3CD Mathematics Specialist WACE Revision

**Matrices**

Questions are taken from VCE Secondary Papers

**2009**

Machine generated alternative text: Question 2
Tickets for the function are sold at the school office, the function hall and online.
Different prices are charged for students, teachers and parents.
Table 1 shows the number of tickets sold at each place and the total value of sales.
Table 1
SchooL Function Online
office hail
Student tickets
283
35
84
Teacher tickets
28
4
3
Parent tickets
5
2
7
Total sales
$8712
$1143
$2609
For this function
• student tickets cost $x
• teacher tickets cost $y
parent tickets cost $:.
a. Use the infomiation in Table 1 to complete the following matrix equation by inserting the missing values
in the shaded boxes.
283 28 5 x 8712
4 y = 1143
84 3 7 z 2609

Machine generated alternative text: b. Use the matrix equation to find the cost of a teacher ticket to the school functioc.

Machine generated alternative text: Question 3
In 2009. the school entered a Rock Eisteddfod coinpetitioa
When rehearsals commenced in February. all students were asked thether they thought the school would make
the state finals. The students responses. ‘yes’. ‘n or ‘undecided are shown in the initial state matrix S0.
160 er
S0= 120 no
220 undecided
a. How many students altend this school?

Machine generated alternative text: iii In total. how many tuðents are not expected to have changed their response at the end of the first
week?Machine generated alternative text: Each week some students aie expected to change their responses. The changes in their responses from one week
to the next are modelled by the transition matrix T shovii below.
re.pon.6 :hi. week
e no undecided
0.85 035 0.60 ye
T= 0.10 040 0.30 no
0.05 025 0.10 undecided
rezponse
next week
The follosng diagram can also be used to display the infonnatioia represented in the transition matrix T.
b. j. Complete the diagram above by writing the missing percentage in the shaded box.
ii. Of the students iio respond ‘yes’ one week. that percentage are expected to respond ‘undecided’
the next week when asked whether they think the school will make the state finals?

Machine generated alternative text: c. Evaluate the product S1 = TS0. ‘1iere S1 is the state matrix at the end of the first week.

Machine generated alternative text: d. How my stu&nts e eected to respond ‘yes’ at the end of the third week dien asked whether they
think the school will make the state finals?

Machine generated alternative text: Question 4
A senes of extra rehearsals cornnnced in Api-il. Each week pafficipants could choose extra dancing rehearsals
or extra singing rehearsals.
A matrix equation used to determine the number of students expected to attend these extra rehearsals is given
10.85 0.251 [5
L,,1 ‘[o.i o7SjL —
where L is the cohmm matrix that lists the number of students attending in week n.
The attendance matrix for the first week of extra rehearsals is given by
dancing
[97j singing
a. Calculate the number of students who are expected to attend the extra singing rehearsals in week 3.

Machine generated alternative text: b. Of the students tho attended extra rehearsals in week 3, how many are not expected to return for any exira
rehearsals in week 4?

**Answers**

2. a. 35 and 2

Machine generated alternative text: $32
283 28 5 8712 27
35 4 2 1143 = 32
84 3 7 2609 35b.

3. a. 500

b. i. 25%

ii. 5%

iii. 206

Machine generated alternative text: 310
S1= 130
‘ 60Machine generated alternative text: 361
3611
T3S0= 9111
47.9.J c. d.

Machine generated alternative text: r51 r085 025]r951 151 11001
L =TxL2 [7j    o.75j[97j L] [so
rg5 0.251r1001 51 rico]
:.L3=TxL,  L3=[015 o3sjLsojL7j=L68j4. a.

Machine generated alternative text: rsl
The subtracted column matnx imhcate that. each week. another 5.7 12 nident will no longer turn up toy
ieheaval.b.

**2008**

Machine generated alternative text: Question 1
Two subjects. Biology and Chemistry. are offered in the first year of a university science course
The inatnxNhsts the number of students enrolled in each subject
_F¾01
j36oj Clieniistiy
The znatrixPlists the proportion of these students expected to be awarded anA. B. C. D orE grade in each
subject.
A B C D E
P= [OE05 OE125 0175 OE45 OE20J
a. Write douni the order of matrix P.

Machine generated alternative text: b. LetthematrixRNP
j. Evaluate the mathx k

Machine generated alternative text: ii. Explain what the matrix element R,4 zptsents.

Machine generated alternative text: c. Students enrolled in Biology have to pay a laboratory fee of Sib, while students enrolled in Cbenústiy
pay a laboratory fee of S95.
j. Wñte doi a clearly labelled row matthc. called F. that lists these fees.

Machine generated alternative text: ü. Show a matrix calculation that will give the total laboratoiy fees. L paid in dollars by the students
enrolled in Biolov and Œemistrv Find this amount.

Machine generated alternative text: Question 2
The following transition matrix. T. is used to help predict class attendance of History students at tfr university
on a lectwe-by-lecture basis
this lecture
attend flot attend
T = [0.90 0.20j attend
0.10 OSOj not attend
S1 is the attendance matrix for the first History lecture.
[540 attend
‘ [36 not attend
S indicates that 540 History students attended the first lecture and 36 History students did not attend the first
lecture.
a. UseTandS1to
j. detemine S, the attendance matrix for the second lecture

Machine generated alternative text: 

Machine generated alternative text: b. Write down a mabix equation for S1, in tenus of T. n and 

Machine generated alternative text: The Hitorv lecture can be trawfened to a ma11ef lecture theae when the nuinbei of students predicted to
attend falls below 400.
c. For ‘..thith lecture can this first be done?

Machine generated alternative text: d. In the 1on tenn how many History students are predicted to attend 1ectuzs?

Machine generated alternative text: Question 3
The bookchop manager at the university has developed a matrix fozmula for determinng the number of
Mathematics and Physics textbooks he should order each yeai
For 2009. the stalling point for the formula is the column matrix S2 This lists the mimber of Mathematics
and Physics textbooks cold in 2008.
1456 Mathematics
s2008 =1
350 Physics
O, is a cohmin matrix listing the number of Mathematics and Physics textbooks to be ordered for 2009
O is given by the matnx formula
°2009 = A S2008 — B where A= [O15  and
a. Determine O,

Machine generated alternative text: The rnatnx fomiula above only allows the manager to predict the number of books he should order one year
ahead Anew matrix fonnula enables him to detennme the nianber of books to be ordered two or maze years
ahead
The new matiix fonnula is
O,,. = CO,, - D
ithere O,, is a column matnx listing the number of Mathematics and Physics textbooks to be ordered for year n.
Here. C = 0.8 0 1 and D = F4°
O 0.8j [38
The ntanber of books ordered in 2008 was given by
F500 Mathematics
02008 = [360 Physics
b. Use the new manix foimula to detennine the number of Mathematics textbooks the bookshop manager
should order in 201OE

Machine generated alternative text: Question 4
By the end of each academic year. students at the university will have either passed. failed or deferred the
year.
Experience has sho’ni that
• 88% of students who pass this year will also pass next year
l0%ofswlentsithopassthisyearwtllfailneflyear
2° à of students who pass this year will defer next year
• 52%’ of students dxo fail this year will pass next year
. 44% of students itho fail this year will fail next year
. 4%ofstudentswhofailthisyearwilldefernextyear
. 65% of students who defer this year will pass next year
. 10 % of students who defer this year will fail next year
• 25% of students who defer this year will defer next yearS
Twelve hundred and this-tv students began a business deree in 2007.
By the end of the 2007 academic year. 880 students had passed. 230 had failed. stle 120 had deferred the year.
No students have dropped out of the business degree penuanently.
Use this information to predict the number of business students ‘ato will defer the 2009 academic year.

**Answers**

1. a. 1 x 5

Machine generated alternative text: r23 57í SOi 207 92
[is 45 63 162 72b. i.

Machine generated alternative text: ii.

Machine generated alternative text: r4601
[110 951L360J = [s4soo]

Machine generated alternative text: BC
[110 95]c. i. ii.

Machine generated alternative text: 421
10.9 0.2]4F5401 E421.46
S =T4S 1o.i o.sj [36 j = 154542. a. i. Machine generated alternative text: r493.2
L 82.8
ro.9 o.21r5401 r493.21
S2 =T S Lo.i o.sj[36j[s2.s! ii.

Machine generated alternative text: S= T  X S1b.

c. Lecture 8

Machine generated alternative text: 3s4
When two different caku1aton Involvwg S = X S, produce the same result, the lone-term state mathx has been
produced High powers for n, such as n = 50 and 51. could be used
50 13840001 51 1384.0001
T S = and T S 1 j both give the ,ame result
L1999J ¡ L191.9J d.

3. a. Machine generated alternative text: r3j
[2j
o  r4561 rig]
Oms !0r5 0.68] [350] [12]

Machine generated alternative text: 248
o 1F°°L 1401
[ o osJ [36oJ [38J [5o
_1°-  1 101 F401_ F248
L o osJ [25oJ Li [162
The added column matiz at each year rs applied after each transition matrix is appheì Consequently, simply squanng
the transition matiz is not appropriate w this question.b.

Machine generated alternative text: 42
018 0.52 0.65 2 880 996.9
S,,,_ 0.10 0.44 0.10 230 = 191.4
0.02 0.G4 0.25 120 41.74.

**2007**

Machine generated alternative text: Question 1
The table below displays the energy content and amounts of fat. carbohydrate and protein contained in a sa-ve
of four foods: bread, margarine, peanut butter and honey
Food
Energy content
(blojoules,sene)
Fat
(gramssene)
Carbohydrate
(graInsserve
Protein
(grams,sen’e)
Bread
531
1.2
20.1
4.2
Margarine
41
6.7
0.4
0.6
Peanut butter
534
10.7
3.5
4.6
Honey
212
0
12.5
0.1
a. Write down a 2 x 3 matrix that displays the fat. carbohydrate and protein content (in columns) of bread
and maiine

Machine generated alternative text: b. A and B are two mainces defined as follows.
A=[2 2 1 1]
L Evaluate the matrix product AB.
531
41
8= 534
212

Machine generated alternative text: n. Deteniiine the ordef of matrix product &4

Machine generated alternative text: Matrix A displays the number of sefvins of the four foods: bread. margarine, peanut butter and honey.
needed to make a peanut butter aixi honey sandwich
Matrix B displays the energy content per sernng of the four foods: bread. margaiine. peanut butter and
honey
iii. Explain the information that the matrix product AB provides.

Machine generated alternative text: c. The number of serves of bread (b). margarine (m), peanut butter (p) and honey (h) that contain. in total.
53 grams offal 101.5 grams of carbohydrate. 28.5 grams of ixotein and 3568 kilojoules of energy can be
determined by solving the matnx equation
1.2 6.7 10.7 0 b 53
20.1 0.4 3.5 12.5 ni 101.5
4.2 0.6 4.6 0.1 p = 28.5
531 41 534 212 h 3568
Solve the matrix equation to find the values b. m.p and h.

Machine generated alternative text: Question 2
To snidy the life-and-death cycle of an insect population. a number of insect eggs (E). juvenile insects (J) and
adult insects (A) are placed in a closed environnent
The initial state of this population can be described by the column niatnx
400 E
200 J
100 A
OD
A row has been included in the state matrix to allow for insects and eggs that die (D).
a. What is the total number of insects in the population (including eggs) at the beginning of the study?

Machine generated alternative text: Ja this population
• eggs may die. or they may live and grow into juveniles
- juveniles may die, or they may live and grow into adults
. adults will live a period of time but they will eventually die.
Ja this populaúoa the adult insects have been steiilised so that no new eggs are produced. In these circumstances.
the life-and-death cycle of the insect can be modelled by the nusition matsit
this week
E JAD
04 0 0 0E
OE5 OE4 0 OJ
next week
o OE5 OES OA
OEl OEl OE2 1 D
b. What proportion of eggs turn into juveniles each week?

Machine generated alternative text: e. j. Evaluate the matrix product S1 = TS0
Si =TS0
ii. Wnte doi the number of live juveniles in the population afier one week.

Machine generated alternative text: iii. Determine the number of live juveni1e in the population after four weeks. Wzite your answer correct
to the nearest whole number

Machine generated alternative text: n After a number of weeks there will be no live eggs (less than one) left in the population.
When does this first occur?

Machine generated alternative text: t Write down the exact steady-state matnx fci this population.
šlare

Machine generated alternative text: d. If t& study is repeated with unstezilised adult insects, eggs will be laid and potentially grow mro adults.
Assuming 30% of adults lay eggs each week. the population iuatnx after one week. S1. is now given by
S1 = TS,+BS0
00 03 0 400E
000 0 200J
±iere B =    and S0 = 100 A
000 0 OD
1. Deteminie S1
E
J
A
D
This pattern continues. The population matrix after n weeks. Si,, is given by
S=TS1+BS1
ii. Deternúne the number of live e.gs in this insect population after two weeks

**Answers**

Machine generated alternative text: 12 20i 4.2
[6.7 04 0.61. a. b. i. [1890]

ii. 4 x 4

Machine generated alternative text: The total enerv content of the servings of these four foods m one sandwichiii.

Machine generated alternative text:   c.

2. a. 700 b. 0.5

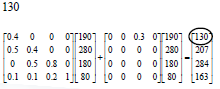
Machine generated alternative text: 0.4 0 0 400 1024
0.5 0.4 0 0 200 56.32
0 0.5 0.8 0 100 312.96
01 01 0.2 1 0 320.48Machine generated alternative text: 160
280
180
80
04 O O O 400 160
Oi 0.4 O O 200 280
O 0.5 0.8 0 100 180
0.1 01 0.2 1 0 80c. i. ii. 280 iii. 56

Machine generated alternative text: 040 0 0400 O
Oi 04 0 0 200 0
o oioso loo O
0.1 0.1 0.2 1 0 700

iv. 7 weeks v.

Machine generated alternative text: 04 O O O4OO D O 03 O 4GO 190
OiGA O OOO DO O 0200 210
O 0.5 0.1 0IO0 D O O O 100 180.
0.1 0.1 0.2 ii 0 D O 0 0 0 10

d. i.

 ii.

**2006**

Machine generated alternative text: Question 2
Anew shopping centre called Shopper Heaven (S) is about to open. It will compete for customers with Eastown
(E) andNoxiand(.
Market research sugests that each shopping centre will have a regular customer base but attract and lose
customers on a weekly basis as follows.
80% of Shopper Heaven customers will return to Shopper Heaven next week
12% of Shopper Heaven customers will shop at Eastown next week
8% of Shopper Heaven customers will shop at Noxiand next week
76% of Eastown customers will return to Eastown next week
9% of Eastown customers will shop at Shopper Heaven next week
15% of Eastown customers will shop at Noxiand next week
85% of Noxiand customers will return to Noxiand next week
I 0Qo of Noxiand customers will shop at Shopper Heaven next week
5% of Noxiand customers will shop at Eastown next week
a. Enter this infonnation into transitiosiniattix Tas indicated below (express percentages as proportions. for
example write 76% as 0.76).
this week
S E N
S
Ï = E nest week
N

Machine generated alternative text: Duiin the ek that Shopper Heaven opened. it had 300000 customers.
In the same ek. Eastoi had 120000 customers and Noxiand had 180000 customers.
b. Write this information in the foini of a column matrix. K0. as indicated below.
s
E
N

Machine generated alternative text: c. Use T and K0 to write and evaluate a matrix product that deteimines the number of cw,toinei-s expected
at each of the shopping centres dining the following week

Machine generated alternative text: d. Show by calculating at least two appropriate state matrices that. in the long terni the number of customers
expected at each centre each week is given by the matrix
194983
K= 150513
254504

Machine generated alternative text: Question 3
Market oesearcheis claim that the ideal number ofbookshops fr). spoils shoe shops (y) and music stores (z) for
a shopping cette can be detenuined by solving the equations
2x+y+z= 12
x—y—:1
2y—:=6
a. Write the equations in matrix fonn using the following template

Machine generated alternative text: b. Do tfr equations have a unique solution? Provide an explanation to justify yow response.

Machine generated alternative text: C. Wzite down an inveme niathx that can be wed to solve these eql1ation.

Machine generated alternative text: d. Solve the equations and hence 1\Tite down the estimated ideal nuniber of bookshops. spoils shoe shops
and music stores for a shoppm centze.

**Answers**

Machine generated alternative text: 300000
LD 120000
180000Machine generated alternative text: 0.80 0.09 0.10
T= 0.12 0.76 0.05
008 0.15 0.85

2. a. b.

Machine generated alternative text: 0.80 0.09 0.10 300000 E26880°
TK: 0.12 0.76 0.05 120000 136200
0.08 0.15 0.85 180000 1195000

c.

Machine generated alternative text: Any two product r K where n 38. For example:
0.80 009 0.10  300000 194983
K1=IK= 0.12 OE76 0.05 120000 = 150513
0.08 0.15 0.85 180000 254504
0.80 0.09 0.10 300000 194983
0.12 0.76 005 120000 = 150513 andthiseameaK3
0.08 0.15 OE85 180000 254504 d.

Machine generated alternative text: 211
Thete r a unique solution znce dut  = 1  O
0 2 —1

Machine generated alternative text: 2 1 lTx] fl2
—1 1 1y—I 1
10 2 —‘Ij [63. a. b.

Machine generated alternative text: 3 bookhop
4 port shoe shops
2 music toreMachine generated alternative text: —1 3 2
1 —2 —1
2 —4 —3c. d.