &= 2x - 3 \\ 2x &= y^2 + 3 \\ x &= \frac{y^2+3}{2} \\ f^{-1}(x) &= \frac{x^2+3}{2} \end{aligned}$$
    Domain $\{x \in \mathbb{R} : x \geq 0\}$ (range of $f(x)$), range $\{y \in \mathbb{R} : y \geq \frac{3}{2}\}$
14. $f^{-1}(x) = \frac{x-5}{2}$
15. $g^{-1}(x) = \frac{x-1}{3}$
16. $$\begin{aligned} y &= 1 + \frac{2}{x} \\ y - 1 &= \frac{2}{x} \\ x &= \frac{2}{y-1} \\ h^{-1}(x) &= \frac{2}{x-1} \end{aligned}$$

17. We expect $\mathrm{f} \circ \mathrm{f}^{-1}(x) = x$ since f and f^{-1} are inverses. Demonstrating that this is true:

$$\begin{aligned}\mathrm{f} \circ \mathrm{f}^{-1}(x) &= f(\frac{x-5}{2}) \\ &= 2(\frac{x-5}{2}) + 5 \\ &= (x-5) + 5 \\ &= x\end{aligned}$$

18. Similarly for $\mathrm{f}^{-1} \circ \mathrm{f}(x)$:

$$\begin{aligned}\mathrm{f}^{-1} \circ \mathrm{f}(x) &= f^{-1}(2x+5) \\ &= \frac{(2x+5)-5}{2} \\ &= \frac{2x}{2} \\ &= x\end{aligned}$$

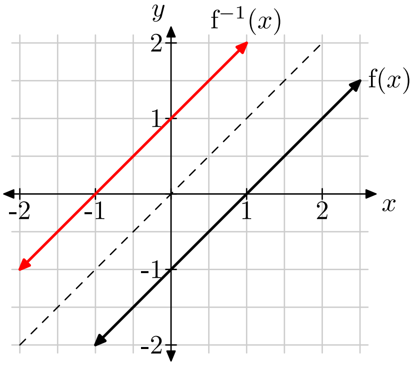
19. $$\begin{aligned}\mathrm{f} \circ \mathrm{h}^{-1}(x) &= f(\frac{2}{x-1}) \\ &= 2(\frac{2}{x-1}) + 5 \\ &= \frac{4}{x-1} + 5\end{aligned}$$

20. $$\begin{aligned}\mathrm{f} \circ \mathrm{g}(x) &= f(3x+1) \\ &= 2(3x+1) + 5 \\ &= 6x + 2 + 5 \\ &= 6x + 7 \\ (\mathrm{f} \circ \mathrm{g})^{-1}(x) &= \frac{x-7}{6}\end{aligned}$$

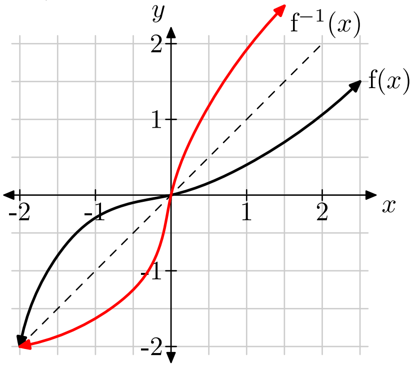
21. $$\begin{aligned}\mathrm{g}^{-1} \circ \mathrm{f}^{-1} &= \mathrm{g}^{-1}(\frac{x-5}{2}) \\ &= \frac{(\frac{x-5}{2}) - 1}{3} \\ &= \frac{(\frac{x-5}{2}) - \frac{2}{2}}{3} \\ &= \frac{\frac{x-7}{2}}{3} \\ &= \frac{x-7}{6}\end{aligned}$$

22. $$\begin{aligned}\mathrm{f} \circ \mathrm{g}(x)^{-1} &= f(\frac{x-1}{3}) \\ &= 2(\frac{x-1}{3}) + 5 \\ &= \frac{2x-2}{3} + 5 \\ &= \frac{2x-2}{3} + \frac{15}{3} \\ &= \frac{2x-2+15}{3} \\ &= \frac{2x+13}{3}\end{aligned}$$

23. (a) Yes, this is a one-to-one function.



(b) Not a function.
(c) Is a function, but not one-to-one.
(d) Not a function.
(e) Not a function.
(f) Yes, this is a one-to-one function.



24. Restricted domain of f(x): $x \geq 0$
$\mathrm{f}^{-1}(x) = \sqrt{x-3}$
Domain and range of $\mathrm{f}^{-1}(x)$:

$$\{x \in \mathbb{R} : x \geq 3\} \quad \{y \in \mathbb{R} : y \geq 0\}$$

Alternatively: restricted domain of f(x): $x \leq 0$
$\mathrm{f}^{-1}(x) = -\sqrt{x-3}$
Domain and range of $\mathrm{f}^{-1}(x)$:

$$\{x \in \mathbb{R} : x \geq 3\} \quad \{y \in \mathbb{R} : y \leq 0\}$$

25. Restricted domain of f(x): $x \geq -3$
$\mathrm{f}^{-1}(x) = \sqrt{x} - 3$
Domain and range of $\mathrm{f}^{-1}(x)$:

$$\{x \in \mathbb{R} : x \geq 0\} \quad \{y \in \mathbb{R} : y \geq -3\}$$

Alternatively: restricted domain of f(x): $x \leq -3$
$\mathrm{f}^{-1}(x) = -\sqrt{x} - 3$
Domain and range of $\mathrm{f}^{-1}(x)$:

$$\{x \in \mathbb{R} : x \geq 0\} \quad \{y \in \mathbb{R} : y \leq -3\}$$

26. Restricted domain of f(x): $x \geq 3$
$\mathrm{f}^{-1}(x) = \sqrt{x-2} + 3$
Domain and range of $\mathrm{f}^{-1}(x)$:

$$\{x \in \mathbb{R} : x \geq 2\} \quad \{y \in \mathbb{R} : y \geq 3\}$$

Alternatively: restricted domain of f(x): $x \leq 3$
$\mathrm{f}^{-1}(x) = -\sqrt{x-2} + 3$
Domain and range of $\mathrm{f}^{-1}(x)$:

$$\{x \in \mathbb{R} : x \geq 2\} \quad \{y \in \mathbb{R} : y \leq 3\}$$

27. Restricted domain of $f(x)$: $0 \le x \le 2$
 $f^{-1}(x) = \sqrt{4 - x^2}$
 Domain and range of $f^{-1}(x)$:

 $\{x \in \mathbb{R} : 0 \le x \le 2\}$ $\{y \in \mathbb{R} : 0 \le y \le 2\}$

 Alternatively:

 restricted domain of $f(x)$: $-2 \le x \le 0$
 $f^{-1}(x) = -\sqrt{4 - x^2}$
 Domain and range of $f^{-1}(x)$:

 $\{x \in \mathbb{R} : 0 \le x \le 2\}$ $\{y \in \mathbb{R} : -2 \le y \le 0\}$

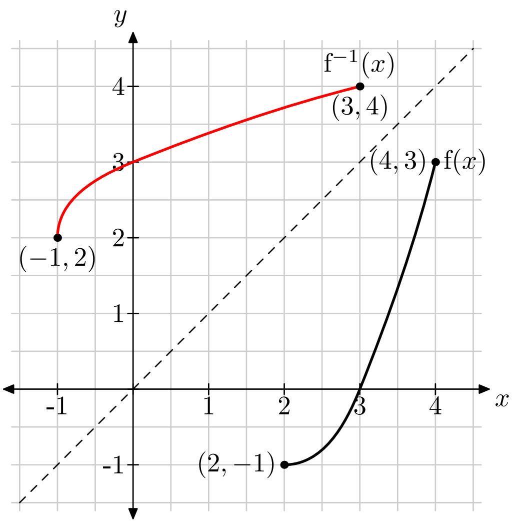
Miscellaneous Exercise 8

1. $C = 2\pi \times 6350 \cos 40^\circ = 30\,564\text{km}$

2. (a) Domain: $\{x \in \mathbb{R} : 2 \le x \le 4\}$
 Range: $\{y \in \mathbb{R} : -1 \le y \le 3\}$

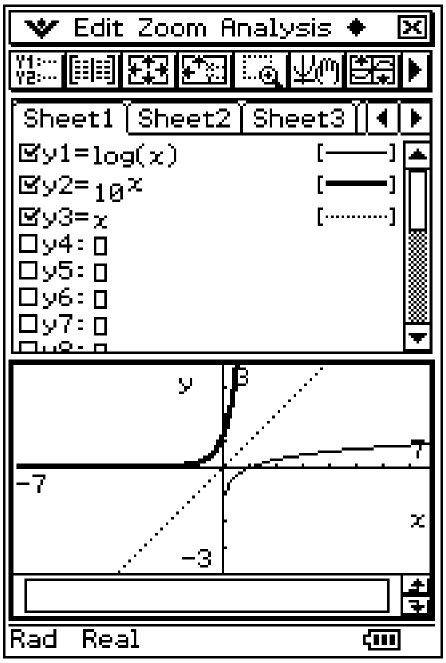
 (b) Domain: $\{x \in \mathbb{R} : -1 \le x \le 3\}$
 Range: $\{y \in \mathbb{R} : 2 \le y \le 4\}$

 (c)

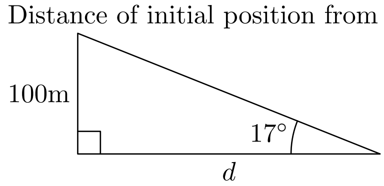


 (d)
$$\begin{aligned} y &= (x-2)^2 - 1 \\ y + 1 &= (x-2)^2 \\ x - 2 &= \sqrt{y+1} \\ x &= \sqrt{y+1} + 2 \\ f^{-1}(x) &= \sqrt{x+1} + 2 \end{aligned}$$

3.
$$\begin{aligned} y &= \log_{10} x \\ 10^y &= x \\ f^{-1}(x) &= 10^x \end{aligned}$$

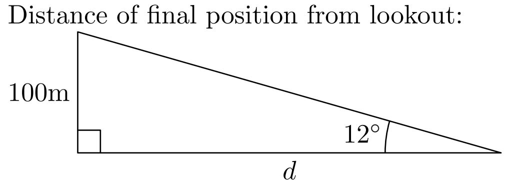


4. (a) Distance of initial position from lookout:



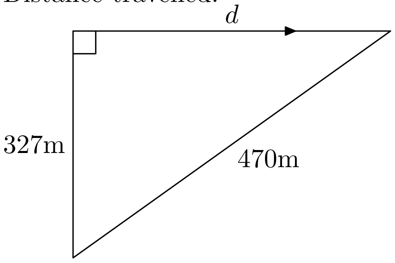
$$\begin{aligned} \tan 17^\circ &= \frac{100}{d} \\ d &= \frac{100}{\tan 17^\circ} \\ &\approx 327\text{m} \end{aligned}$$

 Distance of final position from lookout:



$$\begin{aligned} \tan 12^\circ &= \frac{100}{d} \\ d &= \frac{100}{\tan 12^\circ} \\ &\approx 470\text{m} \end{aligned}$$

Distance travelled:

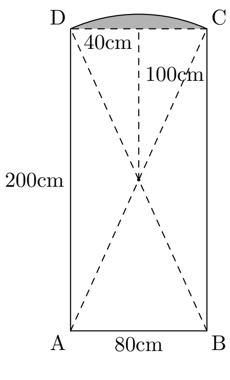


$$d = \sqrt{470^2 - 327^2}$$
$$\approx 338\text{m}$$

(b) $\text{Speed} = \dfrac{338}{10}$
$= 33.8\text{m/min}$
$= 33.8 \times 60$
$= 2029\text{m/hr}$
$= 2.0\text{km/h}$

5. (a) Translate up 5. Maximum t.p. $(-1, 26)$; Minimum t.p. $(3, -6)$.
   (b) Translate down 5. Maximum t.p. $(-1, 16)$; Minimum t.p. $(3, -16)$.
   (c) Reflection in y-axis. Maximum t.p. $(1, 21)$; Minimum t.p. $(-3, -11)$.
   (d) Reflection in x-axis. Minimum t.p. $(-1, -21)$; Maximum t.p. $(3, 11)$.
   (e) y-dilation ($\times 3$ enlargement). Maximum t.p. $(-1, 63)$; Minimum t.p. $(3, -33)$.
   (f) x-dilation ($\div 2$ reduction). Maximum t.p. $(-0.5, 21)$; Minimum t.p. $(1.5, -11)$.

6. Area=area of rectangle + area of segment
   Area of rectangle=$200 \times 80 = 16\,000\text{cm}^2$
   Area of segment: Radius:



$$r = \sqrt{100^2 + 40^2}$$
$$= 107.7\text{cm}$$

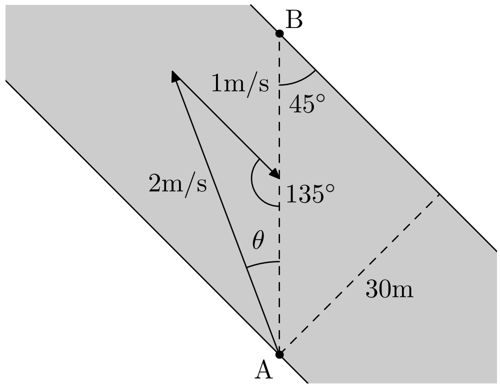
Angle:

$$\tan\frac{\theta}{2} = \frac{40}{100}$$
$$\frac{\theta}{2} = \tan^{-1} 0.4$$
$$\theta = 2\tan^{-1} 0.4$$
$$= 0.761^{\text{R}}$$

$$\text{Area} = \frac{1}{2}r^2(\theta - \sin\theta)$$
$$= \frac{1}{2} \times 107.7^2(0.761 - \sin 0.761)$$
$$= 414\text{cm}^2$$

Total area=$16\,000 + 414 \approx 16\,410\text{cm}^2$

7. (a) $\{y \in \mathbb{R} : y \geq 0\}$
   (b) $\{y \in \mathbb{R} : y \geq 3\}$
   (c) $\{y \in \mathbb{R} : y \geq 0\}$
   (d) $\{y \in \mathbb{R} : y \geq 0\}$
   (e) $\{y \in \mathbb{R} : y \geq 3\}$
   (f) $\{y \in \mathbb{R} : y \geq 0\}$

8. 

$$\frac{\sin\theta}{1} = \frac{\sin 135°}{2}$$
$$\theta = \sin^{-1}\frac{\sin 135°}{2}$$
$$= 20.7°$$
$$360 - 20.7 = 339.3°$$

The person should row on a bearing of 339°.
Distance to travel = $\frac{30}{\sin 45°}$ = 42.43m

Speed:

$$\frac{s}{\sin(180° - 135° - 20.7°)} = \frac{2}{\sin 135°}$$
$$s = \frac{2\sin 44.3°}{\sin 135°}$$
$$= 1.16\text{m/s}$$

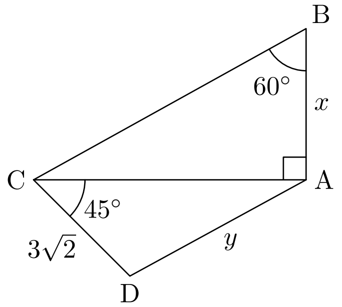
Time=$\frac{42.43}{1.16} = 36.46 \approx 36$s.

9. The critical value of 5 can only be achieved if one of the absolute values changes sign at $x = 5$, implying $a = -5$.

   For $x < 8$ and $x < 5$

$$-(x-8) = -(x-5) + b$$
$$-x + 8 = -x + 5 + b$$
$$8 = 5 + b$$
$$b = 3$$

10.



$$\tan 60^\circ = \frac{\text{AC}}{x}$$
$$\text{AC} = x\tan 60^\circ$$
$$= \sqrt{3}x$$

Using the cosine rule in triangle ACD

$$y^2 = \text{AC}^2 + \text{CD}^2 - 2 \times \text{AC} \times \text{CD}\cos 45^\circ$$
$$= (\sqrt{3}x)^2 + (3\sqrt{2})^2 - 2(\sqrt{3}x)(3\sqrt{2})\left(\frac{\sqrt{2}}{2}\right)$$
$$= 3x^2 + 18 - 6\sqrt{3}x$$
$$= 3(x^2 - 2\sqrt{3}x + 6)$$
$$y = \sqrt{3(x^2 - 2\sqrt{3}x + 6)}$$

Q.E.D.

Chapter 9

Exercise 9A

No working is needed for questions 1–5. Refer to the answers in Sadler.

6. (a) $r = \sqrt{3^2 + 3^2}$
$= 3\sqrt{2}$
θ is in quadrant I.
$\tan\theta = \dfrac{3}{3}$
$\theta = \dfrac{\pi}{4}$
Polar coordinates are $(3\sqrt{2}, \frac{\pi}{4})$.

(b) $r = \sqrt{1^2 + (\sqrt{3})^2}$
$= 2$
θ is in quadrant I.
$\tan\theta = \dfrac{\sqrt{3}}{1}$
$\theta = \dfrac{\pi}{3}$
Polar coordinates are $(2, \frac{\pi}{3})$.

(c) $r = \sqrt{(-2\sqrt{3})^2 + 2^2}$
$= 4$
θ is in quadrant II.
$\tan\theta = \dfrac{2}{-2\sqrt{3}}$
$= -\dfrac{1}{\sqrt{3}}$
$\theta = \dfrac{5\pi}{6}$
Polar coordinates are $(4, \frac{5\pi}{6})$.

(d) $r = \sqrt{(-2\sqrt{3})^2 + (-2)^2}$
$= 4$
θ is in quadrant III.
$\tan\theta = \dfrac{-2}{-2\sqrt{3}}$
$= \dfrac{1}{\sqrt{3}}$
$\theta = \dfrac{7\pi}{6}$
Polar coordinates are $(4, \frac{7\pi}{6})$.

(e) $r = 5$, $\theta = \frac{3\pi}{2}$
Polar coordinates are $(5, \frac{3\pi}{2})$.

(f) $r = \sqrt{7^2 + (-7)^2}$
$= 7\sqrt{2}$
θ is in quadrant IV.
$\tan\theta = \dfrac{-7}{7}$
$= -1$
$\theta = \dfrac{7\pi}{4}$
Polar coordinates are $(7\sqrt{2}, \frac{7\pi}{4})$.

(g) $r = 1$, $\theta = \pi$
Polar coordinates are $(1, \pi)$.

(h) $r = \sqrt{(-5)^2 + (-5\sqrt{3})^2}$
$= 10$
θ is in quadrant III.
$\tan\theta = \dfrac{-5\sqrt{3}}{-5}$
$= \sqrt{3}$
$\theta = \dfrac{4\pi}{3}$
Polar coordinates are $(10, \frac{4\pi}{3})$.

7. (a) $x = 4\cos 30^\circ$
$= 4 \times \dfrac{\sqrt{3}}{2}$
$= 2\sqrt{3}$
$y = 4\sin 30^\circ$
$= 4 \times \dfrac{1}{2}$
$= 2$
Cartesian coordinates are $(2\sqrt{3}, 2)$.

(b) $x = 10\cos 135^\circ$
$= 10 \times -\dfrac{\sqrt{2}}{2}$
$= -5\sqrt{2}$
$y = 10\sin 135^\circ$
$= 10 \times \dfrac{\sqrt{2}}{2}$
$= 5\sqrt{2}$
Cartesian coordinates are $(-5\sqrt{2}, 5\sqrt{2})$.

(c) $x = 3\cos(-90^\circ)$
$= 0$
$y = 3\sin(-90^\circ)$
$= -3$
Cartesian coordinates are $(0, -3)$.

(d) $x = 7\sqrt{2}\cos(-135^\circ)$
$= 7\sqrt{2} \times -\dfrac{\sqrt{2}}{2}$
$= -7$
$y = 7\sqrt{2}\sin(-135^\circ)$
$= 7\sqrt{2} \times -\dfrac{\sqrt{2}}{2}$
$= -7$
Cartesian coordinates are $(-7, -7)$.

(e) $x = 3\cos 40^\circ$
$= 2.30$
$y = 3\sin 40^\circ$
$= 1.93$
Cartesian coordinates are $(2.30, 1.93)$.

(f) $x = 5\cos(-50°)$
$= 3.21$
$y = 5\sin(-50°)$
$= -3.83$
Cartesian coordinates are $(3.21, -3.83)$.

(g) $x = 4\cos 170°$
$= -3.94$
$y = 4\sin 170°$
$= 0.69$
Cartesian coordinates are $(-3.94, 0.69)$.

(h) $x = 10\cos(-100°)$
$= -1.74$
$y = 10\sin(-100°)$
$= -9.85$
Cartesian coordinates are $(-1.74, -9.85)$.

Miscellaneous Exercise 9

1.
$$\begin{aligned} 3^{x-1} &= 5 \\ \log 3^{x-1} &= \log 5 \\ (x-1)\log 3 &= \log 5 \\ x - 1 &= \frac{\log 5}{\log 3} \\ x &= \frac{\log 5}{\log 3} + 1 \end{aligned}$$

2.
$$\begin{aligned} 3^x - 1 &= 5 \\ 3^x &= 6 \\ \log 3^x &= \log 6 \\ x\log 3 &= \log 6 \\ x &= \frac{\log 6}{\log 3} \end{aligned}$$

3. (a)
$$\begin{aligned} \log_x 64 &= 3 \\ x^3 &= 64 \\ x &= \sqrt[3]{64} \\ &= 4 \end{aligned}$$

(b)
$$\begin{aligned} \log_x 64 &= 2 \\ x^2 &= 64 \\ x &= \sqrt{64} \\ &= 8 \end{aligned}$$

(c)
$$\begin{aligned} \log_x 64 &= 6 \\ x^6 &= 64 \\ x &= \sqrt[6]{64} \\ &= 2 \end{aligned}$$

(d)
$$\begin{aligned} \log_{10} 100 &= x \\ x &= 2 \end{aligned}$$

(e)
$$\begin{aligned} \log 17 - \log 2 &= \log x \\ \log \frac{17}{2} &= x \\ x &= \frac{17}{2} \end{aligned}$$

(f)
$$\begin{aligned} \log 17 + \log 2 &= \log x \\ \log(17 \times 2) &= \log x \\ x &= 34 \end{aligned}$$

(g)
$$\begin{aligned} \log\sqrt{2} &= x\log 2 \\ \log 2^{\frac{1}{2}} &= x\log 2 \\ \frac{1}{2}\log 2 &= x\log 2 \\ x &= \frac{1}{2} \end{aligned}$$

(h)
$$\begin{aligned} 3\log 2 &= \log x \\ \log 2^3 &= \log x \\ x &= 8 \end{aligned}$$

4. (a)
$$\begin{aligned} \mathrm{f}(-21) &= 1 - \frac{1}{\sqrt{4-(-21)}} \\ &= 1 - \frac{1}{\sqrt{25}} \\ &= 1 - \frac{1}{5} \\ &= \frac{4}{5} \end{aligned}$$

(b)
$$\begin{aligned} \mathrm{f}\left(\mathrm{f}(3)\right) &= \mathrm{f}\left(1 - \frac{1}{\sqrt{4-(3)}}\right) \\ &= \mathrm{f}\left(1 - \frac{1}{\sqrt{1}}\right) \\ &= \mathrm{f}(1-1) \\ &= \mathrm{f}(0) \\ &= 1 - \frac{1}{\sqrt{4-0}} \\ &= 1 - \frac{1}{\sqrt{4}} \\ &= 1 - \frac{1}{2} \\ &= \frac{1}{2} \end{aligned}$$

(c) Domain: the square root may not be negative so

$$4 - x \geq 0$$
$$x \leq 4$$

In addition, the denominator of the fraction may not be zero so

$$\sqrt{4-x} \neq 0$$
$$4 - x \neq 0$$
$$x \neq 4$$

Combining these we obtain a domain $\{x \in \mathbb{R} : x < 4\}$

(d) Range: the fraction can not be zero, neither can it be negative (since both numerator and denominator are positive), so $y < 1$ and the range is $\{y \in \mathbb{R} : y < 1\}$

(e) Domain and range of $\mathrm{f}^{-1}(x)$ are the range and domain respectively of $\mathrm{f}(x)$. Domain: $\{x \in \mathbb{R} : x < 1\}$; Range: $\{y \in \mathbb{R} : y < 4\}$

$$y = 1 - \frac{1}{\sqrt{4-x}}$$
$$\frac{1}{\sqrt{4-x}} = 1 - y$$
$$\sqrt{4-x} = \frac{1}{1-y}$$
$$4 - x = \left(\frac{1}{1-y}\right)^2$$
$$x = 4 - \left(\frac{1}{1-y}\right)^2$$
$$= 4 - \frac{1}{(1-y)^2}$$
$$\mathrm{f}^{-1}(x) = 4 - \frac{1}{(1-x)^2}$$

5. (a) $\overrightarrow{\mathrm{PQ}} = \overrightarrow{\mathrm{Q}} - \overrightarrow{\mathrm{P}}$

$$= (13\mathbf{i} - 2\mathbf{j}) - (-7\mathbf{i} + 13\mathbf{j})$$
$$= 20\mathbf{i} - 15\mathbf{j}$$

$$\mathrm{PQ} = \sqrt{20^2 + 15^2}$$
$$= 25$$

PR:PQ$= 3 : 5$ so PR$=\frac{3}{5}$PQ$= 15$

RQ=PQ−PR$= 25 - 15 = 10$units

(b) $\overrightarrow{\mathrm{R}} = \overrightarrow{\mathrm{P}} + \frac{3}{5}\overrightarrow{\mathrm{PQ}}$

$$= (-7\mathbf{i} + 13\mathbf{j}) + \frac{3}{5}(20\mathbf{i} - 15\mathbf{j})$$
$$= (-7\mathbf{i} + 13\mathbf{j}) + (12\mathbf{i} - 9\mathbf{j})$$
$$= 5\mathbf{i} + 4\mathbf{j}$$

(c) OR$= \sqrt{5^2 + 4^2} = \sqrt{41} \approx 6.4$units.

6. First solve $|x + 6| = |2x|$:

$x + 6 = 2x$ or $x + 6 = -2x$

$x = 6$ $\qquad$ $3x = -6$

$x = -2$

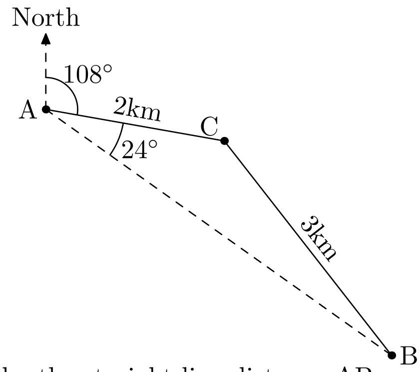
Now test one of the three intervals delimited by these two solutions.

- $x < -2$
  Try a value, say -3:
  Is it true that $|(-3) + 6| \leq |2(-3)|$?
  Yes $(3 \leq 6)$.

Solution set is

$$\{x \in \mathbb{R} : x \leq -2\} \cup \{x \in \mathbb{R} : x \geq 6\}$$

7.



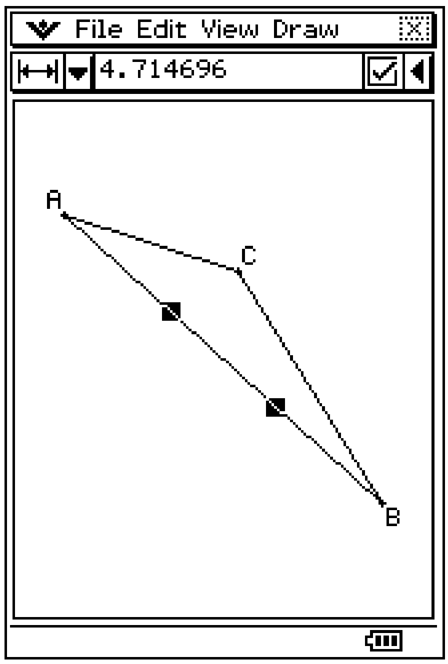
Let x be the straight line distance AB

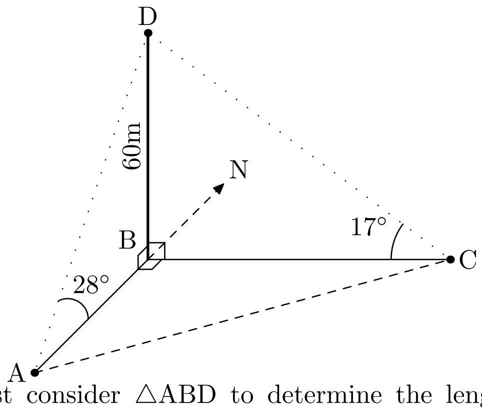
$$3^2 = x^2 + 2^2 - 2x \times 2\cos 24°$$
$$x^2 - (4\cos 24°)x + 4 = 9$$
$$x = 4.715\text{km}$$

(ignoring the negative root.)

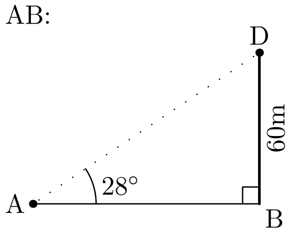
The road route is $2 + 3 - 4.715 = 0.285$km (or about 300m) longer than the straight line distance.

An alternative to solving this algebraically would be to use the geometry app in the ClassPad to construct a scale diagram.



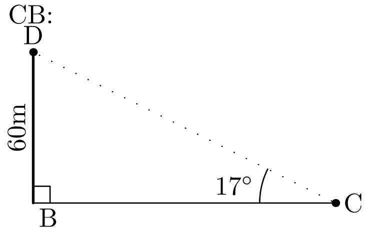
8. 

First consider △ABD to determine the length AB:



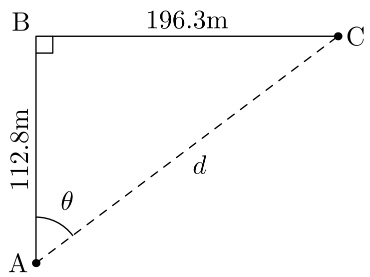
$$\tan 28^\circ = \frac{60}{\text{AB}}$$
$$\text{AB} = \frac{60}{\tan 28^\circ}$$
$$= 112.84\text{m}$$

Next consider △CBD to determine the length CB:



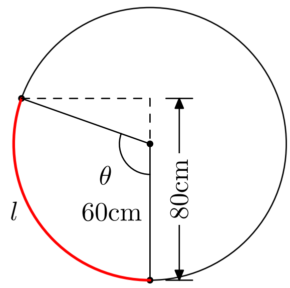
$$\tan 17^\circ = \frac{60}{\text{CB}}$$
$$\text{CB} = \frac{60}{\tan 17^\circ}$$
$$= 196.25\text{m}$$

Finally consider △ABC to determine the length and direction of AC:



$$d = \sqrt{112.84^2 + 196.25^2}$$
$$= 226.38\text{m}$$
$$\tan\theta = \frac{196.25}{112.84}$$
$$\theta = 60.10^\circ$$

C is 226m from A on a bearing of 060°.

9. 

(a) $\cos(\pi - \theta) = \dfrac{80 - 60}{60}$

$$\pi - \theta = \cos^{-1}\frac{20}{60}$$
$$= 1.23$$
$$\theta = \pi - 1.23$$
$$= 1.91$$

(b) $l = r\theta = 60 \times 1.91 \approx 115\text{cm}$

10. $\text{f} \circ \text{g}(x) = \text{f}(2x - 1) = \dfrac{3}{2x - 1}$

$\text{g} \circ \text{f}(x) = \text{g}\left(\dfrac{3}{x}\right) = 2(\dfrac{3}{x}) - 1 = \dfrac{6}{x} - 1$

$\text{f} \circ \text{g}(x)$ has domain determined by $2x - 1 \neq 0$ so the domain is $\{x \in \mathbb{R} : x \neq 0.5\}$
The range of $\text{f} \circ \text{g}(x)$ is $\{y \in \mathbb{R} : y \neq 0\}$.

$\text{g} \circ \text{f}(x)$ has domain $\{x \in \mathbb{R} : x \neq 0\}$.
The range of $\text{g} \circ \text{f}(x)$ is determined by $\frac{6}{x} \neq 0$ so $\frac{6}{x} - 1 \neq -1$ and the range is $\{y \in \mathbb{R} : y \neq -1\}$.

11. Let the original quantity be q. The amount remaining after t years is $q(0.95)^t$.

$$q(0.95)^t = 0.2q$$
$$0.95^t = 0.2$$
$$\log(0.95)^t = \log 0.2$$
$$t \log 0.95 = \log 0.2$$
$$t = \frac{\log 0.2}{\log 0.95}$$
$$= 31.4$$

The company can expect the field to remain profitable for 31 years. It will become unprofitable part-way through the 32nd year.

12. (a) $\log_c 5 = \log_c \dfrac{10}{2}$

$$= \log_c 10 - \log_c 2$$
$$= q - p$$

(b) $\log_c 40 = \log_c(2^2 \times 10)$

$$= 2\log_c 2 + \log_c 10$$
$$= 2p + q$$

(c) $\log_c 200 = \log_c(2 \times 10^2)$

$$= \log_c 2 + 2\log_c 10$$
$$= p + 2q$$

(d) $\log_c(8c) = \log_c(2^3 \times c)$
$= 3\log_c 2 + \log_c c$
$= 3p + 1$

(e)
$$\begin{aligned} 2^{(\log_2 10)} &= 10 \\ log_c 2^{(\log_2 10)} &= \log_c 10 \\ \log_2 10 \log_c 2 &= \log_c 10 \\ \log_2 10 &= \frac{\log_c 10}{\log_c 2} \\ &= \frac{q}{p} \end{aligned}$$

(f)
$$\begin{aligned} 10^{(\log 2)} &= 2 \\ log_c 10^{(\log 2)} &= \log_c 2 \\ \log 2 \log_c 10 &= \log_c 2 \\ \log 2 &= \frac{\log_c 2}{\log_c 10} \\ &= \frac{p}{q} \end{aligned}$$

13. (a) $_{\mathrm{C}}\mathbf{r}_{\mathrm{A}} = {}_{\mathrm{C}}\mathbf{r}_{\mathrm{B}} + {}_{\mathrm{B}}\mathbf{r}_{\mathrm{A}}$
$= (6\mathbf{i} - \mathbf{j}) + (4\mathbf{i} + 5\mathbf{j})$
$= 10\mathbf{i} + 4\mathbf{j}$
$\mathrm{AC} = \sqrt{10^2 + 4^2}$
$= \sqrt{116}$
$= 2\sqrt{29}$

(b) $\mathbf{r}_{\mathrm{C}} = {}_{\mathrm{C}}\mathbf{r}_{\mathrm{A}} + \mathbf{r}_{\mathrm{A}}$
$= (10\mathbf{i} + 4\mathbf{j}) + (-4\mathbf{i} + 6\mathbf{j})$
$= 6\mathbf{i} + 10\mathbf{j}$

(c) $\mathbf{r}_{\mathrm{A}} + 0.5\overrightarrow{\mathrm{AC}} = (-4\mathbf{i} + 6\mathbf{j}) + 0.5(10\mathbf{i} + 4\mathbf{j})$
$= (-4\mathbf{i} + 6\mathbf{j}) + (5\mathbf{i} + 2\mathbf{j})$
$= \mathbf{i} + 8\mathbf{j}$

14. (a) Points P_1 and P_2 lie on the y-axis, so $x = 0$ for both. For P_1, $y = |x - a| = |0 - a| = a$ so the coordinates of P_1 are $(0, a)$.
For P_2, $y = |0.5x - b| = |0.5(0) - b| = b$ so the coordinates of P_2 are $(0, b)$.

(b) Since P_1 is above P_2 we can conclude $a > b$.

(c) For P_4, $|x - a| = 0$ so $x = a$ and the coordinates are $(a, 0)$.
For P_6, $|0.5x - b| = 0$ so $x = 2b$ and the coordinates are $(2b, 0)$.

(d) At P_3,

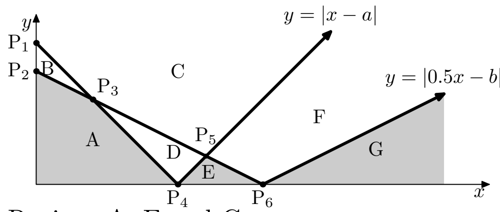
$$\begin{aligned} -(x - a) &= -(0.5x - b) \\ x - a &= 0.5x - b \\ 0.5x &= a - b \\ x &= 2a - 2b \\ y &= -(x - a) \\ &= -(2a - 2b - a) \\ &= -(a - 2b) \\ &= 2b - a \end{aligned}$$

so the coordinates of P_3 are $(2a - 2b, 2b - a)$
At P_5,

$$\begin{aligned} x - a &= -(0.5x - b) \\ x - a &= -0.5x + b \\ 1.5x &= a + b \\ x &= \frac{2a + 2b}{3} \\ y &= x - a \\ &= \frac{2a + 2b}{3} - a \\ &= \frac{2a + 2b - 3a}{3} \\ &= \frac{2b - a}{3} \end{aligned}$$

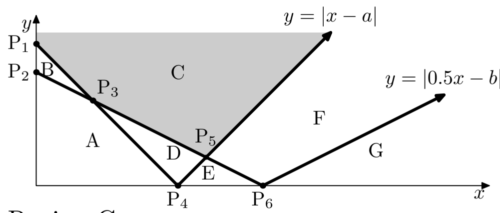
so the coordinates of P_5 are $\left(\frac{2a+2b}{3}, \frac{2b-a}{3}\right)$

(e)



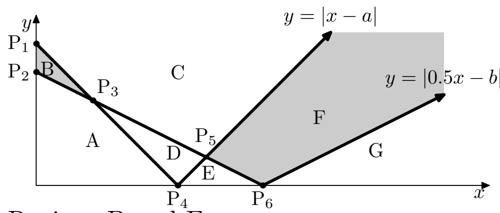
Regions A, E and G.

(f)



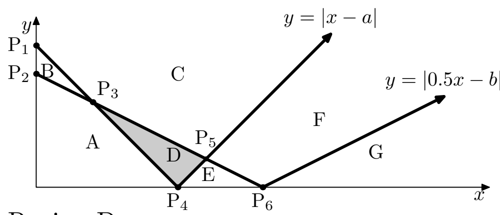
Region C.

(g)



Regions B and F.

(h)



Region D.

15. (a) $\overrightarrow{\mathrm{AC}} = 2\overrightarrow{\mathrm{AB}} = 2\mathbf{b}$

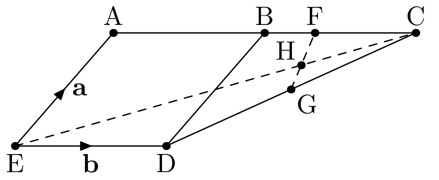
(b) $\overrightarrow{\mathrm{AF}} = \overrightarrow{\mathrm{AB}} + \overrightarrow{\mathrm{BF}} = \mathbf{b} + \frac{1}{3}\mathbf{b} = \frac{4}{3}\mathbf{b}$

(c) $\overrightarrow{\mathrm{DC}} = \overrightarrow{\mathrm{DB}} + \overrightarrow{\mathrm{BC}} = \mathbf{a} + \mathbf{b}$

(d) $\overrightarrow{\mathrm{EC}} = \overrightarrow{\mathrm{ED}} + \overrightarrow{\mathrm{DC}} = \mathbf{b} + \mathbf{a} + \mathbf{b} = \mathbf{a} + 2\mathbf{b}$

(e) $\overrightarrow{\mathrm{EG}} = \overrightarrow{\mathrm{ED}} + \frac{1}{2}\overrightarrow{\mathrm{DC}} = \mathbf{b} + \frac{1}{2}(\mathbf{a} + \mathbf{b}) = \frac{1}{2}\mathbf{a} + \frac{3}{2}\mathbf{b}$

(f) $\overrightarrow{\mathrm{GF}} = \overrightarrow{\mathrm{GC}} + \overrightarrow{\mathrm{CF}} = \frac{1}{2}(\mathbf{a} + \mathbf{b}) + \frac{2}{3}(-\mathbf{b}) = \frac{1}{2}\mathbf{a} - \frac{1}{6}\mathbf{b}$



$$\overrightarrow{GH} = \overrightarrow{GC} + \overrightarrow{CH}$$

$$h\overrightarrow{GF} = \frac{1}{2}\overrightarrow{DC} - k\overrightarrow{EC}$$

$$h\left(\frac{1}{2}\mathbf{a} - \frac{1}{6}\mathbf{b}\right) = \frac{1}{2}(\mathbf{a} + \mathbf{b}) - k(\mathbf{a} + 2\mathbf{b})$$

$$\frac{3h\mathbf{a} - h\mathbf{b}}{6} = \frac{\mathbf{a} + \mathbf{b} - 2k\mathbf{a} - 4k\mathbf{b}}{2}$$

$$3h\mathbf{a} - h\mathbf{b} = 3\mathbf{a} + 3\mathbf{b} - 6k\mathbf{a} - 12k\mathbf{b}$$

$$3h\mathbf{a} - 3\mathbf{a} + 6k\mathbf{a} = 3\mathbf{b} - 12k\mathbf{b} + h\mathbf{b}$$

$$(3h + 6k - 3)\mathbf{a} = (h - 12k + 3)\mathbf{b}$$

$$3h + 6k - 3 = 0$$

$$h + 2k = 1$$

$$h - 12k + 3 = 0$$

$$h - 12k = -3$$

$$14k = 4$$

$$k = \frac{2}{7}$$

$$h + 2\left(\frac{2}{7}\right) = 1$$

$$h = 1 - \frac{4}{7}$$

$$h = \frac{3}{7}$$